LECTURE NOTES
ON

CONTROL SYSTEM ENGINEERING

For
6th sem, Electrical Engg. (Diploma)

GOVERNMENT POLYTECHNIC, BARGARH

Prepared by

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SYLLABUS

CONTROL SYSTEM ENGINEERING
(Elective – C)

Name of the Course: Diploma in Electrical Engineering

Course code: EET 604	Semester 6th
Total Period: 60	Examination 3 hrs
Theory periods: 4 P / week	Class Test: 20
Tutorial: 1 P / week	Teacher's Assessment: 10
Maximum marks: 100	End Semester Examination: 70

A. RATIONALE:

Automatic control has played a vital role in modern Engineering and Science. It has become an indispensable part of modern manufacturing and industrial process. So knowledge of automatic control system is dreadfully essential on the part of an Engineer. Basic approach to the automatic control system has been given in the subjects, so that students can enhance their knowledge in their future professional carrier.

B. OBJECTIVE:

Study of 'Control System' enhances the ability of the student on:

1. Acquire knowledge about time response analysis of control system.
2. Finding out steady state error and error constants.
3. Acquire knowledge about the analysis of stability in Root locus technique.
4. Learning about frequency response analysis of control system.
5. To use Bode plot and Nyquist plot for judgments about stability of a system.

COURSE CONTENTS

1. SIGNAL FLOW GRAPH.

1.1 Review of block diagrams and transfer functions of multivariable systems.
1.2 Construction of signal flow graph.
1.3 Basic properties of signal flow graph.
1.4 Signal flow graph algebra.
1.5 Construction of signal flow graph for control system.

2. TIME RESPONSE ANALYSIS.

2. 1 Time response of control system.
2. 2 Standard Test signal.
2.2.1. Step signal,
2.2.2. Ramp Signal
2.2.3. Parabolic Signal
2.2.4. Impulse Signal
2. 3 Time Response of first order system with:
2.3.1. Unit step response
2.3.2. Unit impulse response.
2. 4 Time response of second order system to the unit step input.
2.4.1. Time response specification.
2.4.2. Derivation of expression for rise time, peak time, peak overshoot, settling time and steady state error.

2.4.3. Steady state error and error constants.
2. 5 Types of control system.[Steady state errors in Type-0, Type-1, Type-2 system]
2. 6 Effect of adding poles and zero to transfer function.
2. 7 Response with P, PI, PD and PID controller.

3. ANALYSIS OF STABILITY BY ROOT LOCUS TECHNIQUE.

3. 1 Root locus concept.
3. 2 Construction of root loci.
3. 3 Rules for construction of the root locus.
3. 4 Effect of adding poles and zeros to G(s) and H(s).

4. FREQUENCY RESPONSE ANALYSIS.

4. 1 Correlation between time response and frequency response.
4. 2 Polar plots.
4. 3 Bode plots.
4. 4 All pass and minimum phase system.
4. 5 Computation of Gain margin and phase margin.
4. 6 Log magnitude versus phase plot.
4. 7 Closed loop frequency response.

5. NYQUIST PLOT

5.1 Principle of argument.
5.2 Nyquist stability criterion.
5.3 Niquist stability criterion applied to inverse polar plot.
5.4 Effect of addition of poles and zeros to G(S) H(S) on the shape of Niquist plot.
5.5 Assessment of relative stability.
5.6 Constant M and N circle
5.7 Nicholas chart.

Learning Resources:

1. A. Ananda Kumar Control System, PHI Publication
2. K. Padmanavan Control System, IK Publication
3. I. J. Nagarath, M. Gopal Control system Engineering, WEN Publication
4. A Natrajan,Ramesh Babu Control system Engineering, Scientific Publication
5. D N Manik Control Systems, Cengage Publication

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CHAPTER-1

SIGNAL FLOW GRAPH

CONTROL SYSTEM

→ control means to regulate a perticulare variable

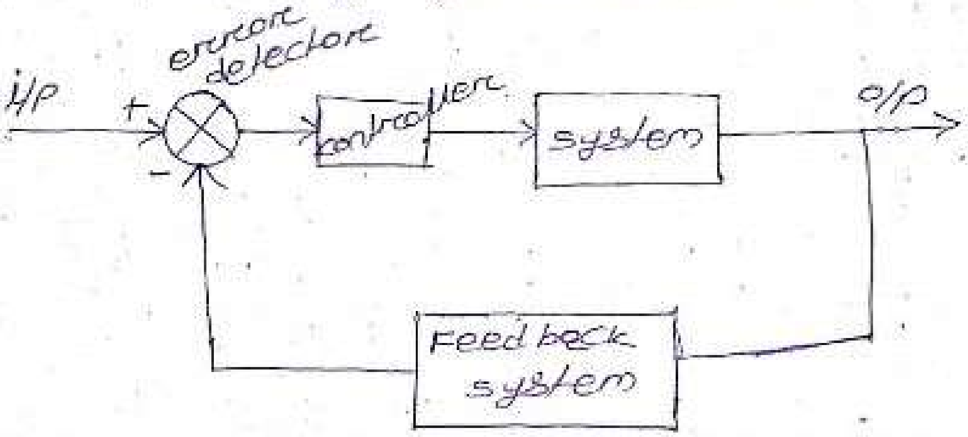
→ control system is a system which is use to regulate or control a perticulare variable at a perticular value. The variable may be temp, pressure, speed, flow rate etc.

→ control system is broadly devided into two types.

(1) open loop control system
(2) close loop control system

Block diagram of open loop control system

→ Errore is very high as output is not measure



Block diagram of close loop control system

→ errore is very less

TRANSFER FUNCTION

Transfer function can be defined mathmetically as the ratio between Laplace transfer of o/p to the Laplace transfer of i/p

$$T.F = \frac{L[o/p]}{L[i/p]}$$

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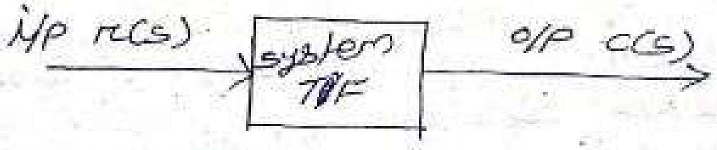
$$T.F = \frac{L[c(s)]}{L[r(s)]}$$



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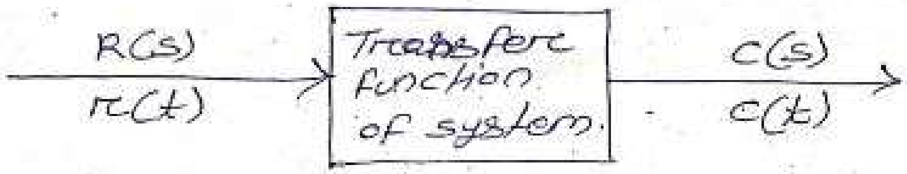
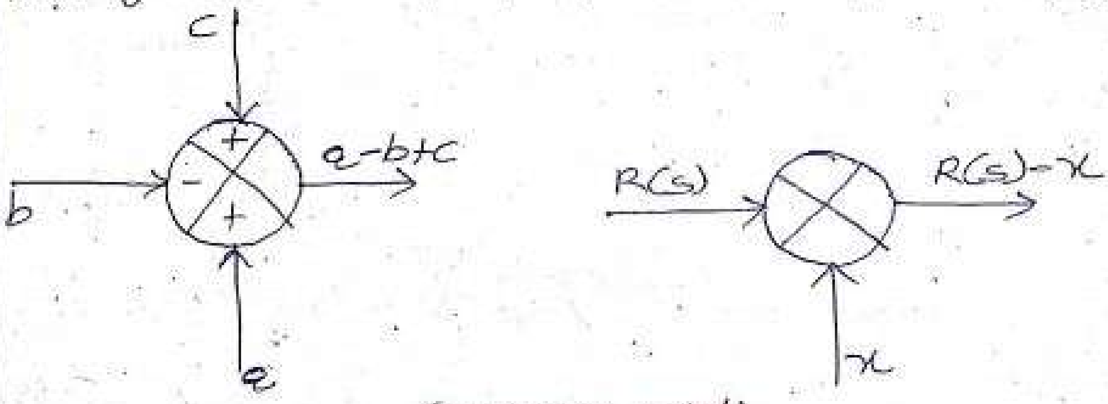
BLOCK DIAGRAM

Block diagram is a ~~pe~~ pictorial representation of the function perform by each component and the flow of signal in a system.

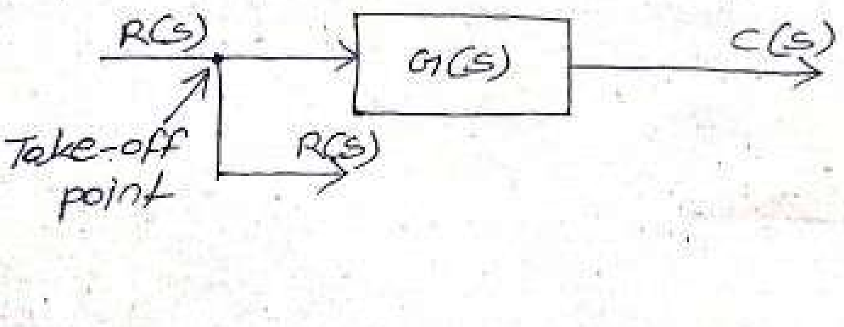


→ The transfer function of component are usually written with in the block which are connected through arrows to indicate the flow of signal.



(summing point)

→ summing point are ~~usually~~ used for adding or subtracting two or more signals.



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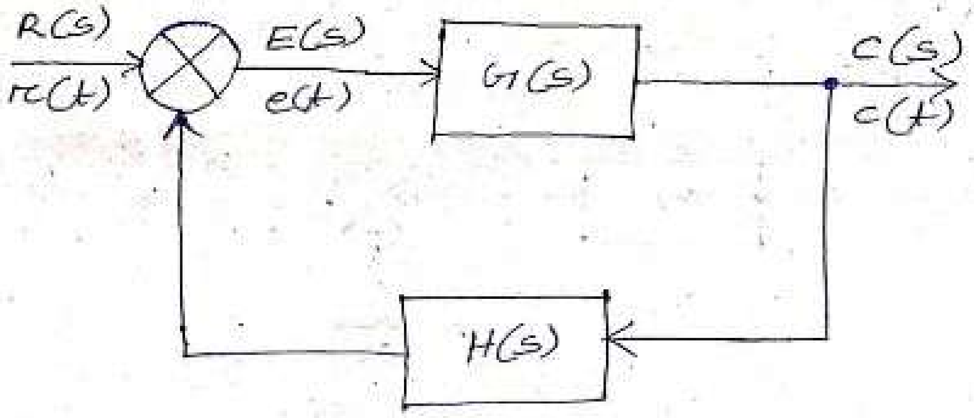
→ Take-off point are used to take a signal from a block or line to feed it to another block.

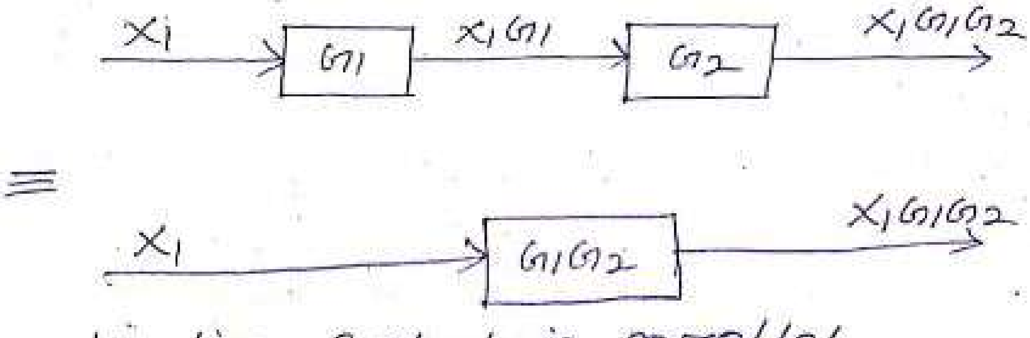
BLOCK DIAGRAM OF CLOSE LOOP SYSTEM



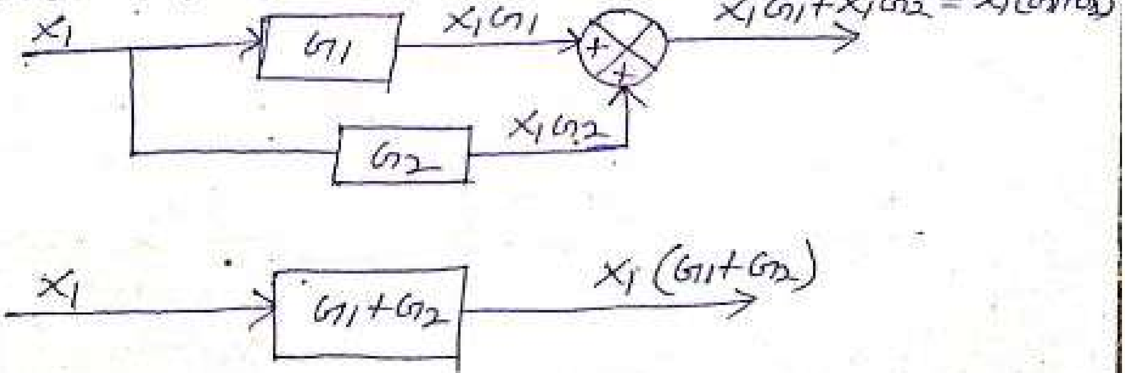
G(s) = Transfer function of forward path
H(s) = Transfer function of feedback path
R(s) = Reference i/p
C(s) = controlled o/p
E(s) = Error signal
B(s) = feed back signal.

BLOCK DIAGRAM REDUCTION RULE

1. combination of Block in cascade



2. combination of Block in parallel



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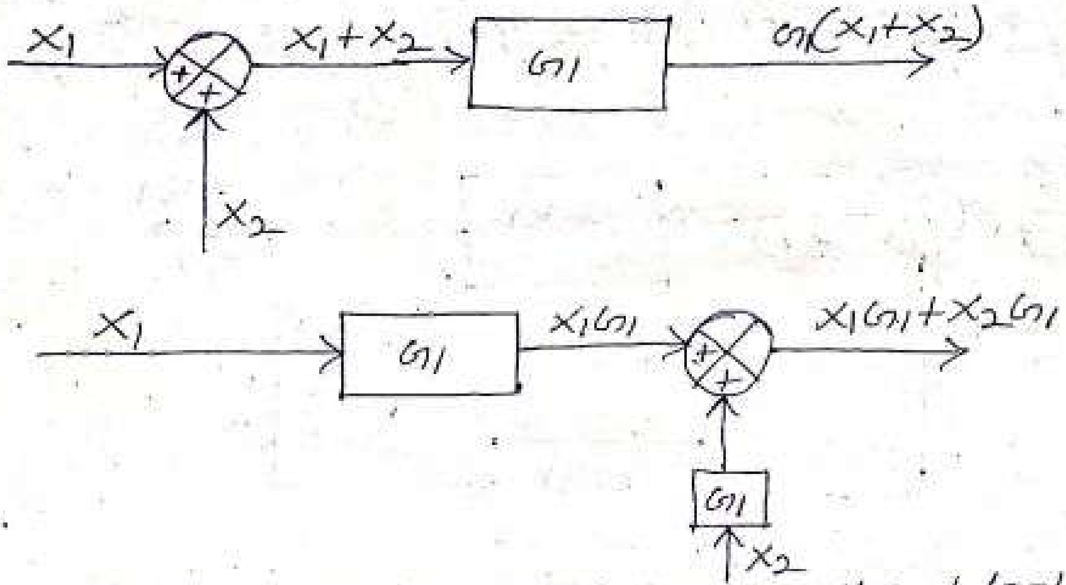
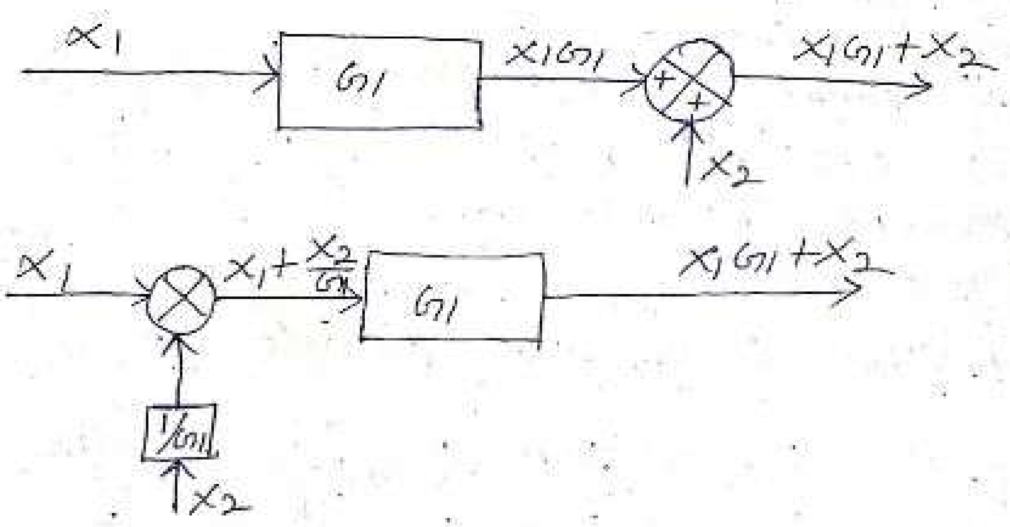
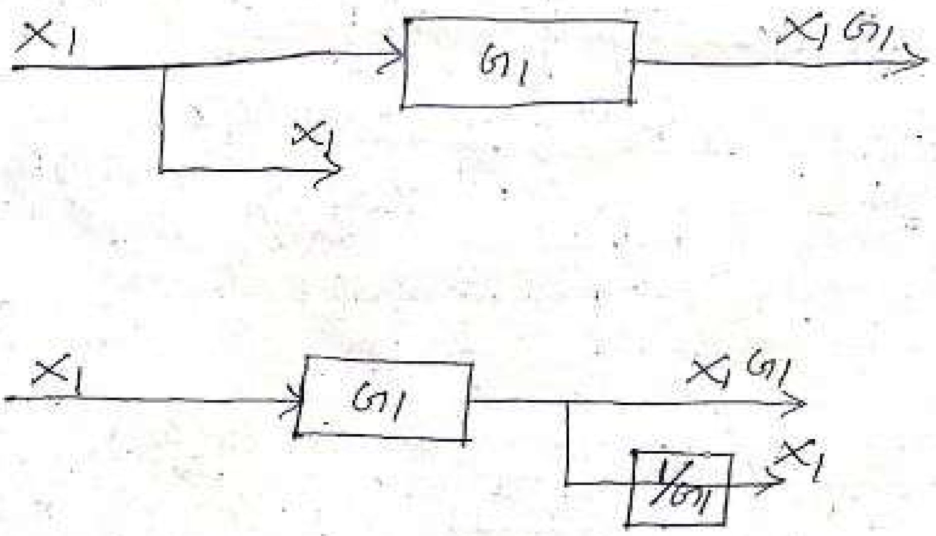
3. Moving the summing point after the block



(4) moving the summing point before the block



(5) moving the take-off point after the block



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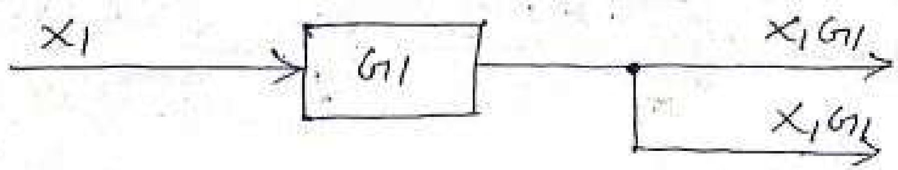
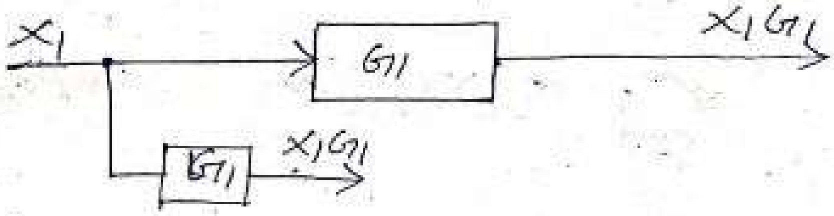
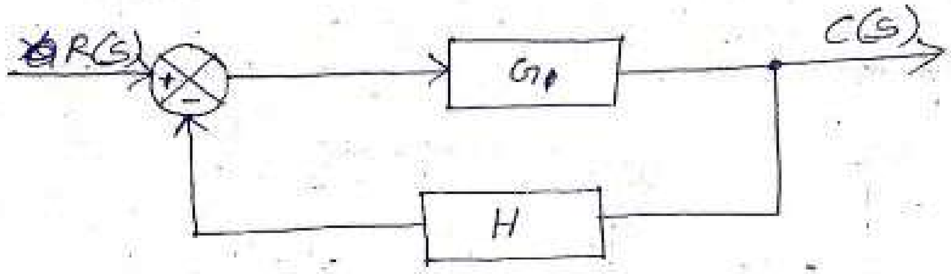
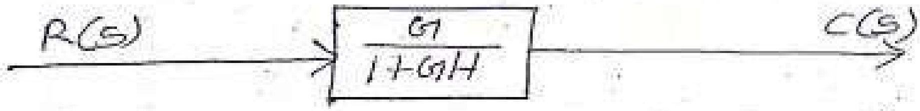
6. Moving take-off point before the block.



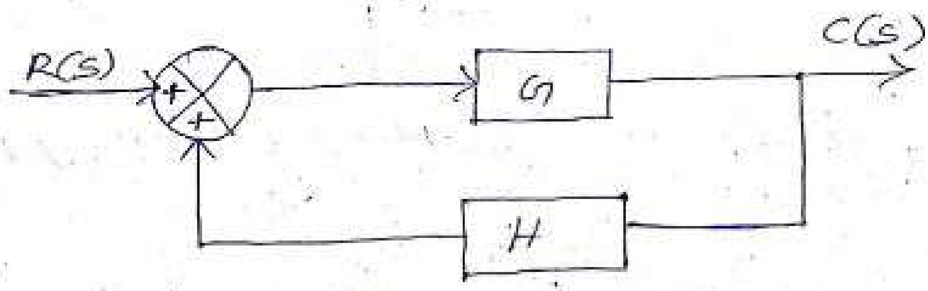


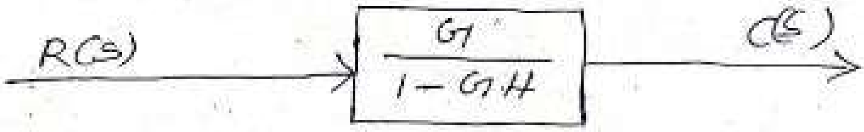
7. Eliminating feed-back loop





For -ve feed back system the equivalent block diagram is $\frac{G}{1+GH}$





→ For +ve feed back system the equivalent block diagram is $\frac{G}{1-GH}$

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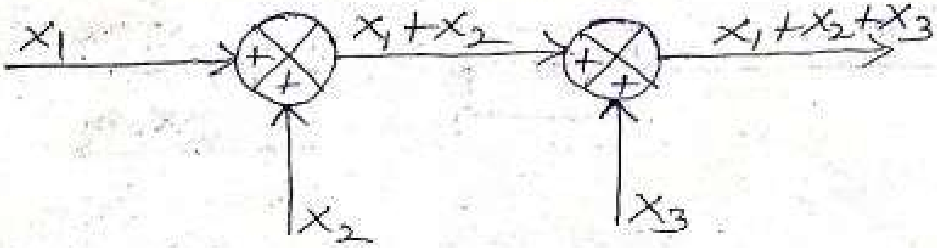
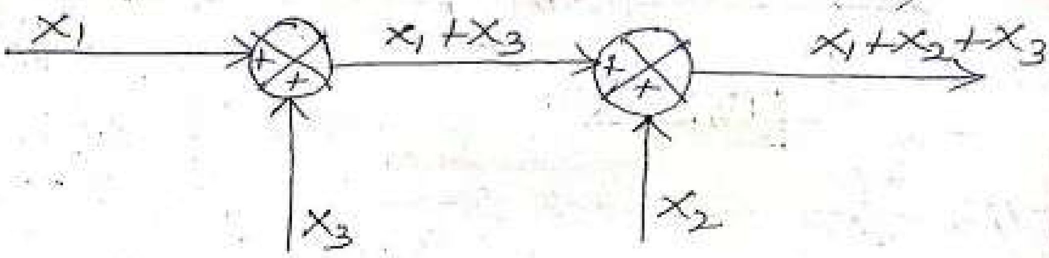
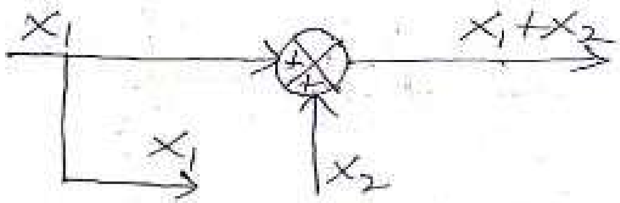
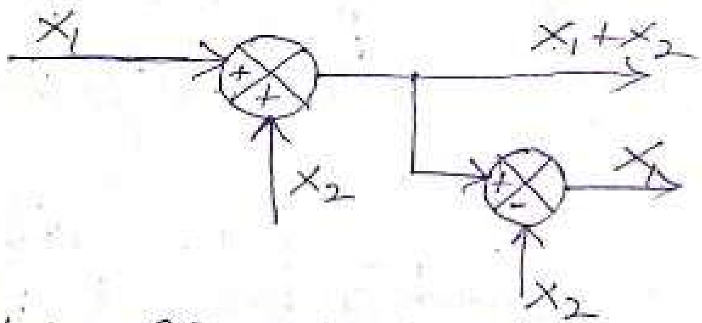
8 Interchanging of summing point



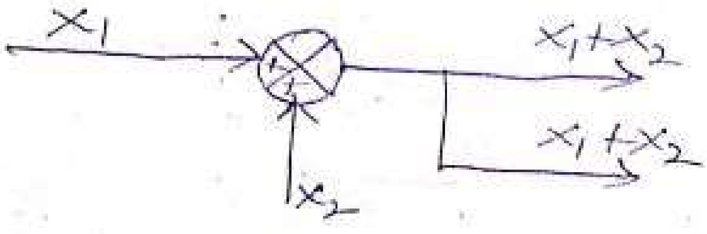


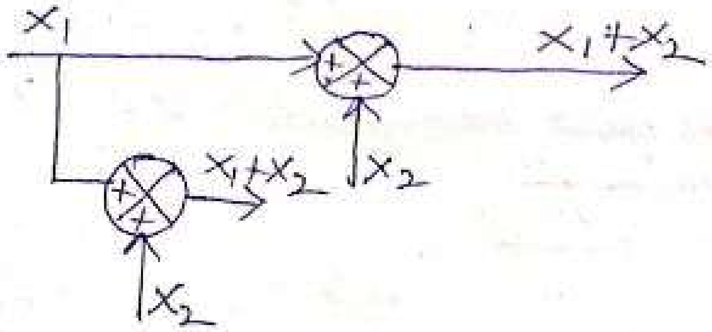
(9) Moving take-off point after a summing point





(10) Moving take-off point before a summing point





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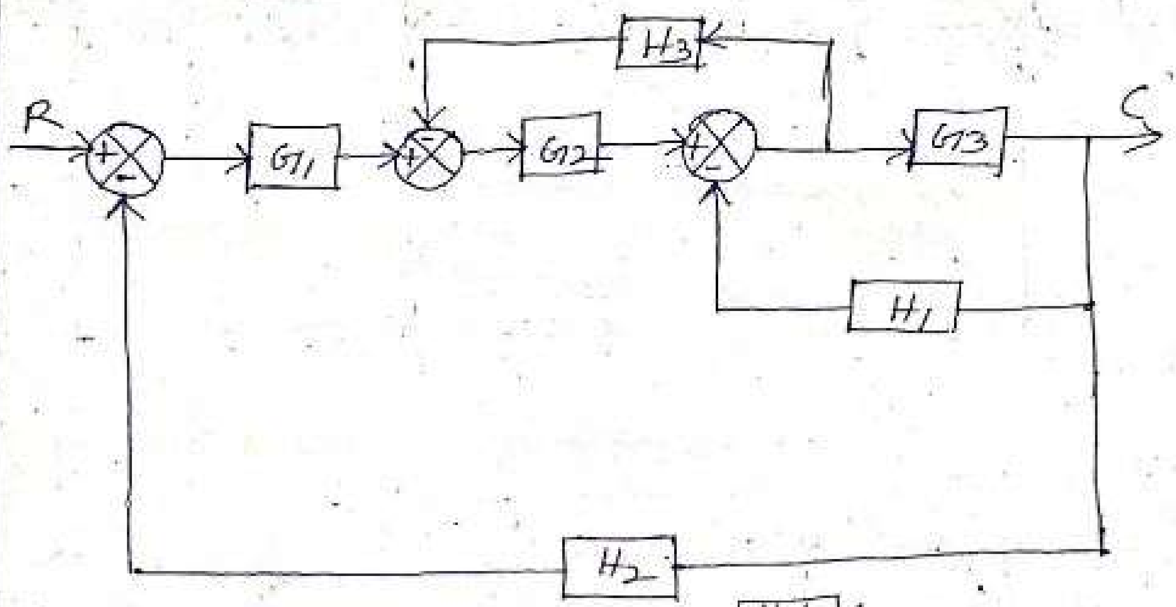
PROBLEM

Findout the overall transfer function of the following block diagram.



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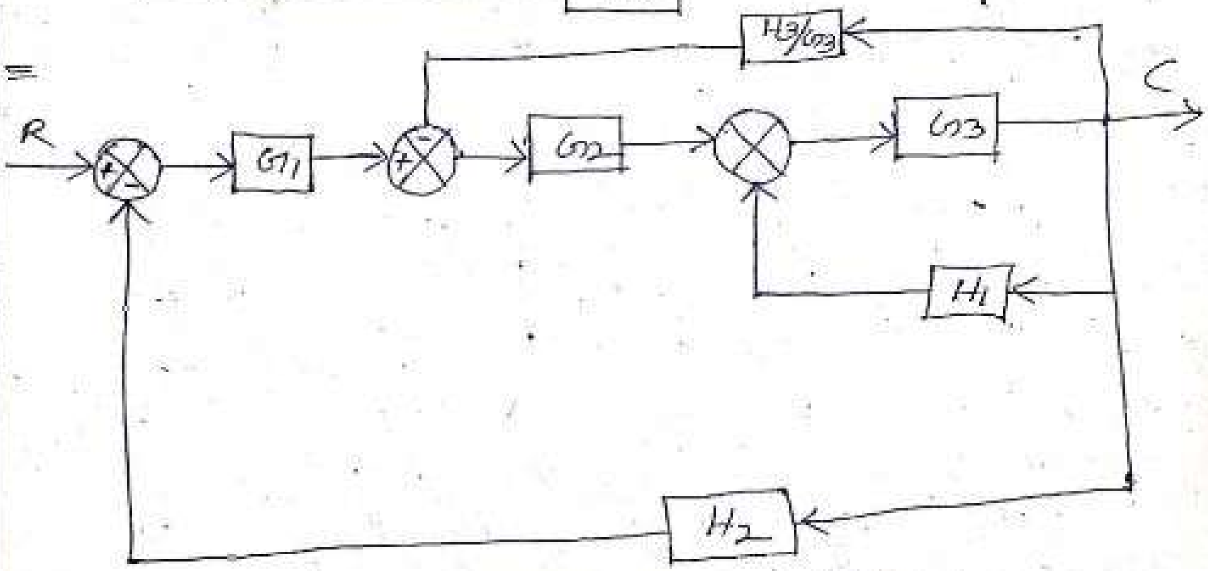

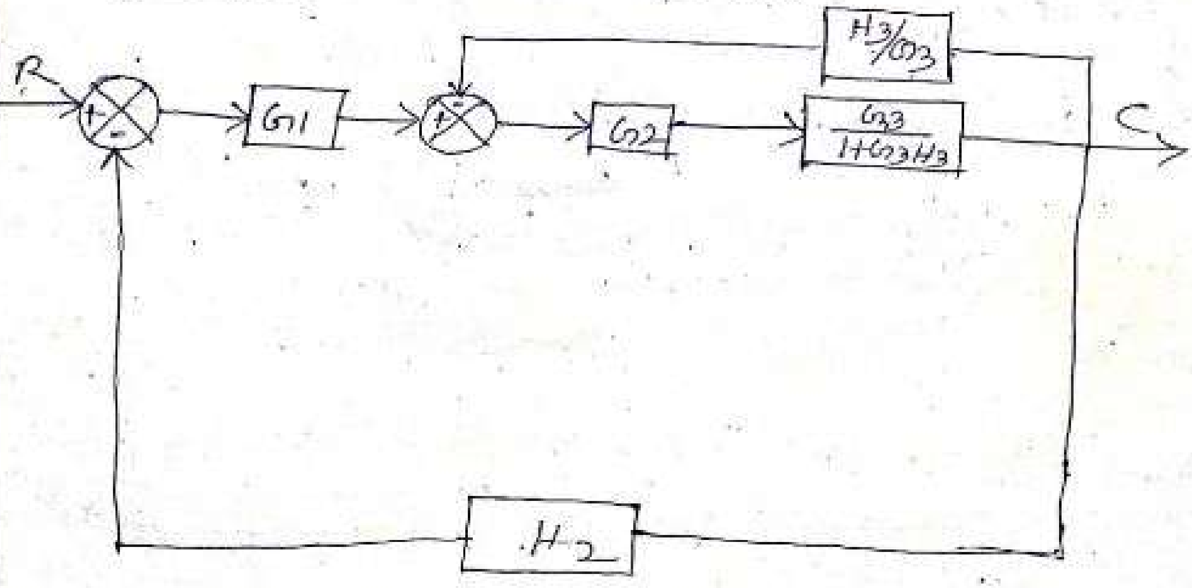

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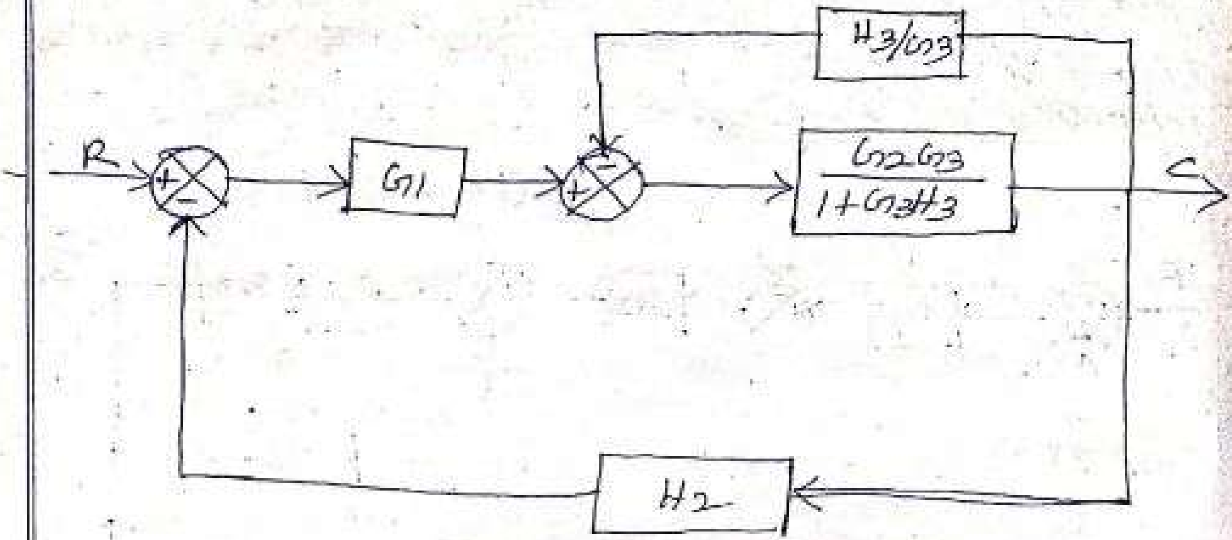
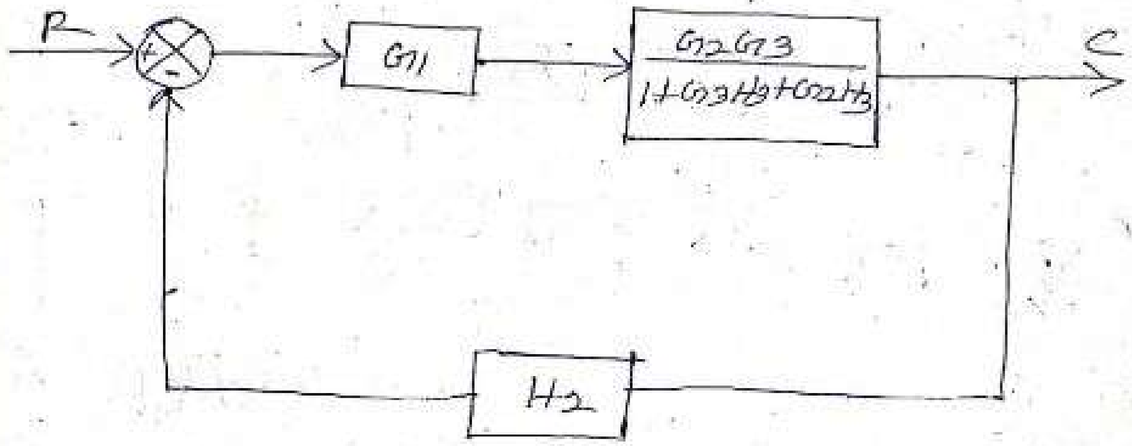
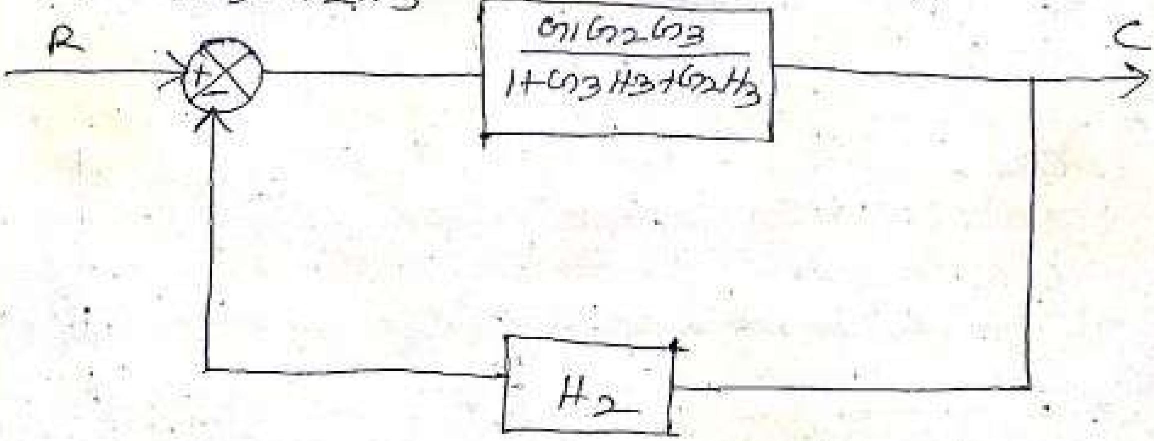




$$\frac{\frac{G_2G_3}{1+G_3H_3}}{1+\frac{G_2G_3}{1+G_3H_3}\times\frac{H_3}{G_3}} = \frac{\frac{G_2G_3}{1+G_3H_3}}{\frac{1+G_3H_3+G_2H_3}{1+G_3H_3}}$$

$$= \frac{G_2G_3}{1+G_3H_3+G_2H_3}$$



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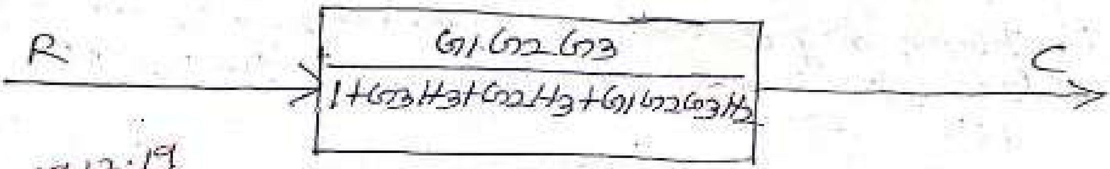
$$\frac{\frac{G_1G_2G_3}{1+G_3H_3+G_2H_3}}{1+\frac{G_1G_2G_3}{1+G_3H_3+G_2H_3}\times H_2} = \frac{\frac{G_1G_2G_3}{1+G_3H_3+G_2H_3}}{\frac{1+G_3H_3+G_2H_3+G_1G_2G_3H_2}{1+G_3H_3+G_2H_3}}$$

$$=\frac{G_1G_2G_3}{1+G_3H_3+G_2H_3+G_1G_2G_3H_2}$$



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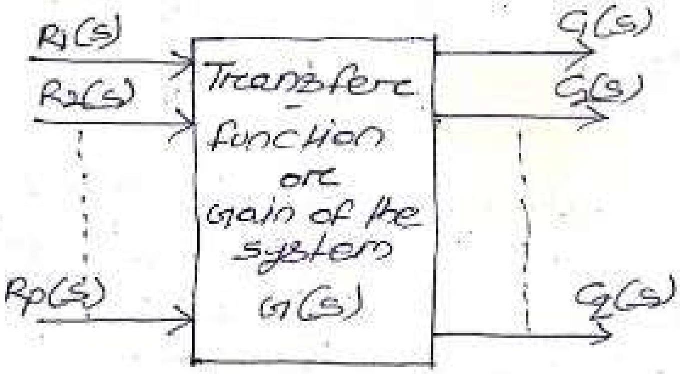
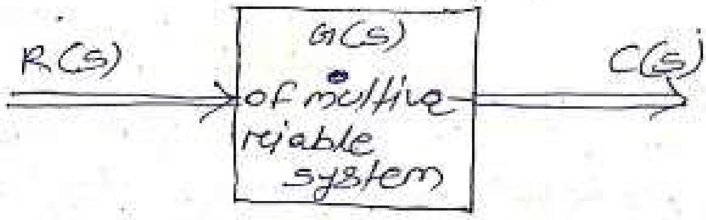
MULTIVARIABLE SYSTEM

when multiple input are present in a linear system then the system is called as multivariable system



→ ~~The above~~



→ The above two diagram represent the block diagram of multivariable system with 'p' no. of input and 'q' no. of o/p.

→ The inputs and outputs can be represented in matrix form system as given below.

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$$\begin{bmatrix} C_1(s) \\ C_2(s) \\ \vdots \\ C_q(s) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1p} \\ G_{21} & G_{22} & \cdots & G_{2p} \\ \vdots & & & \\ G_{q1} & G_{q2} & \cdots & G_{qp} \end{bmatrix} \begin{bmatrix} R_1(s) \\ R_2(s) \\ \vdots \\ R_p(s) \end{bmatrix}$$

→ The above matrix is for multiple input multiple output system.

→ The block diagram of a closeloop multivariable system which has feedback can be given as

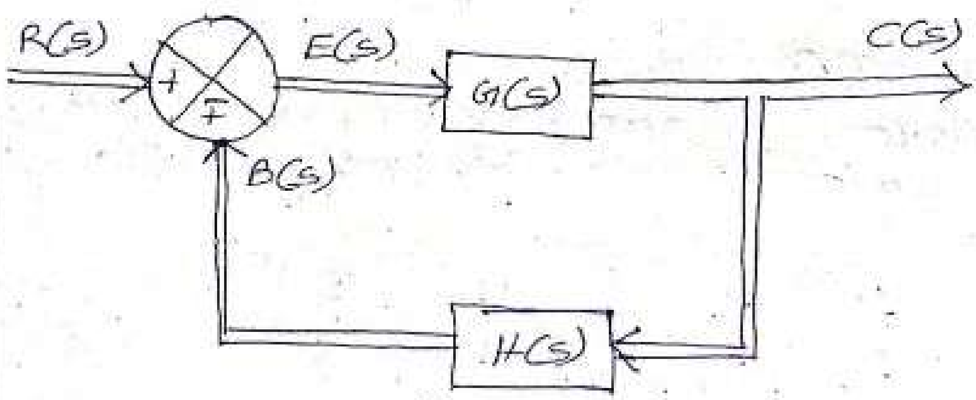

Fig: Block diagram of multivariable feedback (close loop) system

→ The transfer function of the system can be given by

$$T = \frac{G(s)}{1 \pm G(s)H(s)}$$

SIGNAL FLOW GRAPH

signal flow graph is a graphical represen-tation of the relationship between the variables of a set of system equations.

→ The system equation has to be linear algebric equation.

→ signal flow graph of a system can be construc-ted from the system equations which is described below.

→ Let's consider a system described by the following set of equations.

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$x_2 = e_{12}x_1 + e_{22}x_2 + e_{32}x_3 + e_{42}x_4 + e_{52}x_5$

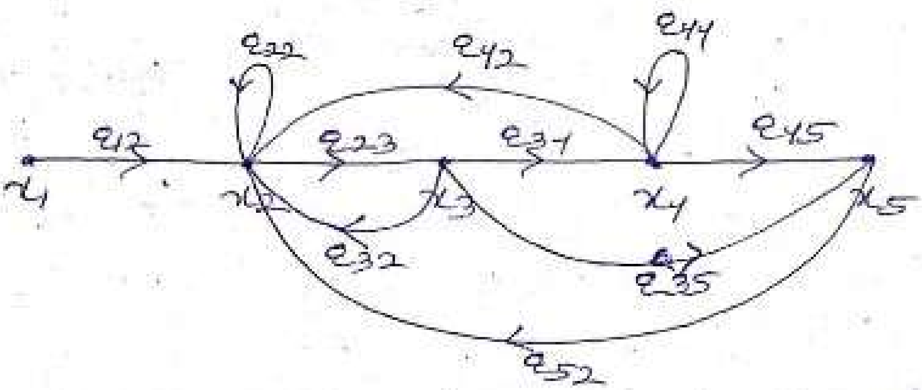
$x_3 = e_{23}x_2$

$x_4 = e_{34}x_3 + e_{44}x_4$

$x_5 = e_{35}x_3 + e_{45}x_4$

then x_1 is the input variable and x_5 is the o/p variable.

→ At first find out the variables and locate the nodes.



→ By putting all the branch gains from the equation we get the above signal flow graph.

→ The overall transfer function of the signal flow graph can be calculated by applying Mason's gain formula.

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MASON'S GAIN FORMULA

$$T = \frac{\sum_K \Delta_K M_K}{\Delta} = \frac{\Delta_1 M_1 + \Delta_2 M_2 + \cdots + \Delta_K M_K}{\Delta}$$

where,

T = overall Transfer function/Gain

Δ = Determination of signal flow graph

= 1 − (sum of loop gains of all individual loop) + (sum of gain product of all possible combination two non touching loop) − (sum of gain product of all possible combination of 3 non touching loop) + (- - - -)

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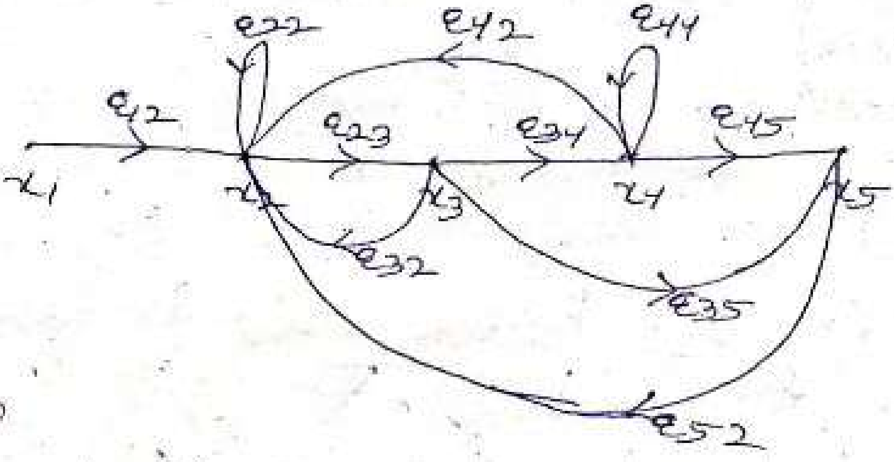
M_k = path gain of k^{th} forword path

Δ_k = value of Δ for the part of the s.f.g. not touching k^{th} forword path.

Ex:



solution

① There are two forword path in the S.F.G which are

$M_1 = (x_1 - x_2 - x_3 - x_4 - x_5) = e_{12} e_{23} e_{34} e_{45}$

$M_2 (x_1 - x_2 - x_3 - x_5) = e_{12} e_{23} e_{35}$

2. There are 6 individual loop in the graph

$L_1 (x_2 - x_2) = e_{22}$

$L_2 (x_2 - x_3 - x_2) = e_{23} e_{32}$

$L_3 (x_2 - x_3 - x_4 - x_2) = e_{23} e_{34} e_{42}$

$L_4 (x_2 - x_3 - x_4 - x_5 - x_2) = e_{23} e_{34} e_{45} e_{52}$

$L_5 (x_4 - x_4) = e_{44}$

$L_6 (x_2 - x_3 - x_5 - x_2) = e_{23} e_{35} e_{52}$

3. The possible combination of two non-touching loops are:-

$L_{15} = e_{22} e_{44}$

$L_{56} = e_{23} e_{35} e_{52} e_{44}$

$L_{25} = e_{23} e_{32} e_{44}$

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4. There are no possible combination of 3-non touching loops so 4 non-touching loop and furthere more does not exist.

5. $\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6) + (L_{15} + L_{56} + L_{25})$

$= 1 - (e_{22} + e_{23} e_{32} + e_{23} e_{34} e_{42} + e_{23} e_{34} e_{45} e_{52} + e_{44} + e_{23} e_{35} e_{52}) + (e_{22} e_{44} + e_{23} e_{35} e_{52} e_{44} + e_{23} e_{32} e_{44})$

6. $\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - L_5 = 1 - e_{44}$

7. By applying Mason's gain formula the overall gain of S.F.G is

$$T = \frac{x_5}{x_1} = \frac{\Delta_1 M_1 + \Delta_2 M_2}{\Delta}$$

$$= \frac{1(e_{12} e_{23} e_{34} e_{45}) + (1 - e_{44})(e_{12} e_{23} e_{35})}{1 - (e_{22} + e_{23} e_{32} + e_{23} e_{34} e_{42} + e_{23} e_{34} e_{45} e_{52} + e_{44} + e_{23} e_{35} e_{52}) + (e_{22} e_{44} + e_{23} e_{35} e_{52} e_{44} + e_{23} e_{32} e_{44}}$$

$$= \frac{e_{12} e_{23} e_{34} e_{45} + e_{12} e_{23} e_{35} - e_{12} e_{23} e_{35} e_{44}}{1 - e_{22} - e_{23} e_{32} - e_{23} e_{34} e_{42} - e_{23} e_{34} e_{45} e_{52} - e_{44} - e_{23} e_{35} e_{52} + e_{22} e_{44} + e_{23} e_{35} e_{52} e_{44} + e_{23} e_{32} e_{44}}$$

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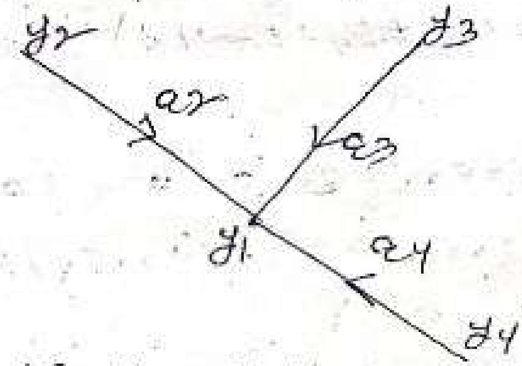
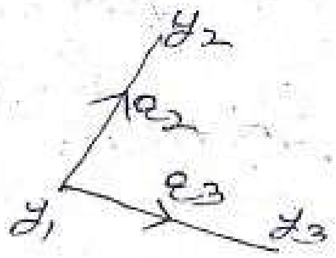
SIGNAL FLOW GRAPH ALGEBRA

1. The value of the variable represented by a node is equal to the sum of all signal entering to the node.



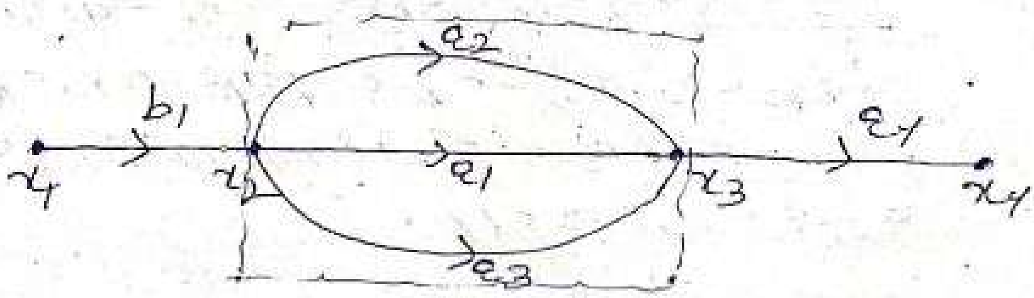
$$y_1 = a_2 y_2 + a_3 y_3 + a_4 y_4$$

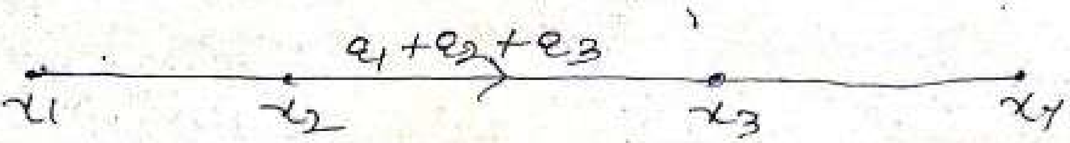
2. The value of the variable represented a node is transmitted through all the branches leaving the node.



$$y_2 = a_2 y_1$$
$$y_3 = a_3 y_3$$

3. parallel branches in the same direction connecting two nodes can be replaced by a single branch with gain equal to sum of all the parallel branch gain.





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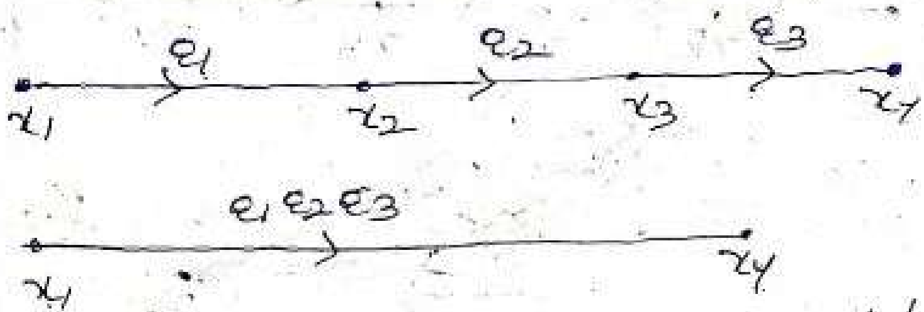
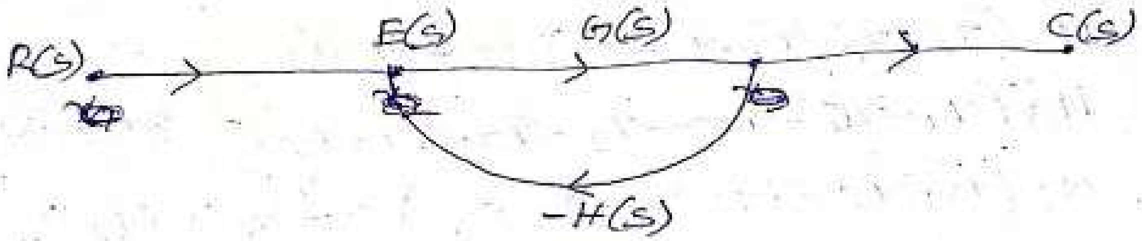
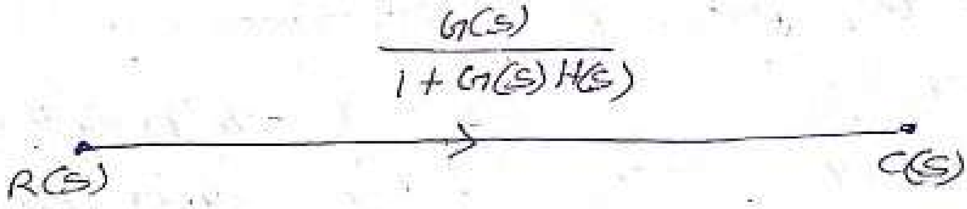
4. A series connection of unidirectional branches can be repleced by a single branch with gain is equal to product of all branch gain



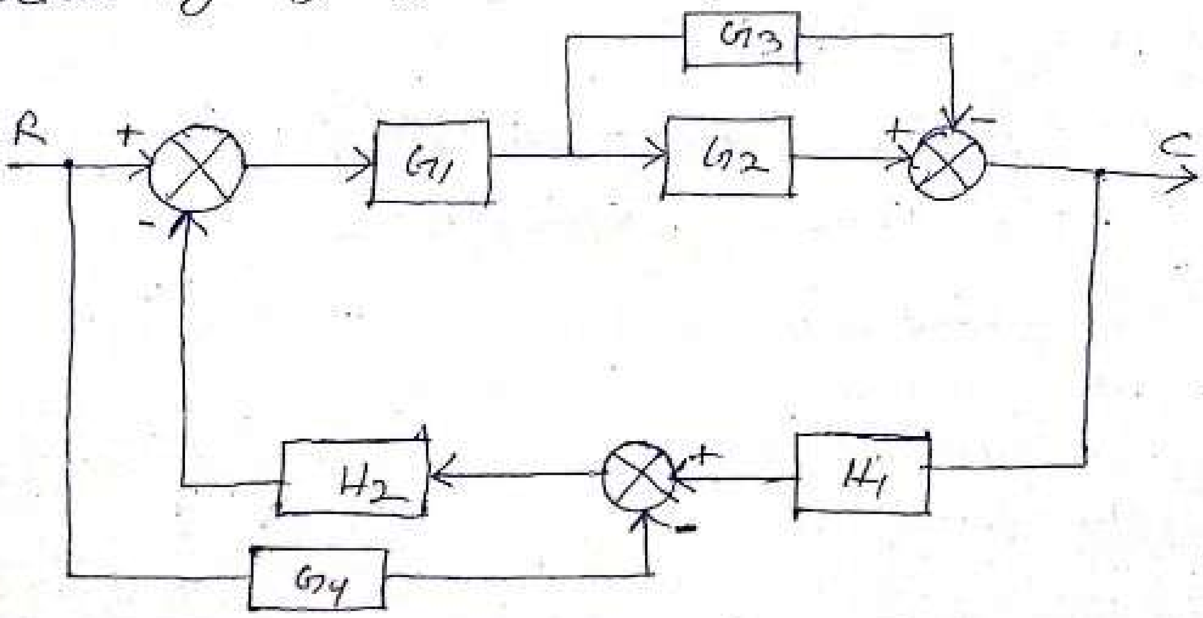
5. A single feedback loop can be repleced by a single branch with gain equal to $\frac{G(s)}{1 \pm G(s)H(s)}$





PROBLEM-2

obtain the transfere function of the control system whose block diagram is shown in the below fig. by signal flow graph:



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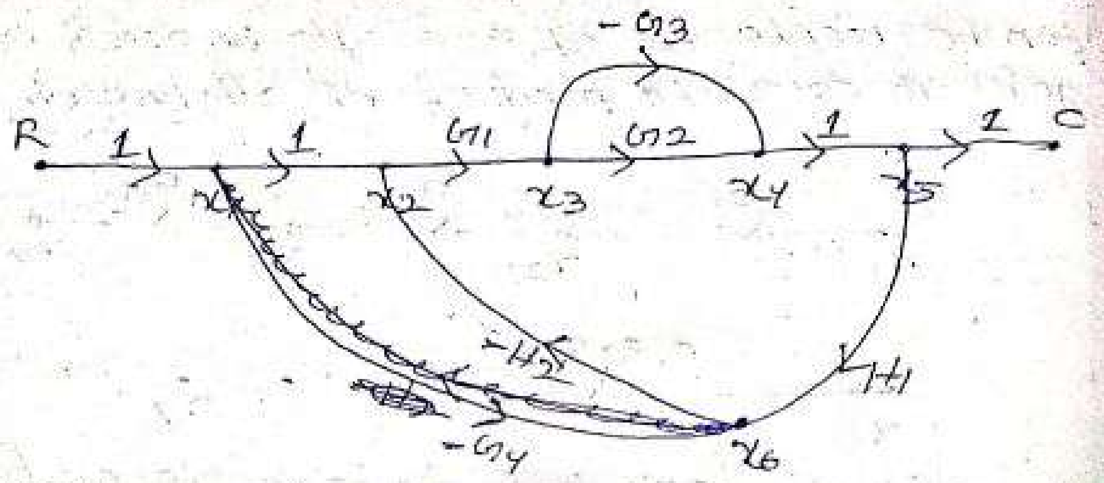
Ans-



→ The forword path of S.F.G are

$M_1 (x_1 - x_2 - x_3 - x_4 - x_5) = G_1 G_2$

$M_2 (x_1 - x_2 - x_3 - x_4 - x_5) = - G_1 G_3$

$M_3 (x_1 - x_6 - x_2 - x_3 - x_4 - x_5) = G_1 G_2 G_4 H_2$

$M_4 (x_1 - x_6 - x_2 - x_3 - x_4 - x_5) = - G_1 G_3 G_4 H_2$

→ The loop present in SFG are

$L_1 (x_2 - x_3 - x_4 - x_5 - x_6 - x_2) = -G_1 G_2 H_1 H_2$

$L_2 (x_2 - x_3 - x_4 - x_5 - x_6 - x_2) = G_1 G_3 H_1 H_2$

→ There are no possible combination of two non-touching loop so 3 non-touching loop and further more does not exist.

→ $\Delta = 1 - (L_1 + L_2)$

$= 1 - (-G_1 G_2 H_1 H_2 + G_1 G_3 H_1 H_2)$

$= 1 + G_1 G_2 H_1 H_2 - G_1 G_3 H_1 H_2$

→ $\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - 0 = 1$

$\Delta_3 = 1 - 0 = 1$

$\Delta_4 = 1 - 0 = 1$

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→ The overall transfer function can be calculated by appling Mason's Gain formula.

$$T = \frac{\sum_k M_k \Delta_k}{\Delta}$$

$$T = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3 + M_4\Delta_4}{\Delta}$$

$$= \frac{G_1G_2 - G_1G_3 + G_1G_2G_4H_2 - G_1G_3G_4H_2}{1 + G_1G_2H_1H_2 - G_1G_3H_1H_2}$$

06.01.29

CHAPTER-2
TIME RESPONSE ANALYSIS

The time response of a system is defind as the o/p of a close loop system as a function of time.

→ Time response of a system is usually devided in to two parts.

1. Transient response
2. steady-state response

→ If $c(t)$ is the time response of a continious data system then it can be written as

$$c(t) = c_t(t) + c_{ss}(t)$$

where,

$c_t(t)$ = Transient response

$c_{ss}(t)$ = steady-state response.

→ $\lim_{t\to\infty} c_t(t) = 0$

so the transient response is zero when time tends to infinity.

→ In control system the transient response is defind as the part of time response that goes

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from zero to a certain value.

→ The steady-state response is the part of time response which remains after the transient response has ended.

STANDRD TEST SIGNAL

To perform the time-domain analysis the following test signals are used

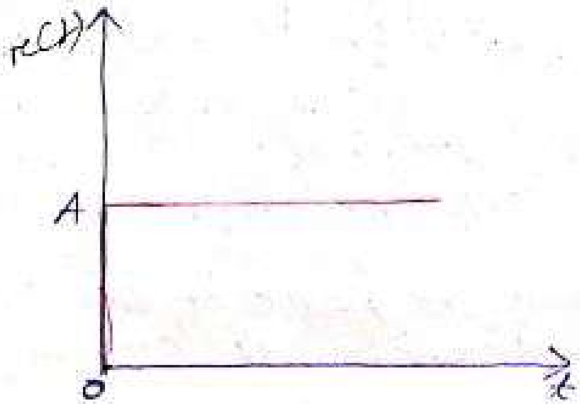
1. step signal
2. Ramp signal
3. parabolic signal
4. Impulse signal

1. STEP SIGNAL

A step signal is a signal whose value changes from one level (zero) to another level (A) at time zero

→ It is denoted as $u(t)$

→ graphical representation of the signal is.



→ Mathematical representation of the signal is

$r(t) = A\,u(t)$

$$r(t) = A \quad , t \geq 0$$
$$r(t) = 0 \quad , t < 0$$

→ If $A=1$ then the signal ~~starts from~~ is called as unit step signal.

→ The Laplace transfer of signal is

$$R(s) = A/s$$

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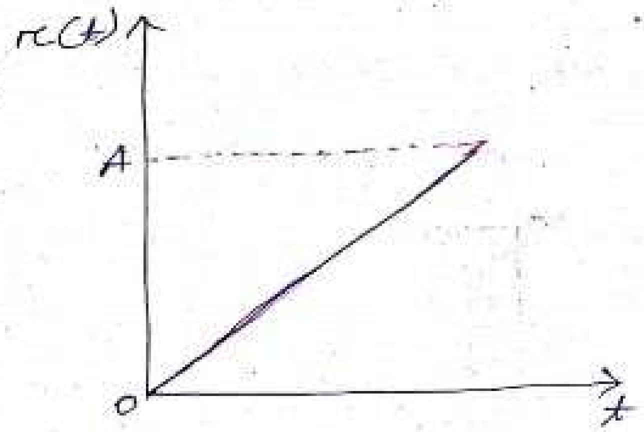
2. RAMP SIGNAL

The ramp signal starts from a zero value and increases linearly with time.

→ Graphical representation of the signal is



→ Mathematical representation of the signal is

$$r(t) = At, \; t \geq 0$$
$$= 0, \; t < 0$$

→ Laplace transfer of a signal is

$$R(s) = A/s^2$$

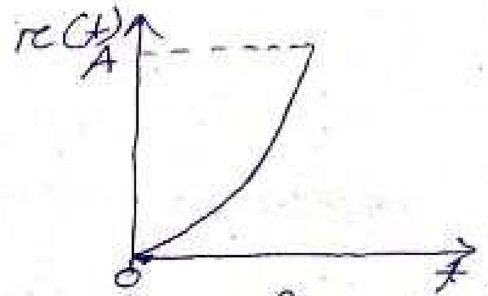
3. PARABOLIC SIGNAL

This signal is one order faster than the ramp signal

→ Mathematical representation of signal is

$$r(t) = At^2, \; t \geq 0$$
$$= 0, \; t < 0$$

→ Graphical representation of signal is



→ Laplace transfer of the signal is

$$R(s) = A/s^3$$

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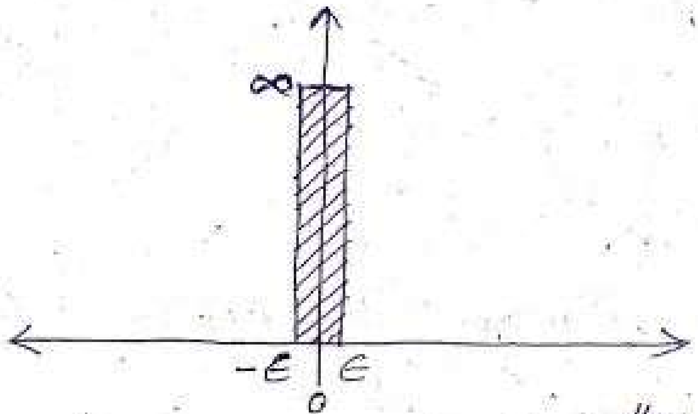
4. IMPULSE SIGNAL

An impulse function is defind as the signal which has zero value every where except at t=0 ; then the magnitude may be considered as infinite.

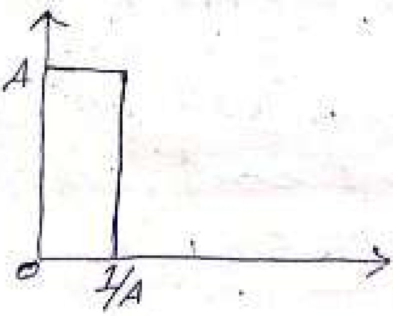
→ Graphical representation of the signal



→ Mathmetical representation of the signal

$$\delta(t) = 0, \; t \neq 0$$
$$= \int_{-\epsilon}^{\epsilon} \delta(t)\,dt \;, t = 0$$

→ For impulse height is equal to A the signal is



$$\int_{-\epsilon}^{\epsilon} \delta(t)\,dt = 1$$

where $\epsilon \rightarrow 0$

→ This impulse function is called as unit impulse function.

→ The Laplace transfer of unit impulse function

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$L[\delta(t)] =$

$$R(s) = 1$$

TIME RESPONSE OF FIRST ORDER SYSTEM

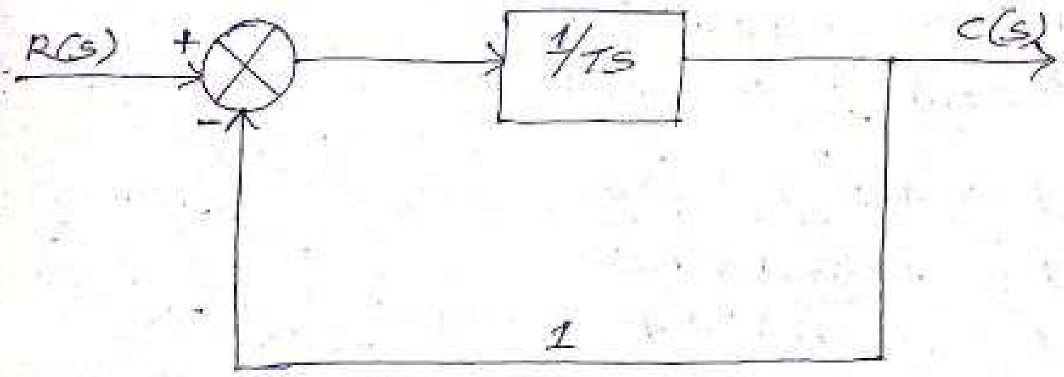
i. unit step Response
ii. unit impulse Response

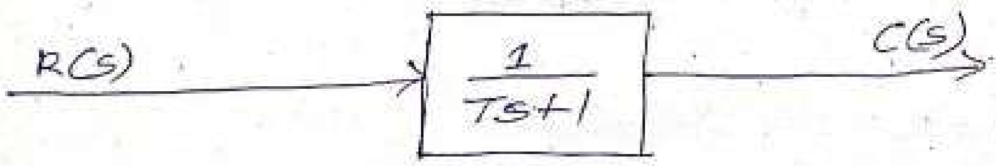
→ Generally the first order systems are R-C ckt and temp measuring thermal system.

→ The transfer function of first order system is

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{Ts+1}$$

→ Block diagram first order close loop control system





→ The first order system will be analysized by providing unit step input and unit impulse input when the initial condition are assumed as zero

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07.01.20

TIME RESPONSE OF FIRST ORDER SYSTEM

1. UNIT STEP RESPONSE

The unit step input $R(s) = \frac{1}{s}$

First order system

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{Ts+1}$$

$$C(s) = R(s)\left(\frac{1}{Ts+1}\right)$$

$$= \frac{1}{s} \times \frac{1}{Ts+1}$$

$$= \frac{1}{s(Ts+1)}$$

By appling partial fraction

$$\frac{1}{s(Ts+1)} = \frac{A}{s} + \frac{B}{Ts+1}$$

$$\frac{1}{s(Ts+1)} = \frac{A(Ts+1)+B(s)}{s(Ts+1)}$$

$\Rightarrow 1 = A(Ts+1) + Bs$

$\Rightarrow 1 = ATs + A + Bs$

$\Rightarrow 1 = (AT+B)s + A$

By compairing co-efficient

$A = 1$

& $AT + B = 0$

$\Rightarrow 1 \times T + B = 0$

$\Rightarrow T + B = 0$

$\Rightarrow B = -T$

By putting the value of A & B

$$\frac{1}{s(Ts+1)} = \frac{1}{s} - \frac{T}{Ts+1}$$

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so $C(s) = \frac{1}{s} - \frac{T}{Ts+1}$

$= \frac{1}{s} - \frac{1}{s+1/T}$

By applying Laplace inverse we get

$c(t) = 1 - e^{-t/T} \left(\because L^{-1}\left[\frac{1}{(s+a)}\right] = e^{-at} \right)$

so from the above equation we found that the output rises exponentially from zero value to unit.

when $t=0$

$c(t) = 1 - e^{0}$

$= 1 - 1 = 0$

when $t = T$

$c(t) = 1 - e^{-T/T}$

$= 1 - e^{-1}$

$= 1 - 0.36 = 0.632$

when $t = 2T$

$c(t) = 1 - e^{-2T/T} = 1 - e^{-2}$

$= 0.864$

when $t = 3T$

$c(t) = 1 - e^{-3T/T} = 1 - e^{-3} = 0.950$

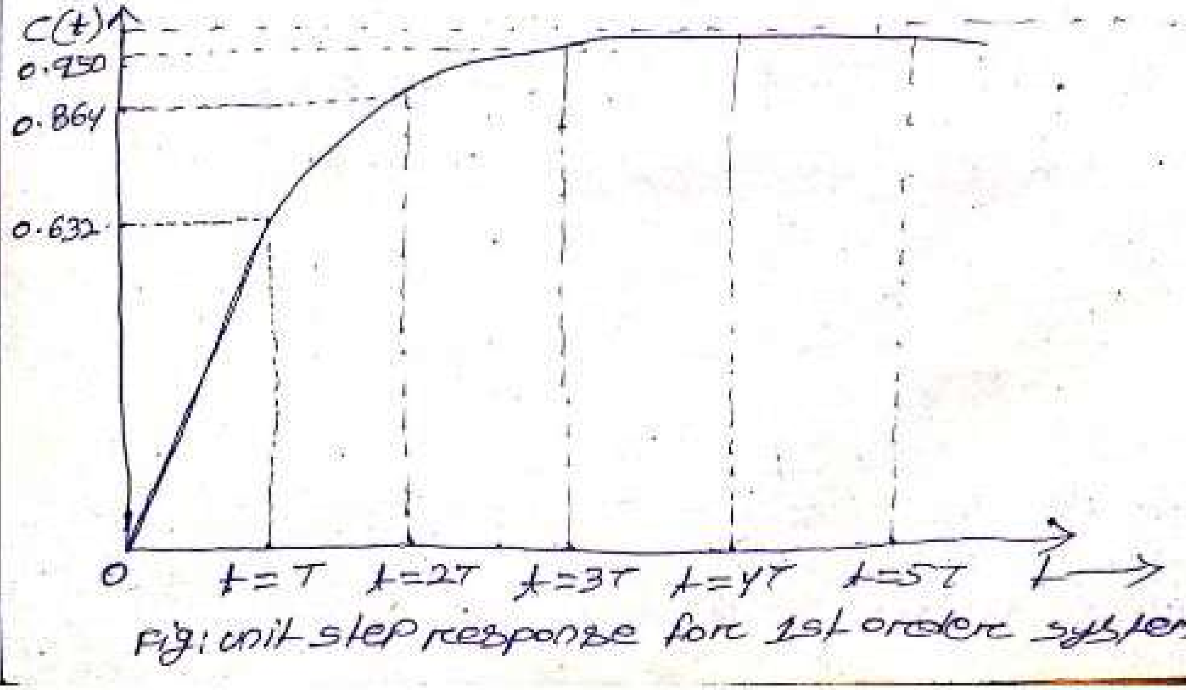

Fig: unit step response for 1st order system

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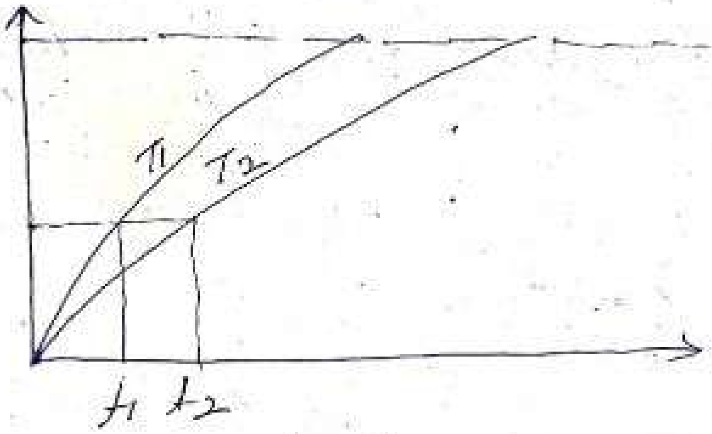
→ T is known as time constant of the system. It shows how fast the system tends to reach the final value.

→ Large time constant (T) corresponds to a sluggish system and small time constant corresponds to a fast system.



$T_1 < T_2$

$t_2 > t_1$

→ The error response of the system is given by

$$e(t) = r(t) - c(t)$$
$$= 1 - (1 - e^{-t/T})$$
$$= 1 - 1 + e^{-t/T}$$
$$\boxed{e(t) = e^{-t/T}}$$

STEADY STATE ERROR

The steady state error is given by

$$e_{ss}(t) = \lim_{t \to \infty} e(t)$$
$$= \lim_{t \to \infty} e^{-t/T}$$
$$\boxed{e_{ss}(t) = 0}$$

→ so first order system provides o/p with zero steady state error if unit step input is provided to it.

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08.01.20

UNIT IMPULSE RESPONSE

The unit impulse input $R(s) = 1$

For 1st order system

$$\frac{C(s)}{R(s)} = \frac{1}{Ts+1}$$

$$\Rightarrow C(s) = R(s) \cdot \frac{1}{Ts+1}$$

$$= 1 \times \frac{1}{Ts+1} = \frac{1}{Ts+1}$$

$$= \frac{1/T}{s+1/T}$$

$$\Rightarrow C(s) = \frac{1/T}{s+1/T} = \frac{1}{T} \times \frac{1}{s+1/T}$$

→ Taking the Laplace inverse of the above equation we get

$$\Rightarrow c(t) = \frac{1}{T} e^{-t/T} \left(\because L^{-1}\left[\frac{1}{s+a}\right] = e^{-at} \right)$$

→ The above expression is for the unit impulse response for first order system.

At $t = 0$

$$c(t) = \frac{1}{T} e^{0} = \frac{1}{T}$$

At $t = T$

$$c(t) = \frac{1}{T} e^{-T/T} = \frac{1}{T} e^{-1}$$

$$= \frac{1}{T} \times 0.36$$

At $t = 2T$

$$c(t) = \frac{1}{T} e^{-2T/T} = \frac{1}{T} e^{-2}$$

$$= \frac{1}{T} \times 0.135$$

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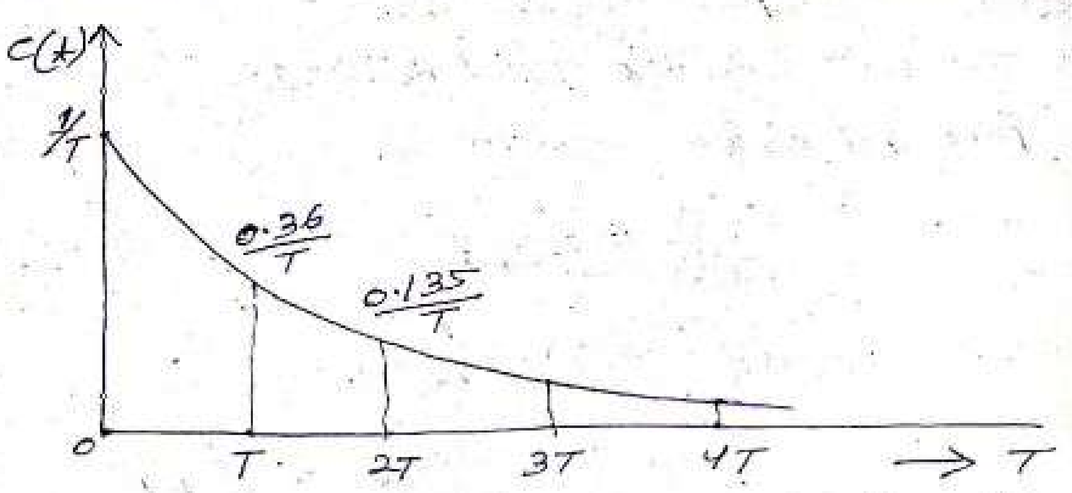

Fig: unit impulse response for 1st order system.

SECOND ORDER SYSTEM

The system which contain s^2 ~~then~~ term in its system equation or the highest power of ~~deriv~~ derivative term is 2 $\left(\frac{d^2}{dt^2}\right)$ then it is called as second order system.

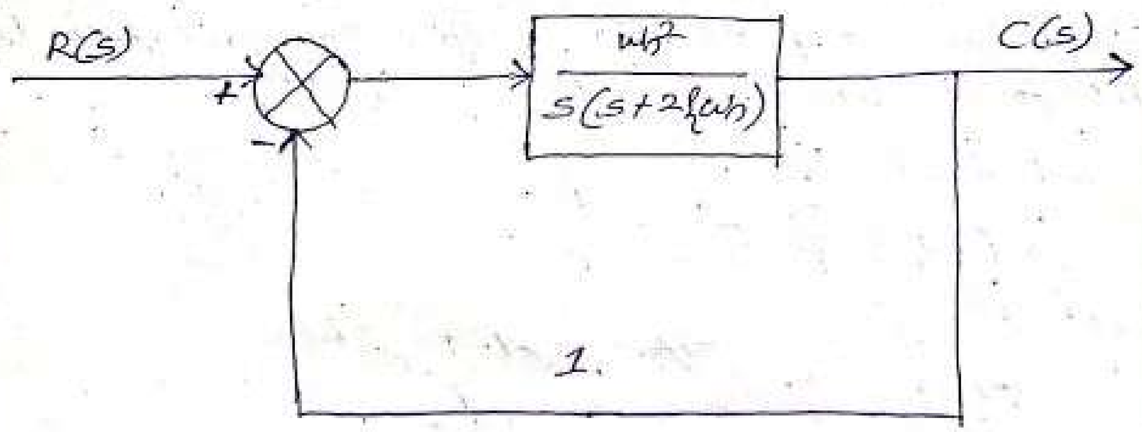

Fig: close loop second order system.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\frac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}}$$

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$$\frac{C(s)}{R(s)} = \frac{\dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}}{\dfrac{s(s+2\zeta\omega_n)+\omega_n^2}{s(s+2\zeta\omega_n)}}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s(s+2\zeta\omega_n)+\omega_n^2}.$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}}$$

where, ζ = damping ratio

ω_n = undamped natural frequency.

→ The dynamic behaviour of the second order system can be described by zeta(ζ) and ω_n.

→ If $\zeta = 0$ then the system is known as undamped system. The transient response does not finised or dies out.

→ If $\zeta = 1$ then the system is known as critically damped.

→ If $0 < \zeta < 1$ then the system is called as under-damped

→ If $\zeta > 1$ then the system is called as overdamped.

→ The underdamped system gives a oscillatory transient response.

→ In over damped system the o/p rises slowly towords the final value. In critically damped system the o/p rises and reaches the final value.

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UNIT STEP RESPONSE

unit step input $R(s) = \frac{1}{s}$

For 2nd order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$C(s) = R(s)\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{1}{s}\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$$

By applying partial fraction.

$$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{A(s^2+2\zeta\omega_n s+\omega_n^2)+(Bs+C)(s)}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$$

$$\omega_n^2 = A(s^2+2\zeta\omega_n s+\omega_n^2)+(Bs+C)(s)$$

$$\omega_n^2 = As^2+2\zeta\omega_n s A+A\omega_n^2+Bs^2+Cs$$

$$\omega_n^2 = s^2(A+B)+s(2\zeta\omega_n A+C)+A\omega_n^2$$

By equating co-efficients.

$$\omega_n^2 A = \omega_n^2$$

$$A = \frac{\omega_n^2}{\omega_n^2} = 1$$

$$A+B=0$$

$$B=-A=-1$$

$$2\zeta\omega_n A+C=0$$

$$C=-2\zeta\omega_n$$

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By putting the value of A, B & C we get

$$C(s) = \frac{1}{s} - \left(\frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}\right)$$

ω_d is called as damped natural frequency

$$\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$$

$$\Rightarrow \boxed{\frac{\omega_d^2}{\omega_n^2} = 1-\xi^2}$$

$$C(s) = \frac{1}{s} - \left(\frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}\right)$$

$$= \frac{1}{s} - \left(\frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\xi^2\omega_n^2-\xi^2\omega_n^2+\omega_n^2}\right)$$

$$= \frac{1}{s} - \left(\frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)}\right)$$

$$= \frac{1}{s} - \left(\frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}\right)$$

$$= \frac{1}{s} - \left(\frac{s+\xi\omega_n+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}\right)$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{\omega_d}\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$\boxed{\begin{aligned}\omega_d &= \omega_n\sqrt{1-\xi^2}\\ \frac{\omega_n}{\omega_d} &= \frac{1}{\sqrt{1-\xi^2}}\end{aligned}}$$

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi}{\sqrt{1-\xi^2}}\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

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By applying Laplace inverse we get

~~$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t \cdot e^{-\xi\omega_n t}\sin\omega_d t$~~

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t}\sin\omega_d t$$

$$L^{-1}\left[\frac{s+a}{(s+a)^2+b^2}\right] = e^{-at}\cos bt$$

$$L^{-1}\left[\frac{b}{(s+a)^2+b^2}\right] = e^{-at}\sin bt$$

Assume

$\sin\theta = \sqrt{1-\xi^2}$

$\cos\theta = \xi$

$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$

$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right]$$

By taking $\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sin\theta\cos\omega_d t + \cos\theta\sin\omega_d t\right)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\theta + \omega_d t)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

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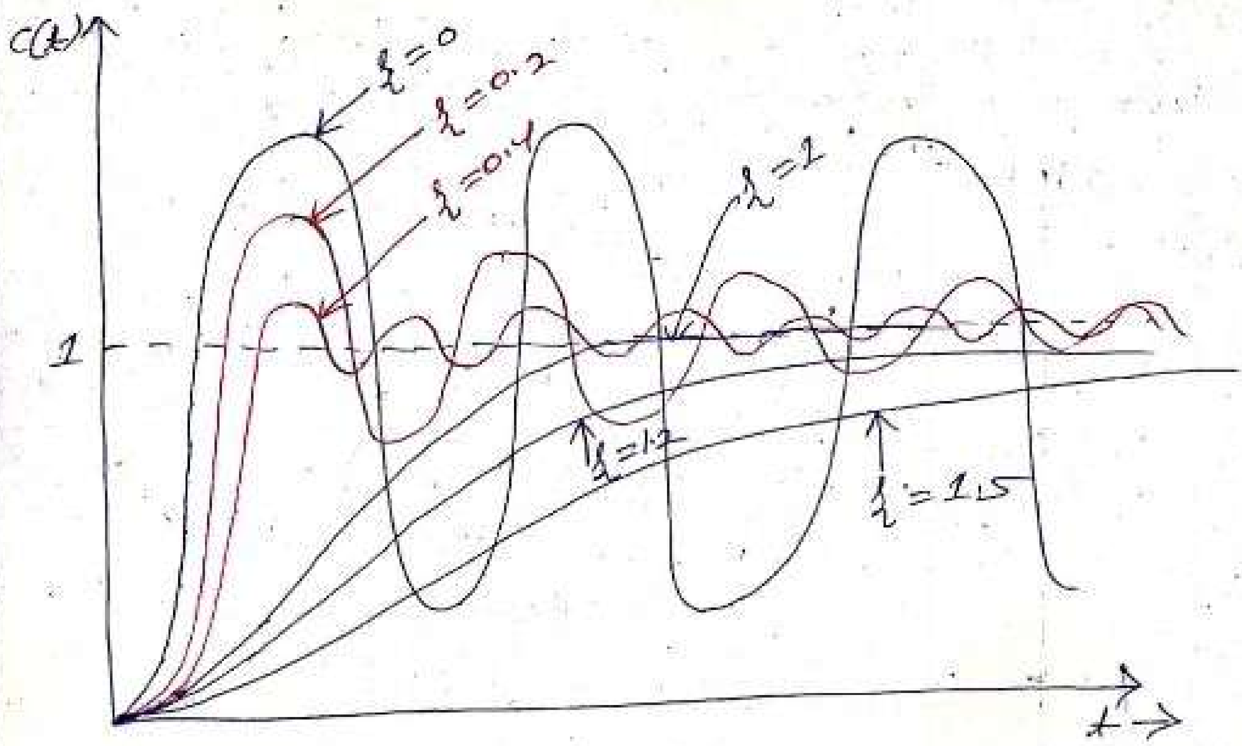

Fig: unit step response of 2nd order system

ξ = 0, undamped system

ξ < 1, underdamped (best)

ξ = 1, critically damped

ξ > 1, overdamped system.

→ As ξ increases the o/p response become sluggish (slowly) and oscillation gradually decreases

ERROR

$$e(t) = r(t) - c(t)$$

$$= 1 - \left\{1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\theta + \omega_d t)\right]\right\}$$

$$\boxed{e(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\theta+\omega_d t)\right]}$$

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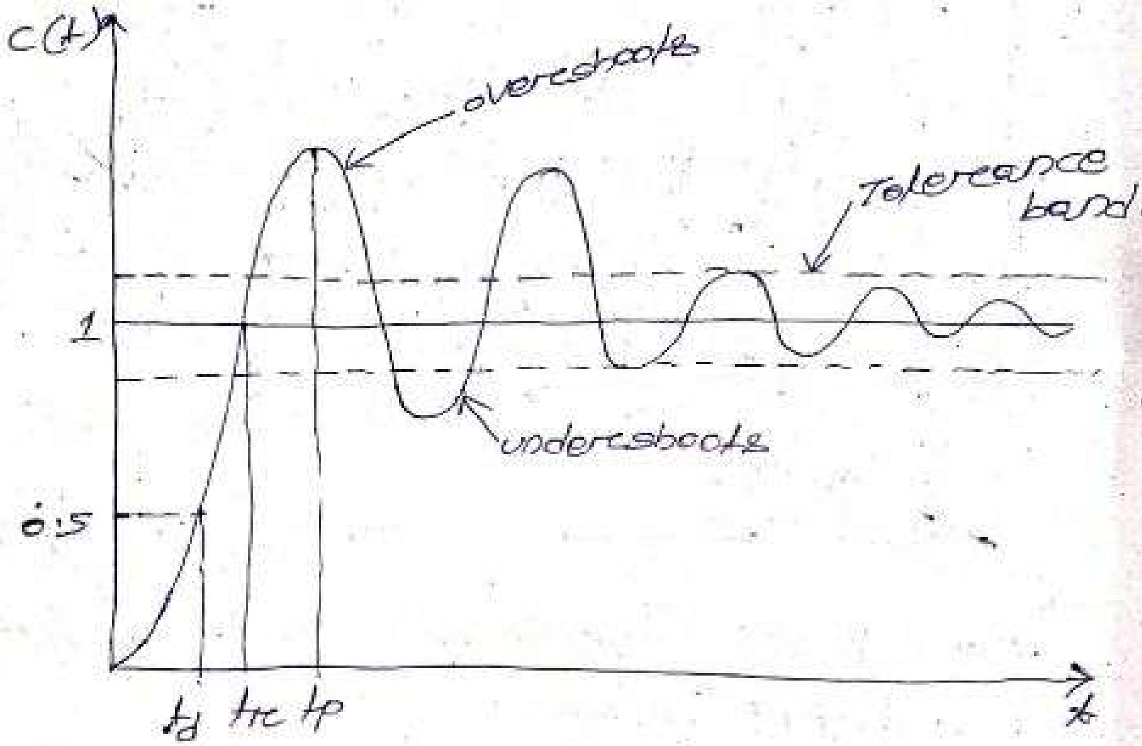
TIME RESPONSE SPECIFICATION

second order control system are designed with damping ratio $\xi < 1$ (underdamped system)



→ The o/p of the underdamped system for unit step input is oscillatory in nature. It contains no. of overshoots and undershoots.

DELAY TIME (t_d)

It is the time required for the response to reach the 50% of the final value for the first time.

RISE TIME (t_r)

It is the time required for the response to reach from zero % to 100% of the final value for a underdamped system.

→ For overdamped system it is the time required for the response to rise from 10% to 90% of its final value.

PEAK TIME (t_p)

It is the time required for the response to reach the first peak of the overshoot.

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PEAK OVERSHOOT (M_p)

The peak or maximum overshoot is defined as the maximum peak value of the response measured from unity (100 %)

→ If the final steady state value is not unity then maximum % overshoot is calculated.

$$\text{Maximum \% overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100 \%$$

SETTLING TIME

The settling time is the time required for the response to reach and stay with in a particular tolerance band limit.

STEADY-STATE ERROR

It is the error between input and the desired o/p as a time tends to infinity

$$e_{ss}(t) = \lim_{t \to \infty} e(t)$$

$$= \lim_{t \to \infty} [r(t) - c(t)]$$

DERIVATION OF EXPRESSION FOR RISE TIME

→ It is the time required for the response to reach from zero % to 100 % of the final value for a underdamped system.

→ For overdamped system it is the time required for the response to rise from 10 % to 90 % of its final value.

→ At rise time $t = t_r$

$$\& \ c(t_r) = 1$$

For second order unit step response

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\theta + \omega_d t)$$

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At rise time

$$c(t_r) = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta)$$

$$\Rightarrow 1 = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta)$$

$$\Rightarrow \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

$$\Rightarrow \sin(\omega_d t_r + \theta) = 0$$

$$\Rightarrow \sin(\omega_d t_r + \theta) = \sin\pi$$

$$\Rightarrow \omega_d t_r + \theta = \pi$$

$$\Rightarrow \omega_d t_r = \pi - \theta$$

$$\Rightarrow \boxed{t_r = \frac{\pi - \theta}{\omega_d}}$$

$$\Rightarrow \boxed{t_r = \frac{\pi - \theta}{\omega_n\sqrt{1-\xi^2}}}$$

$$\Rightarrow \boxed{t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}}$$

→ The above is the expression for ~~to~~ ~~peak~~ rise time(t_r) when unit step input is given to the system.

DERIVATION OF EXPRESSION FOR PEAK TIME (t_p)

→ It is the time required for the response to reach the first peak of the overshoot.

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→ so at peak time $t = t_p$ the slope of $c(t)$ must be zero.

$$\left.\frac{dc(t)}{dt}\right|_{t=t_p} = 0$$

$$\Rightarrow \frac{d}{dt}\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \theta)\right]_{t=t_p} = 0$$

$$\Rightarrow \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cos(\omega_d t_p + \theta)\,\omega_d + \sin(\omega_d t_p + \theta)\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t_p}(-\xi\omega_n) = 0$$

$$\Rightarrow \omega_d \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cos(\omega_d t_p + \theta) - \xi\omega_n \sin(\omega_d t_p + \theta)\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} = 0$$

$$\Rightarrow \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left(\omega_d\cos(\omega_d t_p + \theta) - \xi\omega_n \sin(\omega_d t_p + \theta)\right) = 0$$

$$\Rightarrow \omega_d\cos(\omega_d t_p + \theta) - \xi\omega_n \sin(\omega_d t_p + \theta) = 0$$

⇒ by putting $\omega_d = \sin\theta$ and $\xi\omega_n = \cos\theta$ we get

$$\Rightarrow \sin\theta\cos(\omega_d t_p + \theta) - \cos\theta\sin(\omega_d t_p + \theta) = 0$$

$$\Rightarrow \sin(\theta - (\omega_d t_p + \theta)) = 0$$

$$\Rightarrow \sin(-\omega_d t_p) = 0$$

$$\Rightarrow -\sin(\omega_d t_p) = 0$$

$$\Rightarrow \sin(\omega_d t_p) = 0$$

$$\Rightarrow \sin(\omega_d t_p) = \sin\pi$$

$$\Rightarrow \omega_d t_p = \pi$$

$$\Rightarrow \boxed{t_p = \frac{\pi}{\omega_d}}$$

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$$\Rightarrow \boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

→ The above expression for peak time for a second order unit step response.

$\sin\theta = \omega_d$

$\cos\theta = \xi\omega_n$

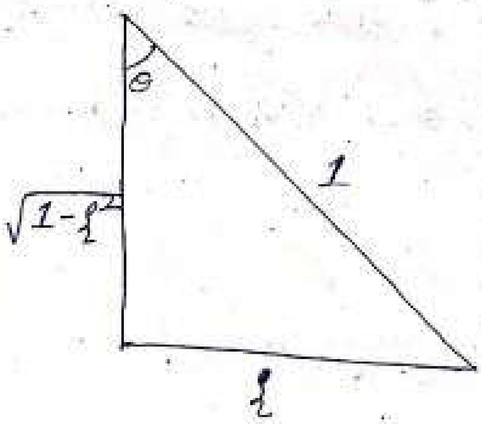
$$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\omega_d}{\xi\omega_n}$$

$$\Rightarrow \tan\theta = \frac{\omega_n\sqrt{1-\xi^2}}{\xi\omega_n}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

NOTE



$\sin\theta = \frac{p}{h} = \sqrt{1-\xi^2}$

$\cos\theta = \frac{b}{h} = \xi$

$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$

EXPRESSION FOR PEAK OVERSHOOT

→ Defination

→ Diagram

→ The peak overshoot is difference between peak value and the reference input (100 %)

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$$M_p = c(t_p) - 1$$

$$= \left[1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)\right] - 1$$

$$= -\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$$

→ At maximum peek overshoot the time is t_p. we have already derive that $t_p = \frac{\pi}{\omega_d}$ or $\frac{\pi}{\omega_n\sqrt{1-\xi^2}}$ substituteing the value of t_p we get.

$$M_p = -\frac{e^{-\xi\omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left(\omega_d \times \frac{\pi}{\omega_d} + \theta\right)$$

$$M_p = -\frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \sin(\pi + \theta)$$

$$= -\frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} (-\sin\theta)$$

$$= \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \times \sin\theta$$

$$= \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \times \sqrt{1-\xi^2} \quad \left(\because \sin\theta = \sqrt{1-\xi^2}\right)$$

$$\boxed{M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

→ The above expression for maximum peek overshoot.

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→ The % of peak overshoot can be given by

% of peak overshoot $= e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$

SETTLING TIME (t_s)

settling time (t_s) is calculated for two tolerance criterion i.e 2% and 5%.

$$t_s = 4T = \frac{4}{\sigma} = \frac{4}{\xi\omega_n} \text{ (2\% criterion)}$$

$$t_s = 3T = \frac{3}{\sigma} = \frac{3}{\xi\omega_n} \text{ (5\% criterion)}$$

→ For 2% criterion t_s reaches minimum value around $\sigma = 0.76$ and for 5% criterion t_s reaches minimum value for $\sigma = 0.68$

→ t_s is ~~~~ inversely propositional to ω_n for a given value of ξ.

~~EXPLAIN~~ EXPRESSION FOR STEADY STATE ERROR

The output of second order underdamped system excited by unit step input signal can be given by.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$e(t) = r(t) - c(t)$$

$$= 1 - \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)\right]$$

$$e(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

→ so steady state error can be given by

$$e_{ss}(t) = \lim_{t\to\infty} e(t)$$

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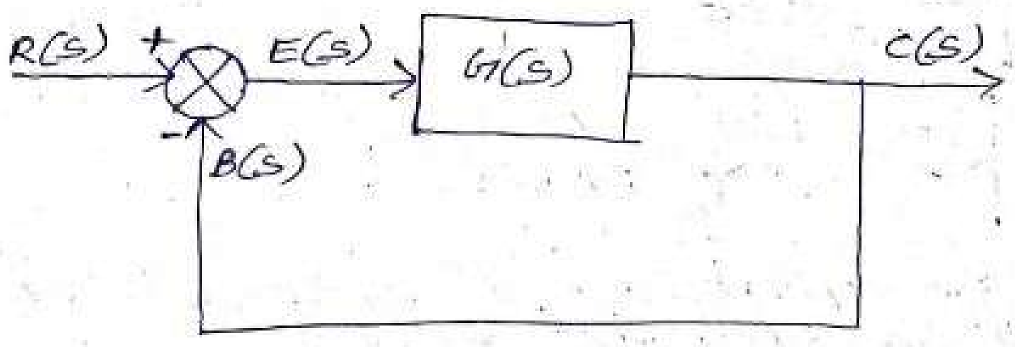
$$= \lim_{t \to \infty} \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$e_{ss}(t) = 0$

→ so for second order system the steady state error for unit step response is zero.

STEADY STATE ERROR & ERROR CONSTANT

The block diagram of a unit feed back system can be given by:-



→ The transfer function of a closeloop system can be given by.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) H(s)}$$

$$= \frac{G(s)}{1 + G(s)} \quad (\because H(s) = 1)$$

$$\Rightarrow C(s) = \frac{R(s) G(s)}{1 + G(s)}$$

C(s) can also be given by

$$C(s) = E(s) G(s)$$

$$\Rightarrow E(s) = \frac{C(s)}{G(s)}$$

$$\Rightarrow E(s) = \frac{\frac{R(s) G(s)}{1 + G(s)}}{G(s)}$$

$$= \frac{R(s)G(s)}{1+G(s)} \times \frac{1}{G(s)}$$

$$\Rightarrow \boxed{E(s) = \frac{R(s)}{1+G(s)}}$$

so the steady state errore can be given by

$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$\boxed{e_{ss} = \lim_{s\to 0} sE(s)}$$

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G(s)}}$$

→ so e_{ss} depends on input R(s) and the foreword path transfere function G(s)

→ so by changing the input R(s) we can get different expression fore ess.

ERROR CONSTANT

(i) STATIC POSITION ERROR CONSTANT

Fore unit step input signal $r(t) = 1$

$$\Rightarrow R(s) = \frac{1}{s}$$

so $e_{ss} = \lim_{s\to 0} sE(s)$

$$= \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

$$= \lim_{s\to 0} \frac{s\times\frac{1}{s}}{1+G(s)} \quad (\because R(s) = \frac{1}{s})$$

$$= \lim_{s\to 0} \frac{1}{1+G(s)}$$

$$\Rightarrow e_{ss} = \frac{1}{1+\lim_{s\to 0} G(s)}$$

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$$= \frac{1}{1+G(0)}$$

$$e_{ss} = \frac{1}{1+k_p}$$

where, k_p is known as position error constant

$$k_p = \lim_{s \to 0} G(s)$$

$$= G(0)$$

ii. STATIC VELOCITY ERROR CONSTANT

The ramp input can be given by

$$r(t) = t$$

$$R(s) = \frac{1}{s^2} \; (\because \text{unit ramp function } A = 1)$$

so steady state error can be given by

$$e_{ss} = \lim_{s \to 0} s\,E(s)$$

$$= \lim_{s \to 0} \frac{s \cdot R(s)}{1+G(s)}$$

$$= \lim_{s \to 0} \frac{s \times \frac{1}{s^2}}{1+G(s)}$$

$$= \lim_{s \to 0} \frac{1/s}{1+G(s)}$$

$$= \lim_{s \to 0} \frac{1}{s+sG(s)}$$

$$\Rightarrow e_{ss} = \frac{1}{0+\lim_{s \to 0} s \cdot G(s)}$$

$$= \frac{1}{\lim_{s \to 0} s \cdot G(s)}$$

$$e_{ss} = \frac{1}{k_v}$$

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where, k_v is known as velocity error constant

and $$k_v = \lim_{s \to 0} s \cdot G(s)$$

3. STATIC ACCLERATION ERROR CONSTANT

The unit parabolic input

$$r(t) = \frac{t^2}{2}$$

$$R(s) = \frac{1}{s^3}$$

so the steady state error can be given by

$$e_{ss} = \lim_{s \to 0} s \cdot E(s)$$

$$= \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s)}$$

$$= \lim_{s \to 0} \frac{s \times \frac{1}{s^3}}{1 + G(s)}$$

$$= \lim_{s \to 0} \frac{1/s^2}{1 + G(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)}$$

$$\Rightarrow e_{ss} = \frac{1}{0 + \lim_{s \to 0} s^2 G(s)}$$

$$= \frac{1}{\lim_{s \to 0} s^2 G(s)}$$

$$e_{ss} = \frac{1}{k_a}$$

where k_a is known as accleration error constant and $$k_a = \lim_{s \to 0} s^2 G(s)$$

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NOTE :-

$$k_p = \lim_{s\to 0} G(s)$$

$$k_v = \lim_{s\to 0} s \cdot G(s)$$

$$k_A = \lim_{s\to 0} s^2 \cdot G(s)$$

TYPES OF CONTROL SYSTEM

The open loop transfer function of a unit feedback system can be written in two forms i.e

$$G(s) = \frac{k(1+T_{z_1}s)(1+T_{z_2}s)(1+T_{z_3}s)\cdots}{s^n(1+T_{P_1}s)(1+T_{P_2}s)(1+T_{P_3}s)\cdots}$$

(Time constant form)

or

$$G(s) = \frac{k'(s+z_1)(s+z_2)(s+z_3)\cdots}{s^n(s+P_1)(s+P_2)(s+P_3)\cdots}$$

(pole-zero form)

→ The term s^n in the denominator of the above equations corresponds to the no. of integrations present in the system.

→ This term helps in determining the steady state error of a system.

→ The control systems are classified w.r.t the no. of integration i.e 'n' present in the open loop transfer function G(s) as below -

Type - 0 system (n = 0, no integration)

Type - 1 system (n = 1, one integration)

Type - 2 system (n = 2, Two integration)

so on - - - -

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1. STEADY STATE ERROR OF TYPE - 0 SYSTEM

For a type-0 system

$$G(s) = \frac{k(1+T_{z_1}s)(1+T_{z_2}s)\cdots}{(1+T_{P_1}s)(1+T_{P_2}s)\cdots}$$

e_{ss} (position)

$$k_p = \lim_{s\to 0} G(s)$$

$$= \lim_{s\to 0} \frac{k(1+T_{z_1}s)(1+T_{z_2}s)\cdots}{(1+T_{P_1}s)(1+T_{P_2}s)\cdots}$$

$$= k$$

$$e_{ss}(\text{position}) = \frac{1}{1+k_p} = \frac{1}{1+k} \text{ (Finite)}$$

e_{ss} (velocity)

$$k_v = \lim_{s\to 0} s \cdot G(s)$$

$$= \lim_{s\to 0} s \times \frac{k(1+T_{z_1}s)(1+T_{z_2}s)\cdots}{(1+T_{P_1}s)(1+T_{P_2}s)\cdots}$$

$$= 0$$

$$e_{ss}(\text{velocity}) = \frac{1}{k_v} = \frac{1}{0} = \infty$$

$$\boxed{e_{ss}(\text{velocity}) = \infty}$$

e_{ss} (acceleration)

$$k_a = \lim_{s\to 0} s^2 \cdot G(s)$$

$$= \lim_{s\to 0} s^2 \times \frac{k(1+T_{z_1}s)(1+T_{z_2}s)\cdots}{(1+T_{P_1}s)(1+T_{P_2}s)\cdots}$$

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$= 0$

$e_{ss}(\text{accleration}) = \frac{1}{k_a} = \frac{1}{0} = \infty$

$$e_{ss}(\text{accleration}) = \infty$$

2 STEADY STATE ERROR TYPE-1 SYSTEM

For a type-1 system

$$G(s) = \frac{k(1+T_{z1}s)(1+T_{z2}s)\cdots}{s(1+T_{P1}s)(1+T_{P2}s)\cdots}$$

e_{ss} (position)

$$k_p = \lim_{s\to 0} G(s)$$

$$= \lim_{s\to 0} \frac{k(1+T_{z1}s)(1+T_{z2}s)\cdots}{s(1+T_{P1}s)(1+T_{P2}s)\cdots}$$

$$= \frac{k}{0} = \infty$$

$e_{ss}(\text{position}) = \frac{1}{1+k_p} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$

$$e_{ss}(\text{position}) = \frac{1}{\infty} = 0$$

e_{ss} (velocity)

$$k_v = \lim_{s\to 0} s \cdot G(s)$$

$$= \lim_{s\to 0} s \times \frac{k(1+T_{z1}s)(1+T_{z2}s)\cdots}{s(1+T_{P1}s)(1+T_{P2}s)\cdots}$$

$$= k$$

$e_{ss}(\text{velocity}) = \frac{1}{k_v} = \frac{1}{k}$

$$e_{ss}(\text{velocity}) = \frac{1}{k} \text{ (finite)}$$

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e_{ss} (accleration)

$$k_a = \lim_{s\to 0} s^2 G(s)$$

$$= \lim_{s\to 0} s^2 \times \frac{k(1+T_{z1}s)(1+T_{z2}s)\cdots}{s(1+T_{p1}s)(1+T_{p2}s)\cdots}$$

$$= 0$$

$$e_{ss}(\text{accleration}) = \frac{1}{k_a} = \frac{1}{0} = \infty$$

$$\boxed{e_{ss}(\text{accleration}) = \infty}$$

3. STEADY STATE ERROR TYPE-2 SYSTEM

For a type-2 system

$$G(s) = \frac{k(1+T_{z1}s)(1+T_{z2}s)\cdots}{s^2(1+T_{p1}s)(1+T_{p2}s)\cdots}$$

e_{ss} (position)

$$k_p = \lim_{s\to 0} G(s)$$

$$= \lim_{s\to 0} \frac{k(1+T_{z1}s)(1+T_{z2}s)\cdots}{s^2(1+T_{p1}s)(1+T_{p2}s)\cdots}$$

$$= \frac{k}{0} = \infty$$

$$e_{ss}(\text{position}) = \frac{1}{1+k_p} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$$

$$\boxed{e_{ss}(\text{position}) = \frac{1}{\infty} = 0}$$

e_{ss} (velocity)

$$k_v = \lim_{s\to 0} s \cdot G(s)$$

$$= \lim_{s\to 0} s \times \frac{k(1+T_{z1}s)(1+T_{z2}s)\cdots}{s^2(1+T_{p1}s)(1+T_{p2}s)\cdots}$$

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$= \frac{k}{0} = \infty$

$e_{ss}(\text{velocity}) = \frac{1}{\infty} = 0$

ess (accleration)

$$k_a = \lim_{s \to 0} s^2 G(s)$$

$$= \lim_{s \to 0} s^2 \times \frac{k(1+T_{z1}s)(1+T_{z2}s)\cdots}{s^2(1+T_{p1}s)(1+T_{p2}s)\cdots}$$

$$= k$$

$$e_{ss}(\text{accleration}) = \frac{1}{k_a} = \frac{1}{k}$$

$$e_{ss}(\text{accleratio}) = \frac{1}{k} \text{ (finite)}$$

STEADY STATE ERROR FOR VARIOUS INPUTS & SYSTEM

Inputs	Type-0	Type-1	Type-2	
unit step (1/s)	$\frac{1}{1+k_p} = \frac{1}{1+k}$	0	0	position error
unit Ramp ($1/s^2$)	∞	$\frac{1}{k_v} = \frac{1}{k}$	0	velocity error
unit parabolic ($1/s^3$)	∞	∞	$\frac{1}{k_a} = \frac{1}{k}$	accleration error

→ For a type-0 system, it has constant position error and infinite velocity and accleration error.

→ For a type-1 system, it has zero position error, finite velocity error and infinite accleration error.

→ For a type-2 system, it has zero position error and velocity error and finite accleration error.

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EFFECTS OF ADDING POLES AND ZEROES TO TRANSFER FUNCTION

→ By adding poles to the char. equation the transient response and stability of the system can be varied. zeroes of transfer function are also very important which may be added to the transfer function to achive the satisfactory performance.

1. ADDING POLES TO OPEN LOOP TRANSFER FUNCTION

→ If pole is added in the forward path transfer function then

* overshoot increases
* stability decreases
* It also increase rise time of step response

2. ADDING POLE ~~ZERO~~ TO CLOSE LOOP TRANSFER FUNCTION

→ By adding pole to the close loop transfer function

* over~~-~~shoot decreases
* increases rise time of step response

3. ADDING ZERO TO OPEN LOOP TRANSFER FUNCTION

→ when zero is added which is very far away from imaginary axis then the overshoot is large and the damping is poor.

→ The overshoot is reduced and ~~imaginary~~ damping improves when zero moves towards right and closer to the origin.

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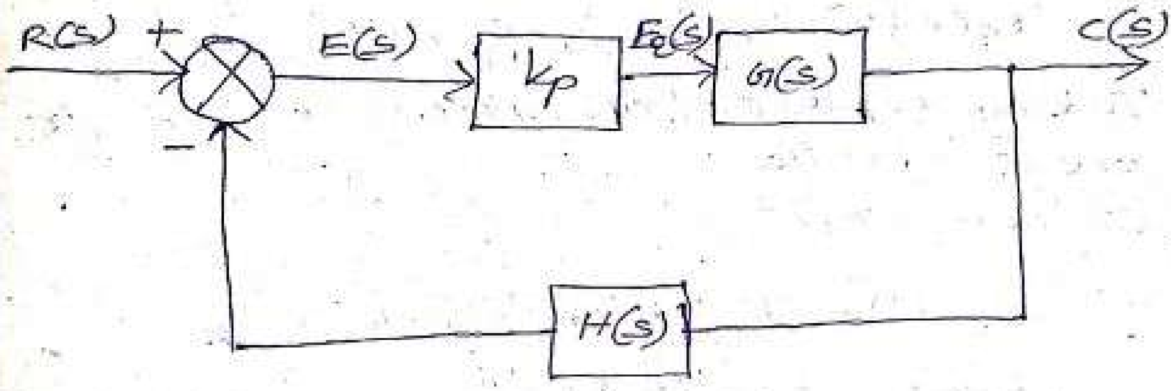
4 ADDING OF ZERO TO CLOSE LOOP TRANSFER FUNCTION

When zero is added to close loop transfer function rise time decreases but maximum overshoot increases.

RESPONSE WITH P, PI, PD & PID CONTROLLER

PROPORTIONAL CONTROL (P-CONTROLLER)

In this type of control the actuating signal $E_a(s)$ is proportional to the error signal $E(s)$. Hence this control system is known as proportional control system.



$$E_a(s) \propto E(s)$$

$$\Rightarrow E_a(s) = k_p E(s)$$

$$\Rightarrow \boxed{k_p = \frac{E_a(s)}{E(s)}}$$

→ where k_p is known as proportional gain

→ By adding k_p in the forward path then forward path gain is increased. The sluggish over damped system can be made faster by increasing the forward path gain.

→ This type of controller is known as 'p' ~~can reduce steady state error~~ controller or proportional control

→ 'p' controller can reduce steady state error

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DERIVATIVE CONTROL (PD-CONTROLLER)

→ For derivative error compensation, the actuating signal $E_a(s)$ consist of proportional error signal and the actuating signal is also propositional to derivative of the error signal.

→ A controller producing such type of signal is known as propositional plus derivative controlled or PD control.

→ we can write

$$e_a(t) = e(t) + T_d \frac{d}{dt} e(t)$$

Taking Laplace transformation of the above equation
we can write

$$E_a(s) = E(s) + K_d s \cdot E(s)$$

$$\Rightarrow E_a(s) = [1 + s K_d] E_s$$

→ The block diagram of a derivative control system with PD-controlled can be given by.

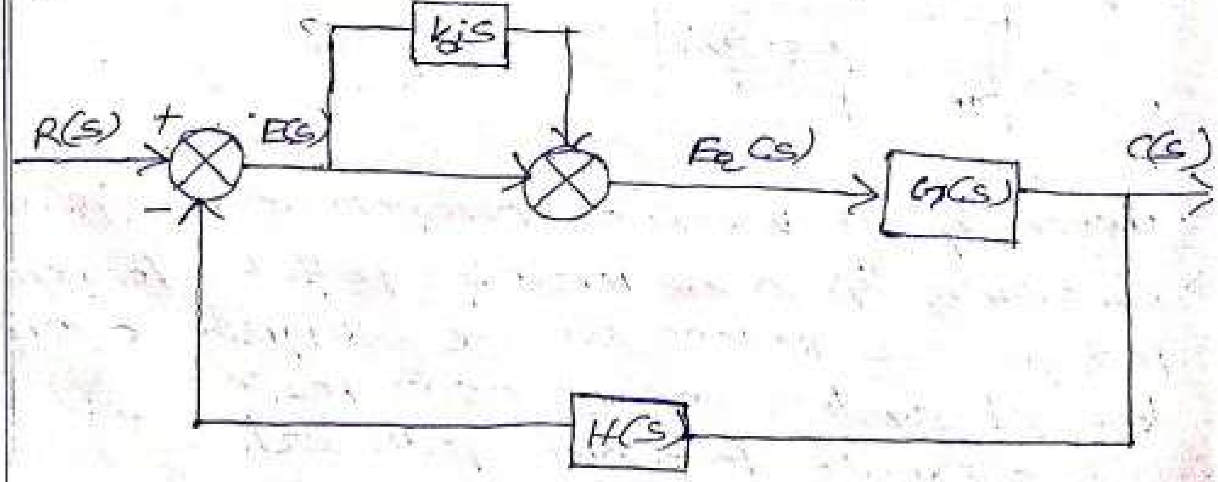

Fig: Block diagram of PD controller/ derivative control system.

→ k_d is known as derivative control gain

→ so derivative control can increase the damping ratio of the system for this region. The maximum overshoot decreases by using derivative control.

INTEGRAL CONTROL (PI CONTROLLER)

→ In this controller the actuating signal consist of proportional error signal and the ~~term~~ term proportional to intrigration of the error.

→ This type of controller is known as proportional plus integral controller or PI controller.

~~Hence~~,

$$e_a(t) \propto e(t)$$

$$e_a(t) \propto \int e(t)\,dt$$

Here,

$$e_a(t) = e(t) + k_i \int e(t)\,dt$$

→ By taking Laplace transform~~ation~~ of the above equation.

$$\Rightarrow E_a(s) = E(s) + \frac{k_i}{s} E(s)$$

$$\Rightarrow E_a(s) = \left[1 + \frac{k_i}{s}\right] E(s)$$

→ The block diagram of integral control system can be given by

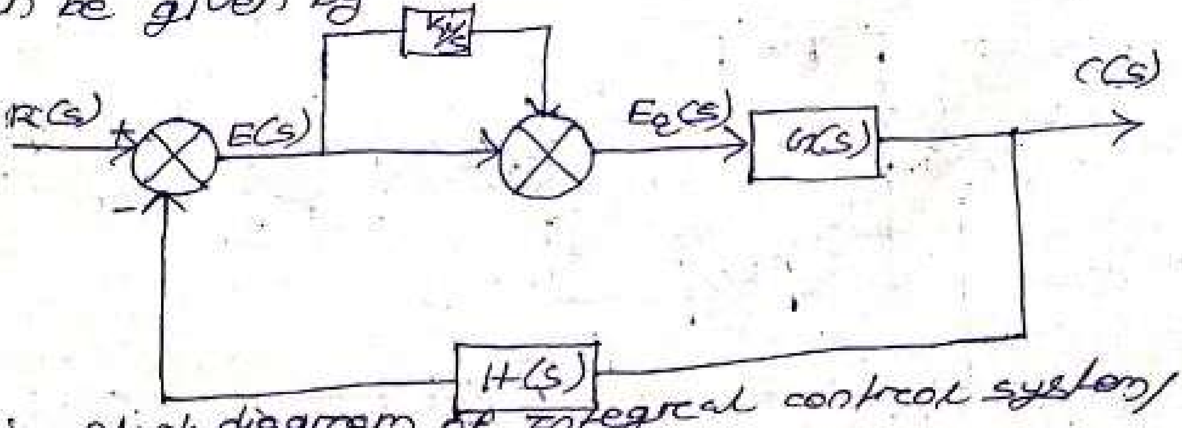

Fig: Block diagram of integral control system/ PI controller

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→ k_i is known as integral controller gain.

→ The chare. equation of the system is changed by adding intergal control.

→ The chare. equation degree or order is increase by one to intergal control.

→ The integral controller is used to meet high accuracy requirement.

→ The steady state error value is reduced by this controller.

PID CONTROLLER

→ In this type of controller the actuating signal consist of three terms i.e proportional error, propositional to the ~~integral~~ integration of the error and propositional to the derivative of the error signal.

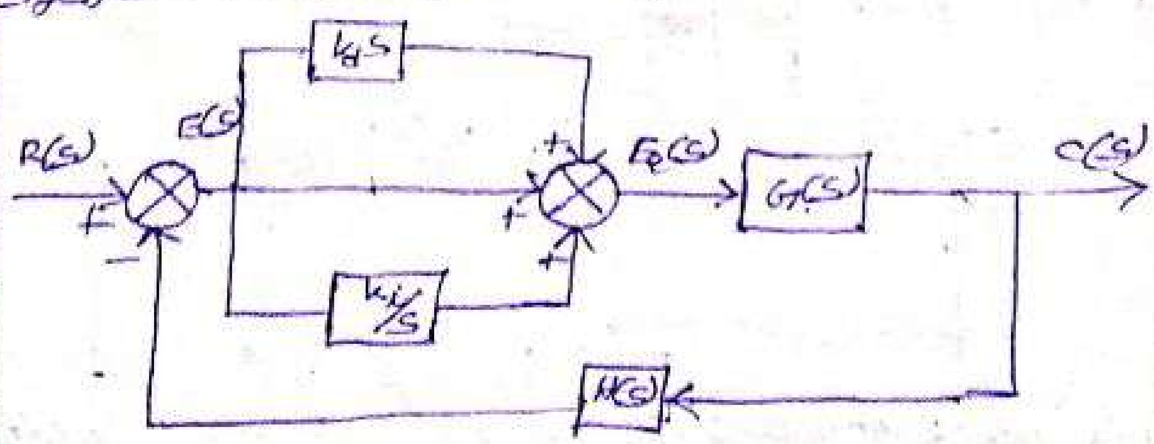
$$e_a(t) \propto e(t) + k_d \frac{d}{dt} e(t) + k_i \int e(t)\, dt$$

→ By taking Laplace transfer of the above equation we get.

$$E_a(s) = E(s) + s k_d E(s) + \frac{k_i}{s} E(s)$$

$$\Rightarrow E_a(s) = \left[1 + s k_d + \frac{k_i}{s}\right] E(s)$$

→ The block diagram of PID controller system can be given by.



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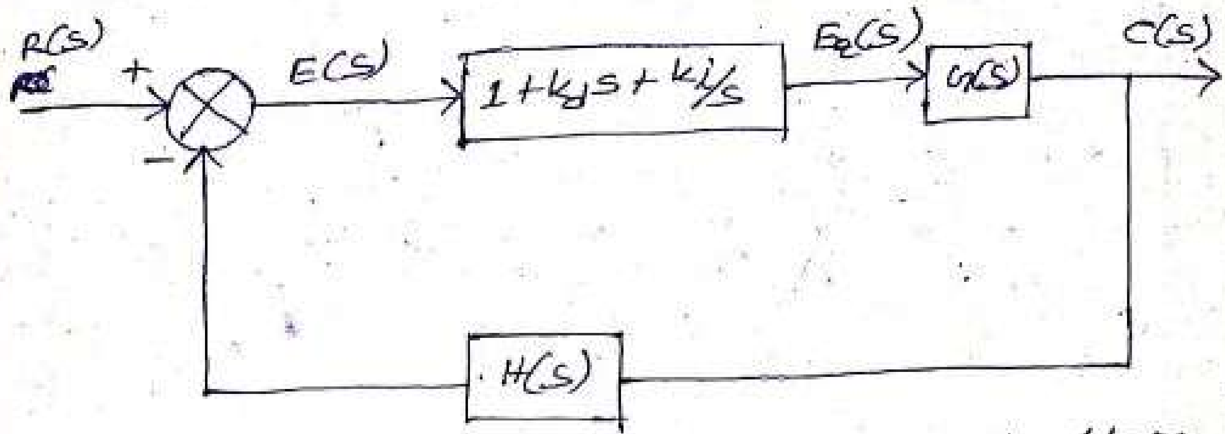

Fig: Block diagram of PID controller system.

→ Here k_d is known as derivative controller gain and k_i is known as integral controller gain.

→ This type controller where k_p, k_d and k_i are present in the controller algorithm is known as PID controller.

→ PID controller has the advantage of proportional, integral and derivative controller so it is the most efficient one.

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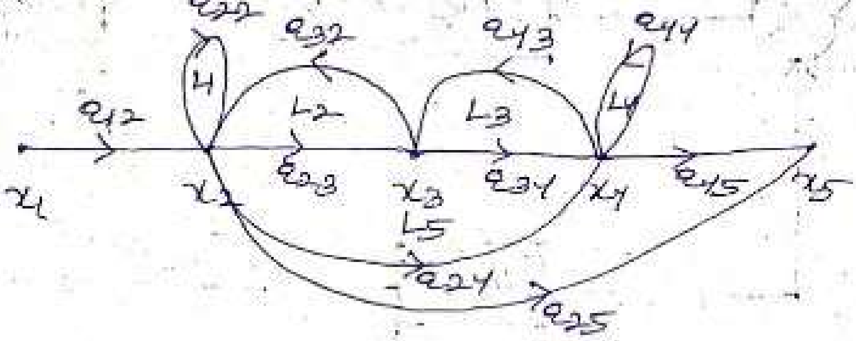
Problem



→ The forward path in SFG are-

$M_1 (x_1 - x_2 - x_3 - x_4 - x_5) = a_{12} a_{23} a_{34} a_{45}$

$M_2 (x_1 - x_2 - x_4 - x_5) = a_{12} a_{24} a_{45}$

$M_3 (x_1 - x_2 - x_5) = a_{12} a_{25}$

→ The individual loop present in SFG are-

$L_1 (x_2 - x_2) = a_{22}$

$L_2 (x_2 - x_3 - x_2) = a_{23} a_{32}$

$L_3 (x_3 - x_4 - x_3) = a_{34} a_{43}$

$L_4 (x_4 - x_4) = a_{44}$

$L_5 (x_2 - x_4 - x_3 - x_2) = a_{24} a_{43} a_{32}$

→ The possible combination of two non-touching loops are.

$L_{13} = a_{22} a_{34} a_{43}$

$L_{14} = a_{22} a_{44}$

$L_{42} = a_{44} a_{23} a_{32}$

→ There are no possible combination of 3 non-touching loop so four non touching loop and furthermore does not exist.

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_{13} + L_{14} + L_{42})$

$$\Delta = 1 - e_{22} - e_{23}e_{32} - e_{34}e_{43} - e_{44} - e_{24}e_{43}e_{32} + e_{22}e_{34}e_{43} + e_{22}e_{44} + e_{44}e_{23}e_{32}$$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - 0 = 1$

$\Delta_3 = 1 - (L_3 + L_4) = 1 - ~~e_{34}e_{43}e_{44}~~ \; e_{34}e_{43} - e_{44}$

→ By applying Mason's gain formula

$$\Delta = \frac{\sum_K \Delta_K M_K}{\Delta} = \frac{\Delta_1 M_1 + \Delta_2 M_2 + \Delta_3 M_3}{\Delta}$$

$$= \frac{e_{12}e_{23}e_{34}e_{45} + e_{12}e_{24}e_{45} + (1 - e_{34}e_{43} - e_{44})(e_{12}e_{25})}{1 - e_{22} - e_{23}e_{32} - e_{34}e_{43} - e_{44} - e_{24}e_{43}e_{32} + e_{22}e_{34}e_{43} + e_{22}e_{44} + e_{44}e_{23}e_{32}}$$

$$\Delta = \frac{e_{12}e_{23}e_{34}e_{45} + e_{12}e_{24}e_{45} + e_{12}e_{25} - e_{12}e_{25}e_{34}e_{43} - e_{44}e_{12}e_{25}}{1 - e_{22} - e_{23}e_{32} - e_{34}e_{43} - e_{44} - e_{24}e_{43}e_{32} + e_{22}e_{34}e_{43} + e_{22}e_{44} + e_{44}e_{23}e_{32}}$$

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CH-3

ROOT LOCUS TECHNIQUE

CONCEPT

It is a technique which provides graphical method of plotting the locus of the roots on the s-plane.

$$s = \sigma \pm j\omega$$

where, σ = Real part

$j\omega$ = Imaginary part

→ The root locus is plot from the roots of the characteristics equation of a close loop system. In the s-plane where the parameter (k) varied from zero to ∞.

→ The valu of the parameters for a desire root location can be determined from the root locus.

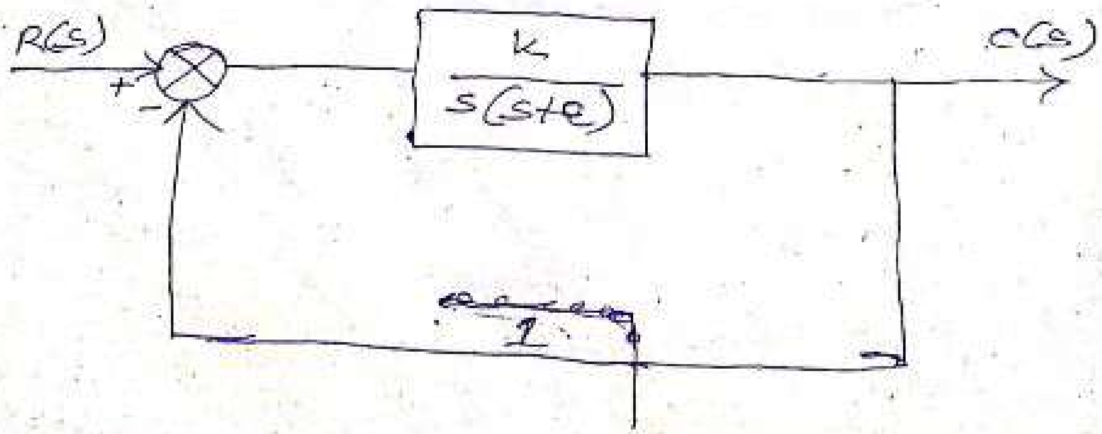
→ The designer can easyly visualise the effect of varing the system parameters on the root locus.

ROOT LOCUS

Let's consider a simple system having open loop transfer function.

$$G(s) = \frac{k}{s(s+a)}$$

→ The block diagram of close loop system can be given by.



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$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)R(s)}$$

$$= \frac{\frac{k}{s(s+a)}}{1+\frac{k}{s(s+a)}} = \frac{\frac{k}{s(s+a)}}{\frac{s(s+a)+k}{s(s+a)}}$$

$$= \frac{k}{s(s+a)+k}$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{k}{s^2+sa+k}$$

→ so the characteristics equation is

$$s^2+as+k=0$$

→ The roots of the char. equation are

$$= \frac{-b \pm\sqrt{b^2-4ac}}{2a} = \frac{-a \pm\sqrt{a^2-4\times1\times k}}{2\times1}$$

$$= \frac{-a\pm\sqrt{a^2-4k}}{2} = -a/2 \pm\sqrt{\frac{a^2}{4}-k}$$

$$= -a/2 \pm\sqrt{\left(\frac{a}{2}\right)^2-k}$$

so the roots are

$$s_1 = -a/2 +\sqrt{\left(\frac{a}{2}\right)^2-k}$$

$$s_2 = -a/2 -\sqrt{\left(\frac{a}{2}\right)^2-k}$$

→ It is seen that if the parameter k of the system changes then the system char. equation roots also changes.

→ Let's consider a variable open loop gain k while 'a' is constant.

→ k is varied from '0' to infinity and the two roots s_1 and s_2 are ploted on the root locus over the s-plane.

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(1) $0 \le k < a^2/4$

For this range of k, the roots s_1 and s_2 are real and distict.

→ If $k=0$, then $s_1 = 0$ &
$s_2 = -a$

→ In this condition the system is over damped

(2) $k = a^2/4$

If $k = a^2/4$ then the roots are real & same

→ If $k = a^2/4$ then $s_1, s_2 = -a/2$

→ In this condition the system is called as critically damped system.

(3) $k > a^2/4$ ($a^2/4 < k \le \infty$)

If k value is greater than $a^2/4$ then the roots are complex conjugate numbers.

→ For this condition it has unvarying real part i.e $-a/2$

→ In this condition the system is known as under-damped system.

pole → X
zero → O

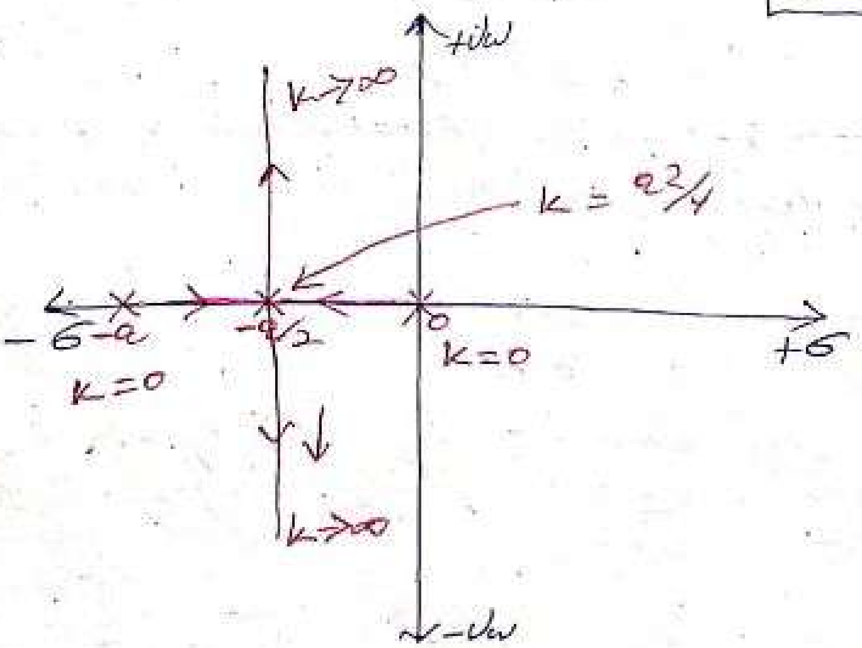
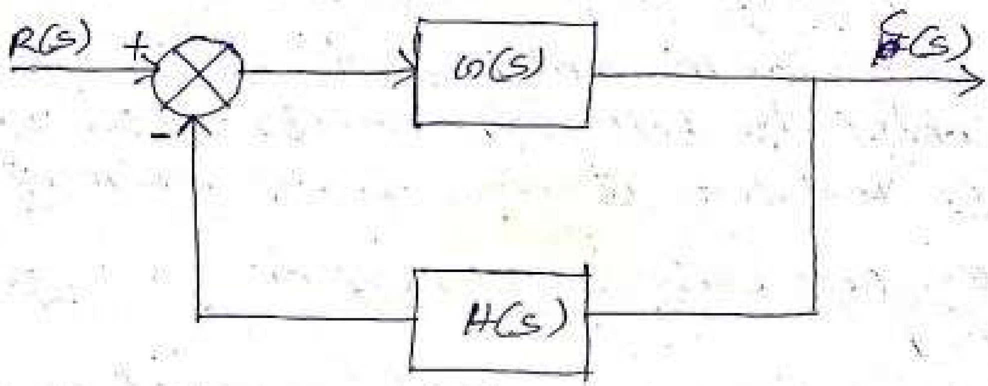


Fig: Root locus for $s^2+as+k=0$ as a function of k

CONSTRUCTION OF ROOT LOCI



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

→ The char: equation of the above system is

$$1+G(s)H(s) = 0$$

$$\Rightarrow G(s)H(s) = -1$$

→ since s is a complex variable the equation $G(s)H(s) = -1$ can be converted into two Evan's condition. which are magnitude condition and phase angle condition.

Magnitude condition (Imp)

$$|G(s)H(s)| = 1$$

phase angle condition (Imp)

$$\angle G(s)H(s) = \pm(2q+1)\pi$$

where $q = 0, 1, 2 \ldots$

→ The root locus point can be determine from magnitude criterion and it can be drawn by using phase angle condition on the s-plane.

$$k = \frac{\text{product of phasor lengths from } s_0 \text{ to all open loop poles}}{\text{product of phasor lengths from } s_0 \text{ to all open loop zeroes.}}$$

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where,

s_0 = any point on the root locus

→ An approximate sketch of the root locus can be obtain by following certain rules known as rules ~~for~~ for the construction of root locus

RULES FOR CONSTRUCTION OF ROOT LOCUS

RULE-1

The root locus is symmetrical about the real axis.

RULE-2

At open loop poles $k=0$ & at open loop zeros $k=\infty$

Explanation

As k varies from zero to infinity, each branch of the root locus originates from an open loop pole $k=0$ and terminates on an open loop zero at $k=\infty$

→ open loop transfer function

$$G(s)H(s) = \frac{k(s+z_1)(s+z_2)------}{(s+P_1)(s+P_2)-------}$$

$$= \frac{k\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+P_j)}$$

→ The charc. equation can be given by

$$1+G(s)H(s) = 0$$

$$\Rightarrow 1+\frac{k\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+P_j)} = 0$$

$$\Rightarrow k\prod_{i=1}^{m}(s+z_i) = -\prod_{j=1}^{n}(s+P_j)$$

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$$\Rightarrow \prod_{j=1}^{n}(s+P_j) + k\prod_{i=1}^{m}(s+z_i) = 0$$

At $k=0$,

$$\prod_{j=1}^{n}(s+P_j) + 0 \times \prod_{i=1}^{m}(s+z_i) = 0$$

$$\Rightarrow \prod_{j=1}^{n}(s+P_j) = 0$$

$$\Rightarrow s+P_j = 0$$

$$\Rightarrow \boxed{s = -P}$$

so at $k=0$ we get open loop poles and the root locus starts from the pole.

At $k=\infty$,

$$\prod_{j=1}^{n}(s+P_j) + k\prod_{i=1}^{m}(s+z_i) = 0$$

$$\Rightarrow k\prod_{i=1}^{m}(s+z_i) = -\prod_{j=1}^{n}(s+P_j)$$

$$\Rightarrow \prod_{i=1}^{m}(s+z_i) = -\frac{\prod_{j=1}^{n}(s+P_j)}{k}$$

$$\Rightarrow \prod_{i=1}^{m}(s+z_i) = 0$$

$$\Rightarrow s+z_i = 0$$

$$\Rightarrow \boxed{s = -z_i}$$

so at $k=\infty$ we get open loop zeros and the root locus terminates at zero

→ If $n>m$ then the open loop transfer function has $(n-m)$ branches of root locus which terminates at zeros.

RULE-3

segment of the real axis having odd no. of real axis open loop poles and zeros to theire right are part of the root locus.

EXPLANATION

→ For root locus to exist at any point on the s-plane the phase angle condition

$\angle G(s)H(s) = \pm(2q+1)\pi$ need to be satisfied.

→ Each pole and zeros on the real axis to the right of any point contribute 180° (π) w.r.t that point.

→ Each pole and zero on the real axis to the left of any point contribut 0° w.r.t that point.

→ complex conjugate poles and zeros contribut 0°.

→ so to satisfy phase angle condition, odd no. of poles and zeros should be present at the right side of the point.

RULE-4

$(n-m)$ branches of root locus goes to infinity along straight line asymptotes whose angle are -

$$\theta_q = \pm\frac{(2q+1)\pi}{(n-m)}$$

where,

$q = 0, 1, 2, \ldots\ldots (n-m-1)$

RULE-5

The asymptotes cross the real axis at a point know as centroid.

$$\text{centroid} = -\sigma = \frac{\text{sum of real parts of open loop poles} - \text{sum of real parts of open loop zeros}}{\text{no. of poles} - \text{no. of zeros}}$$

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RULE-6

The break away point and break-in points of the root locus are the solution of

$$\frac{dk}{ds} = 0$$

→ The root locus must approach or leave the break point on the real axis at angle $\pm \frac{180}{r}$,

where, r = no. of branch approaching or leaving.

RULE-7

Angle of departure from open loop pole is given by $\theta_d = \pm(2q+1)\pi + \varphi$

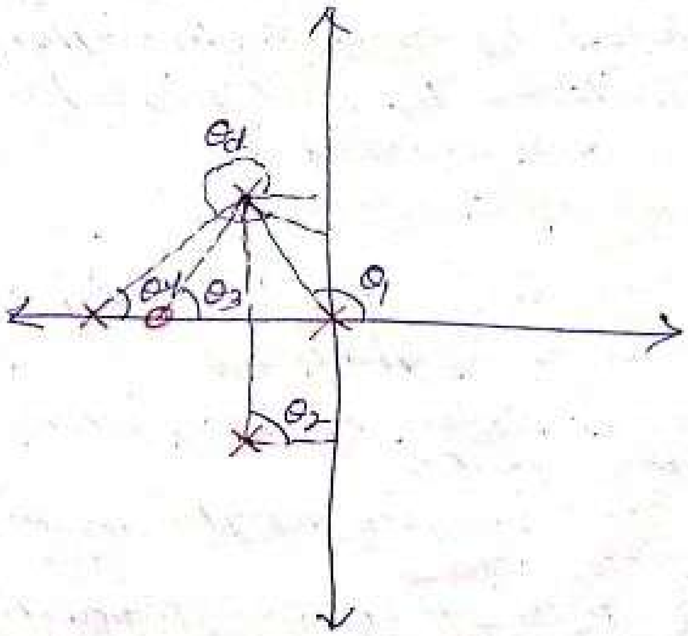
where, $q = 0, 1, 2, 3$ . . . .

→ Angle of arrival can be given by

$$\theta_a = \pm(2q+1)\pi - \varphi$$

where $q = 0, 1, 2$ . . . .

φ = net angle contribution at open loop poles and zeros for all other open loop poles and zeros.



$$\varphi = \theta_3 - (\theta_1 + \theta_2 + \theta_4)$$

$$\theta_d = \pm(2q+1)\pi + \varphi$$

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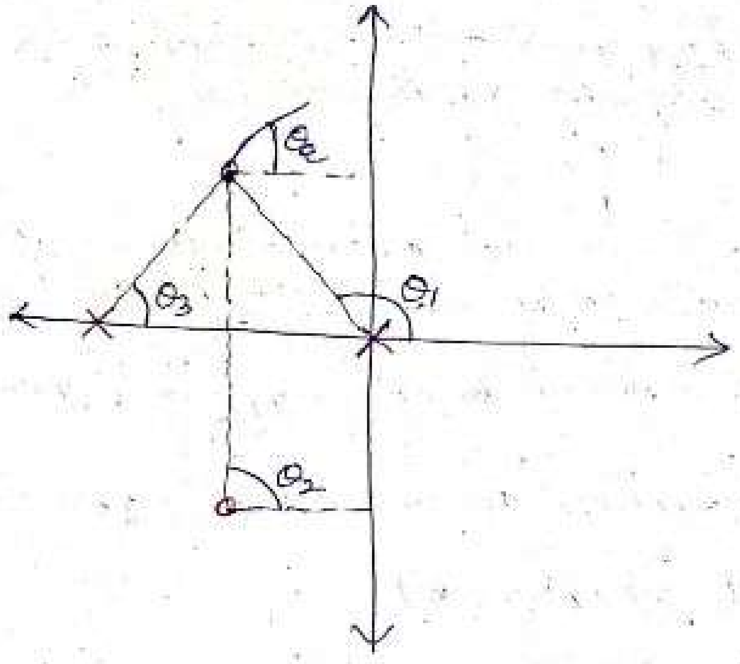

$$\phi = \theta_2 - (\theta_1 + \theta_3)$$

$$\theta_e = \pm(2q+1)\pi - \phi$$

→ The angle of departure and angle of arrival need to be calculated only when there are complex conjugate poles and zeros

RULE-8

The point of intersection of root locus branches with imaginary axis and the critical value of k can be calculated by using Routh criterion or it can be calculated by putting $s = j\omega$ in the char. equation and solving it.

RULE-9

The value of open loop gain k at any point S_0 on the root locus is given by

$$k = \frac{\text{product of phasor lengths from } S_0 \text{ to all open loop poles}}{\text{product of phasor lengths from } S_0 \text{ to all open loop zeros.}}$$

$$= \frac{\text{product of length of vector drawn from } S_0 \text{ to all open loop poles}}{\text{product of length of vector drawn from } S_0 \text{ to all open loop zeros.}}$$

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Q/
considere the system with the following open loop transfere function.

$$G(s)\,H(s) = \frac{k(s+3)(s+4)}{(s+1)(s+5)(s+6)}$$

Draw a root locus plot fore it.

Solution

RULE-1

The open loop transfere function has poles
$s = +1$, $s = -5$ & $s = -6$
zeros $s = -3$ & $s = -4$

→ since all the poles and zeros lie on the real axis so root locus is symmetrical about the real axis.

RULE-4

no. of poles $(n) = 3$
no. of zeros $(m) = 2$

since $n > m$, so $n-m = 3-2 = 1$ breench of root locus will travel to infinity (∞)

RULE-5

→ angle of as asymptotes

$$\theta_q = \pm \frac{(2q+1)\pi}{n-m}$$

where $q = 0, 1 \ldots\ldots (n-m-1)$

$$\theta_q = \pm \frac{(2q+1)\pi}{1} \quad \text{where } q = 0$$

$$= \pm (2 \times 0 + 1)\pi$$

$$= \pm \pi = 180^\circ$$

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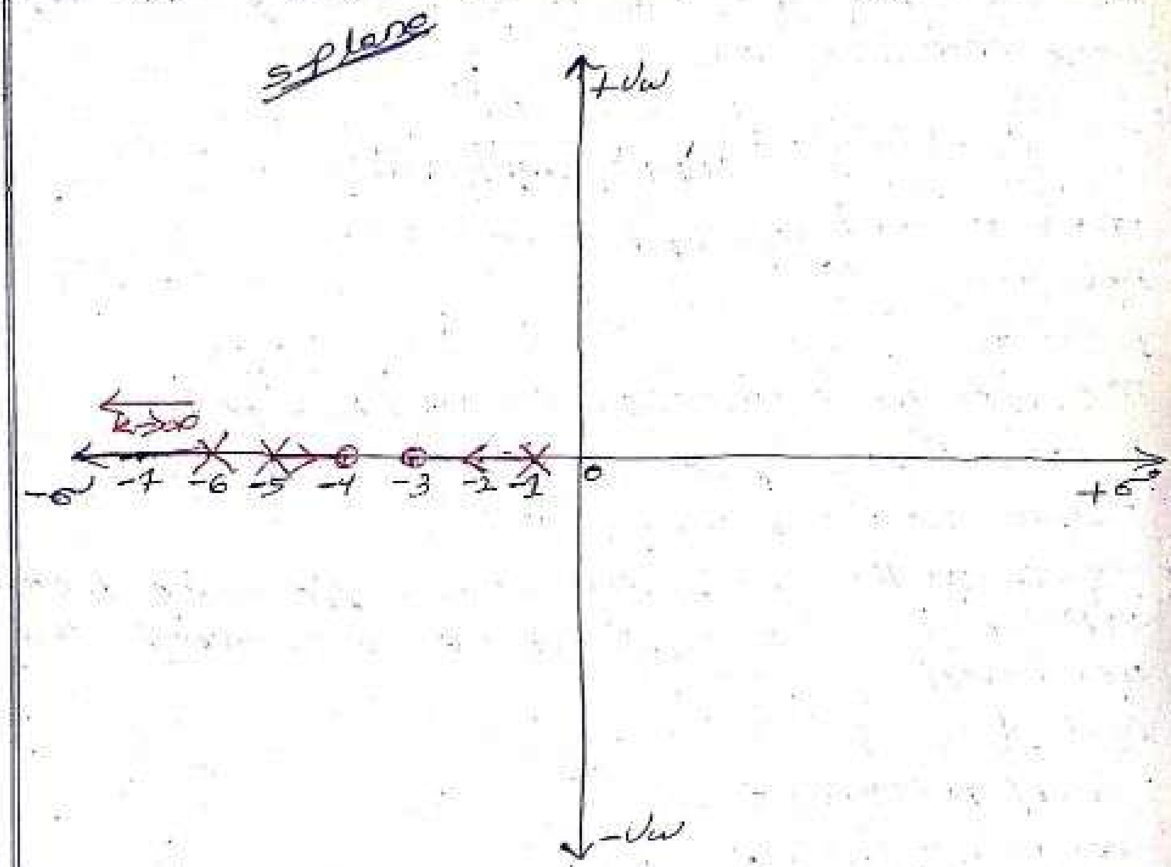
Root locus plot



→ The root locus branches starts from open loop poles and terminates at open loop zeros

→ In the above root locus plot 3-segement of root locus branches present i.e between

$s = -1$ & $s = -3$

between $s = -5$ & $s = -4$

between $s = -6$ & $s = -\infty$

Q-2

Draw a root locus plot for

$$G(s)\,H(s) = \frac{k}{s(s+1)(s+3)}$$

solution

The open loop transfer function contain 3 no. of open loop poles i.e $s = 0$, $s = -1$ and $s = -3$ and it contain no zeros.

so $n = 3$, $m = 0$

→ since $n > m$, $n - m = 3 - 0 = 3$, so 3 branches of root ~~loc~~ locus starts from open

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loop poles end at infinity.

→ There will be 3 asymptotes angle of asymptotes

$$\theta_q = \frac{\pm(2q+1)\pi}{n-m}$$

where $q = 0, 1, 2 \cdots\cdots (n-m-1)$

$= 0, 1, 2$

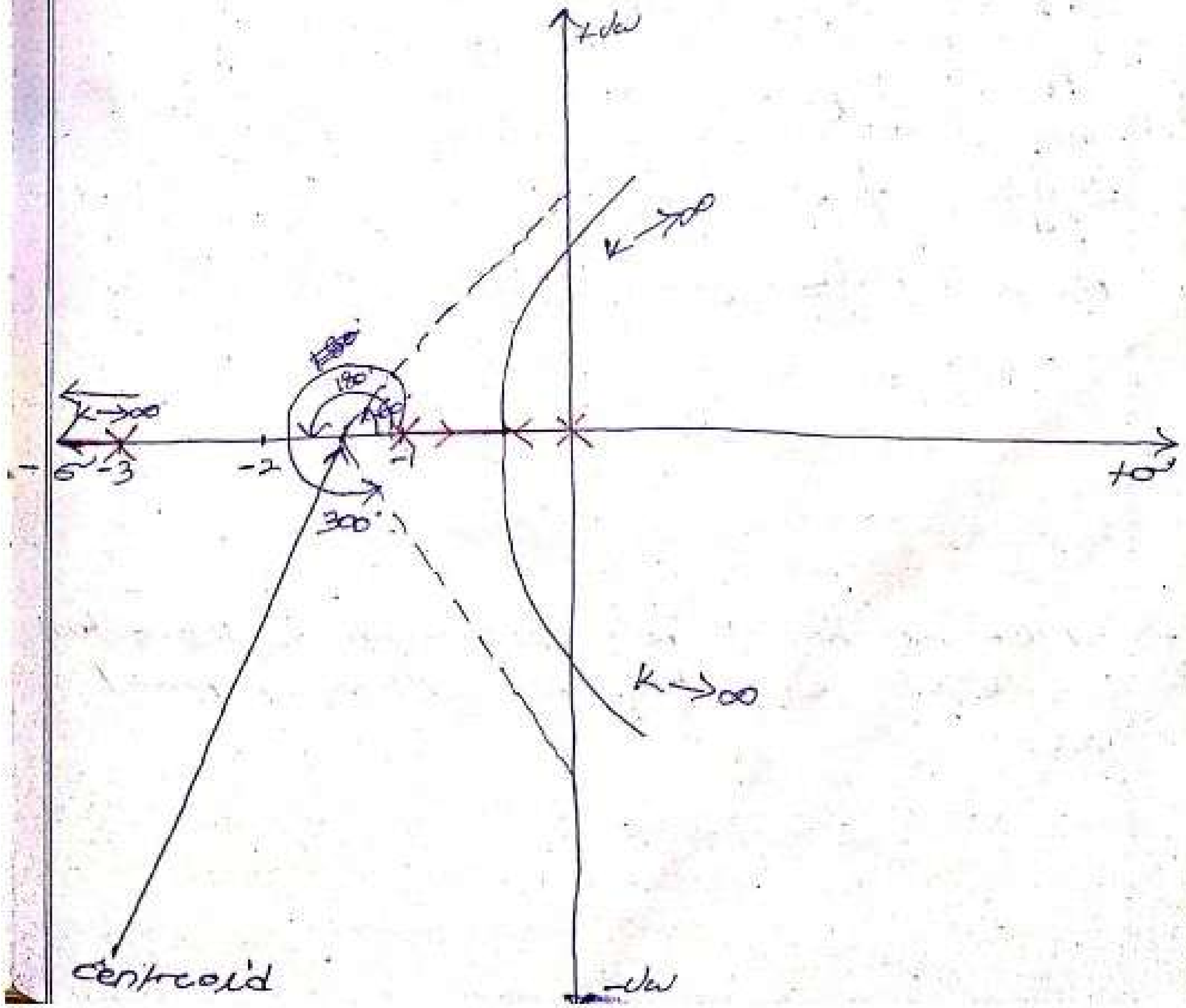
$$\theta_0 = \frac{\pm(2\times0+1)\pi}{3-0} = \frac{\pi}{3} = 60^\circ$$

$$\theta_1 = \frac{\pm(2\times1+1)}{3-0} = \frac{3\pi}{3} = 180^\circ$$

$$\theta_2 = \frac{\pm(2\times2+1)}{3-0} = \frac{5\pi}{3} = 300^\circ$$

$$\text{centroid} = \frac{\text{sum of real part of open loop poles} - \text{sum of real part of open loop zeros}}{\text{No. of poles} - \text{No. of zeros}}$$

$$= \frac{(0-1-3)-0}{3-0} = \frac{-4}{3} = -1.33$$



→ The break away point can be found from

$$\frac{dk}{ds} = 0$$

char. equation

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{k}{s(s+1)(s+3)} = 0$$

$$\Rightarrow \frac{s(s+1)(s+3)+k}{s(s+1)(s+3)} = 0$$

$$\Rightarrow s(s+1)(s+3)+k = 0$$

$$\Rightarrow k = -s(s+1)(s+3)$$

$$\Rightarrow k = (-s^2 - s)(s+3)$$

$$\Rightarrow k = -s^3 - 3s^2 - s^2 - 3s$$

$$\Rightarrow k = -s^3 - 4s^2 - 3s$$

$$\frac{dk}{ds} = \frac{d}{ds}(-s^3 - 4s^2 - 3s)$$

$$\Rightarrow \frac{dk}{ds} = -3s^2 - 8s - 3$$

By putting $\frac{dk}{ds} = 0$

$$-3s^2 - 8s - 3 = 0$$

$$3s^2 + 8s + 3 = 0$$

$$s = -0.45$$

$$s = -2.21$$

→ out of the break point $s = -0.45$ is the actual break point as it ~~has~~ lies with in the root locus.

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PROBLEM

1) For the system represented by the following equations, find the transfer function $\frac{X(s)}{U(s)}$ by the S.F.G technique.

$$x = x_1 + \alpha_0 u \quad \text{---} (1)$$

$$\frac{dx_1}{dt} = -\alpha_1 x_1 + x_2 + \alpha_2 u \quad \text{---} (2)$$

$$\frac{dx_2}{dt} = -\alpha_2 x_1 + \alpha_1 u \quad \text{---} (3)$$

Ans

Taking the Laplace transform of the system equation we get.

$$X(s) = X_1(s) + \alpha_0 U(s) \quad \text{---} (4)$$

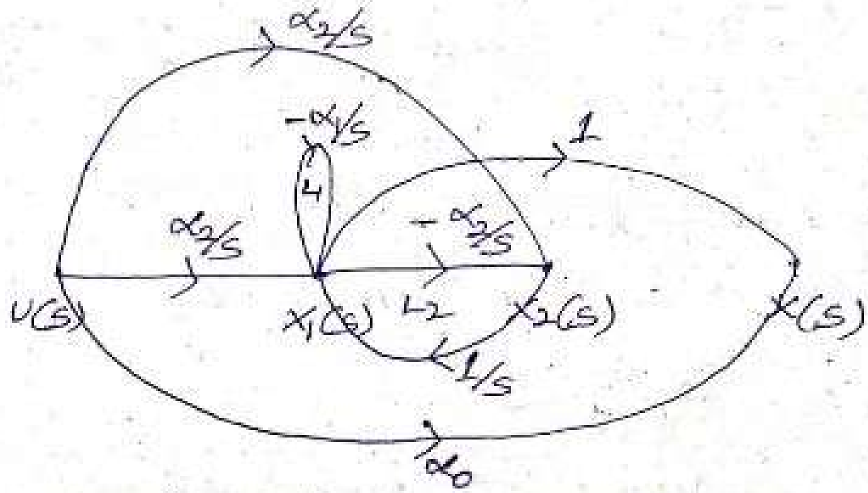
$$sX_1(s) = -\alpha_1 X_1(s) + X_2(s) + \alpha_2 U(s)$$

$$\Rightarrow X_1(s) = -\frac{\alpha_1}{s} X_1(s) + \frac{1}{s} X_2(s) + \frac{\alpha_2}{s} U(s) \quad \text{---} (5)$$

$$sX_2(s) = -\alpha_2 X_1(s) + \alpha_1 U(s)$$

$$\Rightarrow X_2(s) = -\frac{\alpha_2}{s} X_1(s) + \frac{\alpha_2}{s} U(s) \quad \text{---} (6)$$

equation 4, 5 & 6 are used to draw the S.F.G



The Forward path in S.F.G are

$M_1(U(s) - X(s)) = \alpha_0$

$M_2(U(s) - X_1(s) - X_2(s) - X_1(s) - X(s)) = \frac{\alpha_2}{s} \times -\frac{\alpha_2}{s} \times \frac{1}{s} \times 1$

$= -\frac{\alpha_2^2}{s^3}$

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$M_3 (U(s) - X_2(s) - X_1(s) - X(s))$

$$= \alpha_1/s \times 1/s \times 1 = \frac{\alpha_1}{s^2}$$

$M_4 (U(s) - X_1(s) - X(s)) = \alpha_2/s \times 1$

$$= \alpha_2/s$$

The indivisual loop present in S.F.G are

$L_1 = -\alpha_1/s$

$L_2 = -\alpha_2/s \times 1/s = \frac{-\alpha_2}{s^2}$

→ There are no possible combination of 2 non touching loop so 3 non touching loop and further ~~more and~~ more does not exist

$\Delta = 1 - (L_1 + L_2)$

$= 1 - \left(-\alpha_1/s - \frac{\alpha_2}{s^2}\right)$

$= 1 + \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2}$

$\Delta_1 = 1 - \left(\frac{-\alpha_1}{s} - \frac{\alpha_2}{s^2}\right) = 1 + \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2}$

$\Delta_2 = 1 - 0 = 1$

$\Delta_3 = 1 - 0 = 1$

$\Delta_4 = 1 - 0 = 1$

By using Mason's gain formula

$$T = \frac{\Sigma_k \Delta_k M_k}{\Delta}$$

$$= \frac{\Delta_1 M_1 + \Delta_2 M_2 + \Delta_3 M_3 + \Delta_4 M_4}{\Delta}$$

$$= \frac{\left(1 + \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2}\right)\alpha_0 + \left(-\frac{\alpha_2^2}{s^3}\right) + \frac{\alpha_1}{s^2} + \frac{\alpha_2}{s}}{1 + \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2}}$$

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$$= \frac{\left(1 + \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2}\right)\alpha_0 - \frac{\alpha_2^2}{s^3} + \frac{\alpha_1}{s^2} + \frac{\alpha_2}{s}}{1 + \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2}}$$

Root locus

PROBLEM

Draw the root locus plot for the following open loop transfer function

$$G(s)H(s) = \frac{k}{s(s+2)(s^2+2s+5)}$$

Solution

There is no zeros in the open loop transfer function. The poles of the transfer function are

$s = 0$

$s+2 = 0 \Rightarrow s = -2$

$s^2+2s+5 = 0$

$\Rightarrow s = -1 \pm 2j$

There are four no. of poles which are at $s=0, s=-2, s=-1+2j$ & $s=-1-2j$.

So the root locus is symmetrical about the real axis.

no. of poles(n) = 4

no. of zeros(m) = 0

since $n>m$, $n-m = 4-0 = 4$, so 4 branches of root locus starts from open loop poles end at infinity.

There will be 4 asymptotes angle of asymptotes

$$\theta_q = \frac{\pm(2q+1)\pi}{n-m}$$

where $q = 0, 1, 2 \ldots (n-m-1)$

$= 0, 1, 2, 3$

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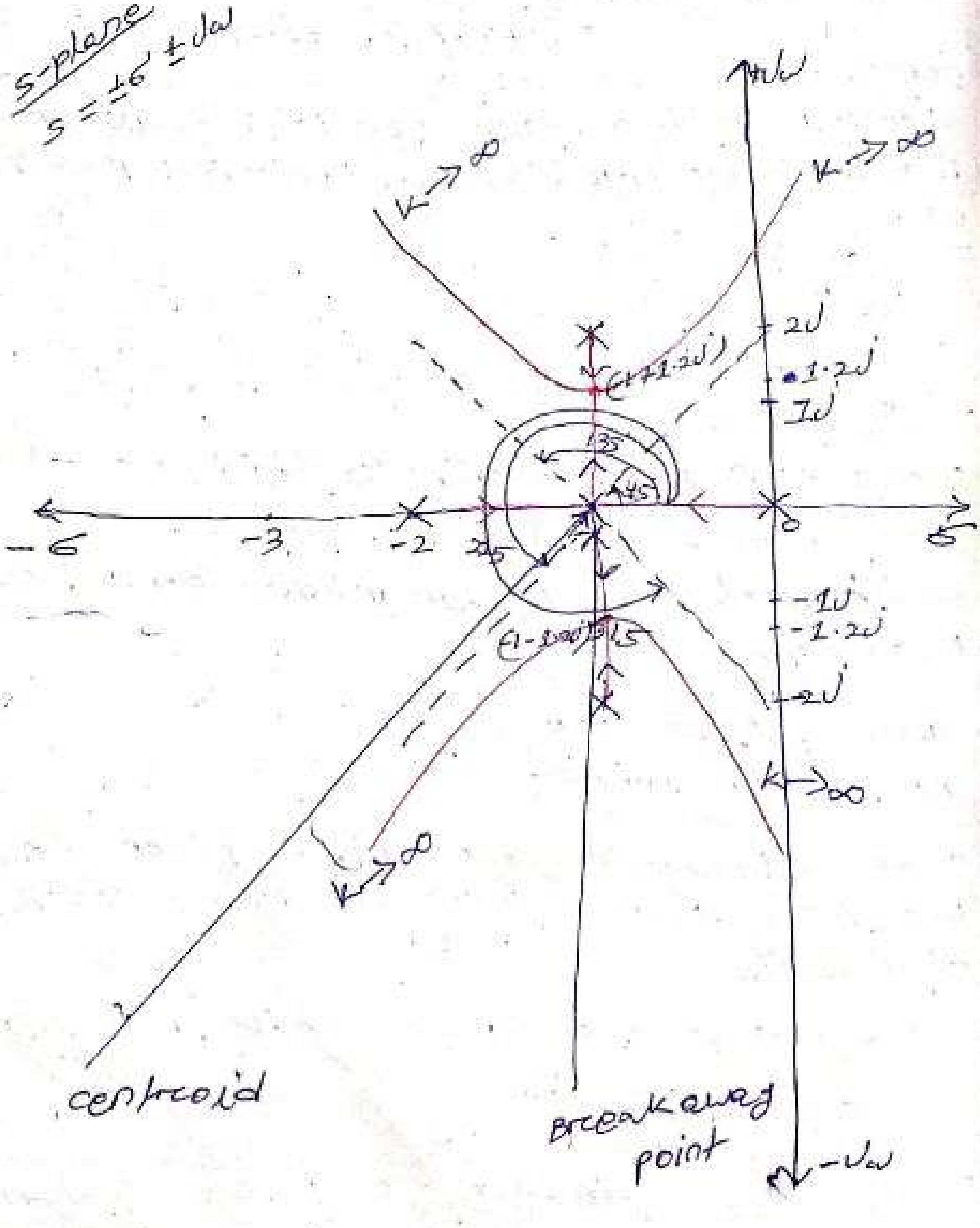
$$\theta_0 = \pm\frac{(2\times0+1)\pi}{4-0} = \frac{\pi}{4} = 45^\circ$$

$$\theta_1 = \pm\frac{(2\times1+1)\pi}{4} = \frac{3\pi}{4} = \frac{3\times180}{4\pi} = 135^\circ$$

$$\theta_2 = \pm\frac{(2\times2+1)\pi}{4} = \frac{5\pi}{4} = 225^\circ$$

$$\theta_3 = \pm\frac{(2\times3+1)\pi}{4} = \frac{7\pi}{4} = 315^\circ$$



→ All the asymptotes inserts each other at centroid.

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$$\text{centroid} = \frac{\text{sum of real part of open loop pole} - \text{sum of real part of open loop zeros}}{\text{No. of poles} - \text{No. of zeros}}$$

$$= \frac{(0-2-1-1)-0}{4-0} = \frac{-4}{4} = -1$$

→ The Breakaway point can be calculated by solving $\frac{dk}{ds} = 0$

→ The char. equation of the system is

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{k}{s(s+2)(s^2+2s+5)} = 0$$

$$\Rightarrow \frac{s(s+2)(s^2+2s+5) + k}{s(s+2)(s^2+2s+5)} = 0$$

$$\Rightarrow s(s+2)(s^2+2s+5) + k = 0$$

$$\Rightarrow (s^2+2s)(s^2+2s+5) + k = 0$$

$$\Rightarrow s^4 + 2s^3 + 5s^2 + 2s^3 + 4s^2 + 10s + k = 0$$

$$\Rightarrow s^4 + 4s^3 + 9s^2 + 10s + k = 0$$

$$\Rightarrow k = -s^4 - 4s^3 - 9s^2 - 10s$$

$$\Rightarrow \frac{dk}{ds} = -4s^3 - 12s^2 - 18s - 10$$

putting $\frac{dk}{ds} = 0$, we get

$$4s^3 + 12s^2 + 18s + 10 = 0$$

By solving this equation we get

$s = -1$

$s = -1 + 1.2j$

$s = -1 - 1.2j$

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Finding value of k

Already we get that

$$1+G(s)H(s)=0$$

$$\Rightarrow 1+\frac{k}{s(s+2)(s^2+2s+5)}=0$$

$$\Rightarrow s(s+2)(s^2+2s+5)+k=0$$

$$\Rightarrow s^4+4s^3+9s^2+10s+k=0$$

using Routh array criterion

s^4	1	9	k
s^3	4	10	
s^2	$13/2$	k	
s^1	$\frac{65-4k}{13/2}$		
s^0	k		

⇒ According to routh criterion the system will be stable if all the roots of the char. equation lies on the left half of the s-plane. For this all the elements in the first column of the routh array should be have same sign

so, we can write

$$k>0 \quad \text{———} (1)$$

&

$$\frac{65-4k}{13/2}>0$$

$$\Rightarrow 65-4k>0$$

$$\Rightarrow 4k<65$$

$$\Rightarrow k<65/4 \Rightarrow k<16.25 \quad \text{———} (2)$$

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From the equation (1) and (2) the value of k will fall in the following range $0 < k < 16.25$

→ The critical value of k is given by

$$65 - 4k = 0$$

$$\Rightarrow 4k = 65$$

$$\Rightarrow k = 65/4 = 16.25$$

→ The root locus plot 4 branches of root locus terminates at $k = \infty$ and the root locus lie between $s = \infty$ to $s = -2$ and in between $s = -1 + 2j$ to $s = -1 - 2j$

EFFECT OF ADDING POLES AND ZEROS TO OPEN LOOP T.F [G(S) H(S)]

1. ADDING POLES TO G(S)H(S)

→ Adding pole to G(S) H(S) shifts the root locus towards the right half of the s-plane.

→ The angle of asymptotes reduces and the centroid is shifted towards left.

→ The system becomes and the stability reduces by adding poles to G(S)H(S)

2. ADDING ZEROS TO G(S)H(S)

→ Adding zeros to G(S)H(S) shifts the root locus towards left half of the s-plane.

→ The angle of asymptotes increases and the centroid is shifted towards rights.

→ so the stability of the system improved by adding zeros to G(S) H(S)

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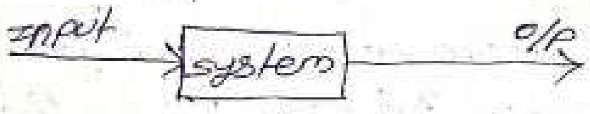
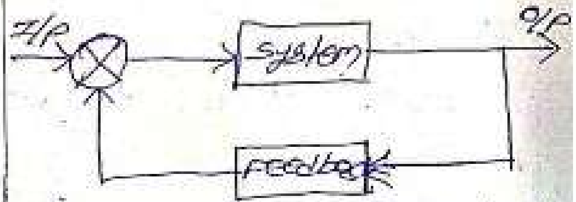
DIFFERENCE BETWEEN OPEN LOOP T.F AND CLOSE LOOP T.F

OPEN LOOP CONTROL SYSTEM	CLOSE LOOP CONTROL SYSTEM
→ Feedback is not present in this type of system	→ Feed back path is present in this type of system output is measured & feedback to the controller to modify the input
→ 	
→ Errorr is this type of system is very high	→ Errorr in close loop system is less
→ Example of open loop system cooler	→ Example of close loop system Air conditioner

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CHAPTER-4

FREQUENCY RESPONSE ANALYSIS

→ The response of a system may be categorized as time response and frequency response.

→ For frequency response analysis is generally the sinosodial input given to the system

$$r(t) = A\sin\omega t$$

→ under steady state the o/p may be written as $B\sin(\omega t + \phi)$

→ The magnitude and phase relationship between the sinosodial input and steady state o/p of a system is termed as frequency response.

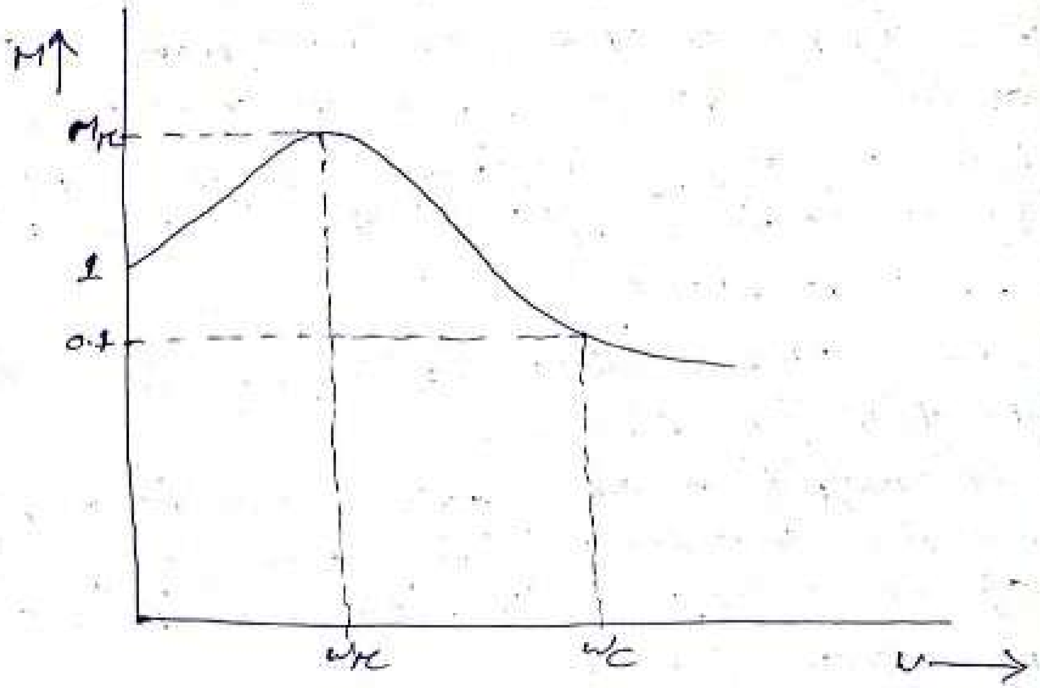
→ The frequency response test is performed by keeping amplitude 'A' constant and determining 'B' and ϕ for suitable range of frequencies.

CORELATION BETWEEN TIME RESPONSE & FREQUENCY RESPONSE

→ In time domain the relative stability is measured by parameters such as maximum peak overshoot, damping ratio etc. In frequency domain ~~the relative~~ resonant peak (M_r) is use to measure relative stability.

→ In time domain, if rise time (t_r) is less then system is faster. In frequency domain larger bandwidth corresponds to faster system.

→ Increasing zeta (ξ) bandwidth decrease and rise time (t_r) increases. rise time and bandwidth are inversly proportional to each other



→ Reasonant peek (Mr) = $\frac{1}{2\xi\sqrt{1-\xi^2}}$

→ When $M = \frac{1}{2}$ then the perticulare freequency is called as cutoff freequency.

→ The reange of freequeny fore which $M \geq \frac{1}{\sqrt{2}}$ is definded as Bandwidth.

POLAR PLOT

→ The sinosodial T.F G(jω) is a complex functio which can be given by

$$G(j\omega) = Re[G(j\omega)] + j\,Im[G(j\omega)]$$

$$= |G(j\omega)| \angle G(j\omega)$$

$$= M \angle \phi$$

→ ~~The~~ This G(jω) may be represented as phaso of magnitude 'M' and phase angle φ.

→ As the input freequency 'ω' is veried from '0' to '∞', the magnitude 'M' and phase angle 'φ' change and hence the tip of the ~~tedder~~ phasore traces a locus on the s-plane.

→ The Locus traced by the tip of the phasore G(jω) as frequency 'ω' veried from '0' to

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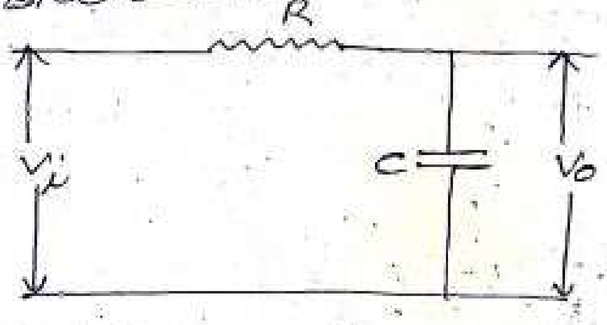
"∞" is called as polar plot.

Ex-1
Consider a RC ~~ckt~~ filter ckt shown in the below fig.



Transfer Function

$$= \frac{V_o(s)}{V_i(s)} = \frac{1/sC}{R+1/sC} = \frac{1/Cs}{R+1/Cs} = \frac{1/Cs}{\frac{RCs+1}{Cs}}$$

$$= \frac{1}{1+RCs} = \frac{1}{1+Ts}$$

where, $T = RC$

→ so the T.F of the system is $1/1+Ts$

polar plot

substituting $s = j\omega$ we get

$$G(j\omega) = \frac{1}{1+T(j\omega)}$$

$$= \frac{1}{1+jT\omega}$$

$$= \frac{1}{\sqrt{1+(\omega T)^2}} \angle -\tan^{-1}\left(\frac{\omega T}{1}\right)$$

$$= M \angle \phi$$

$$M = \frac{1}{\sqrt{1+(\omega T)^2}}, \quad \phi = -\tan^{-1}\left(\frac{\omega T}{1}\right)$$

→ For $\omega = 0$

$M = 1$, $\phi = 0°$

$\omega = 1/T$, $M = \frac{1}{\sqrt{2}}$, $\phi = -45°$

$\omega = \infty$, $M = 0$, $\phi = -90°$

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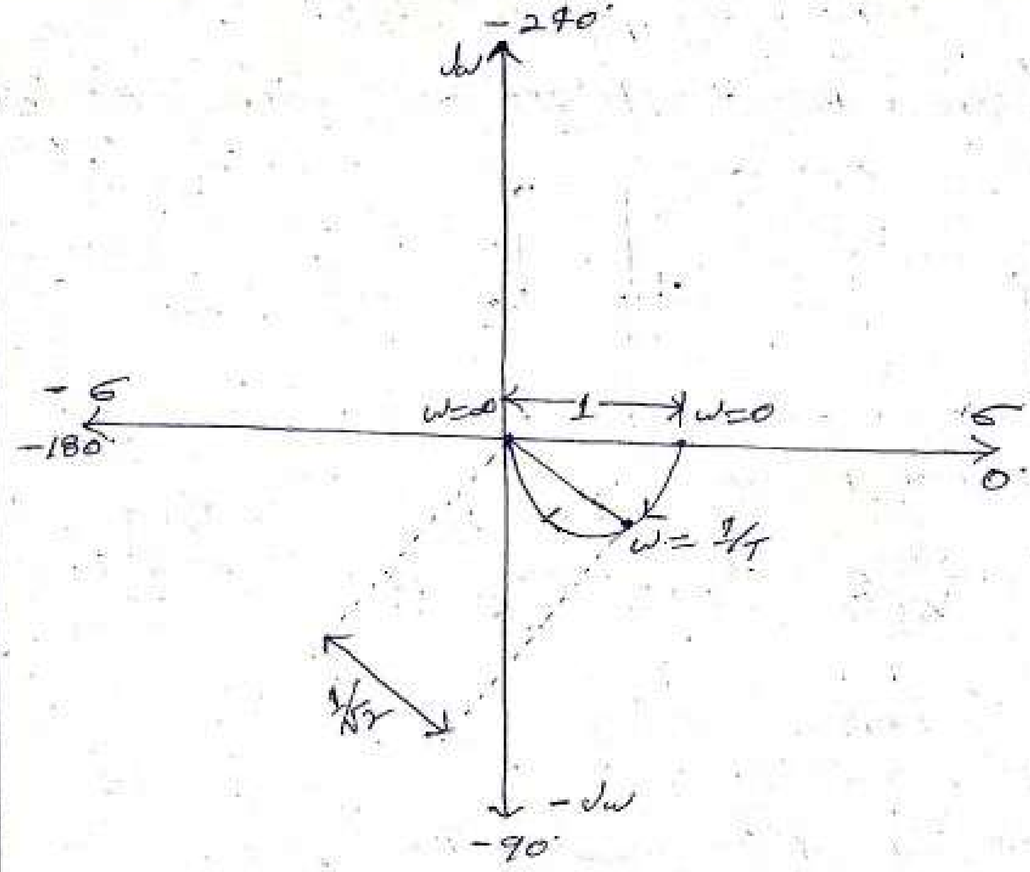

→ The polar plot is drawn by drawing a smooth curve of the tip of the phasor when 'ω' is varied from zero to ∞ gradually.

EX-3

Draw ~~as~~ the polar plot ~~is drawn by drawing a smooth curve~~ of the following Transfer function.

$$G(s) = \frac{1}{s(1+Ts)}$$

Ans

$$G(s) = \frac{1}{s(1+Ts)}$$

substituting $s = j\omega$, then transfer function is

$$G(j\omega) = \frac{1}{j\omega(1+T\times j\omega)}$$

→ The magnitude and phase angle of the T.F are

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$$G(j\omega) = \frac{1}{j\omega + Tj^2\omega^2}$$

$$= \frac{1}{-T\omega^2 + j\omega}$$

$$M = \frac{1}{\sqrt{(-T\omega^2)^2 + \omega^2}} = \frac{1}{\sqrt{\omega^2 + T^2\omega^4}}$$

$$= \frac{1}{\sqrt{\omega^2(1+T^2\omega^2)}}$$

$$\phi = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega}{-T\omega^2}\right)$$

$$= 0 - \tan^{-1}\left(\frac{1}{-T\omega}\right)$$

$$= -\tan^{-1}\left(\frac{1}{-\omega T}\right)$$

By varying 'ω' from '0' to '∞' we can get different value of 'M' & 'ϕ'.

For

$\omega = 0$, $M = \frac{1}{0} = \infty$, $\phi = -\tan^{-1}\left(\frac{1}{0}\right)$

$$= -90^\circ$$

$\omega = \frac{1}{T}$, $M = \frac{1}{\sqrt{\frac{1}{T^2}\left(1 + T^2 \times \frac{1}{T^2}\right)}} = \frac{1}{\sqrt{2/T^2}}$

$$= \frac{T}{\sqrt{2}}$$

$$\phi = -\tan^{-1}\left(\frac{1}{-\frac{1}{T}\times T}\right) = -\tan^{-1}(1)$$

$$= -45^\circ \text{ or } -135^\circ = -135^\circ$$

$\omega = \infty$, $M = \frac{1}{\infty} = 0$, $\phi = -\tan^{-1}\left(\frac{1}{\infty}\right)$

$$= -\tan^{-1}(0)$$

$$= 0^\circ \text{ or } 180^\circ$$

$$= -180^\circ$$

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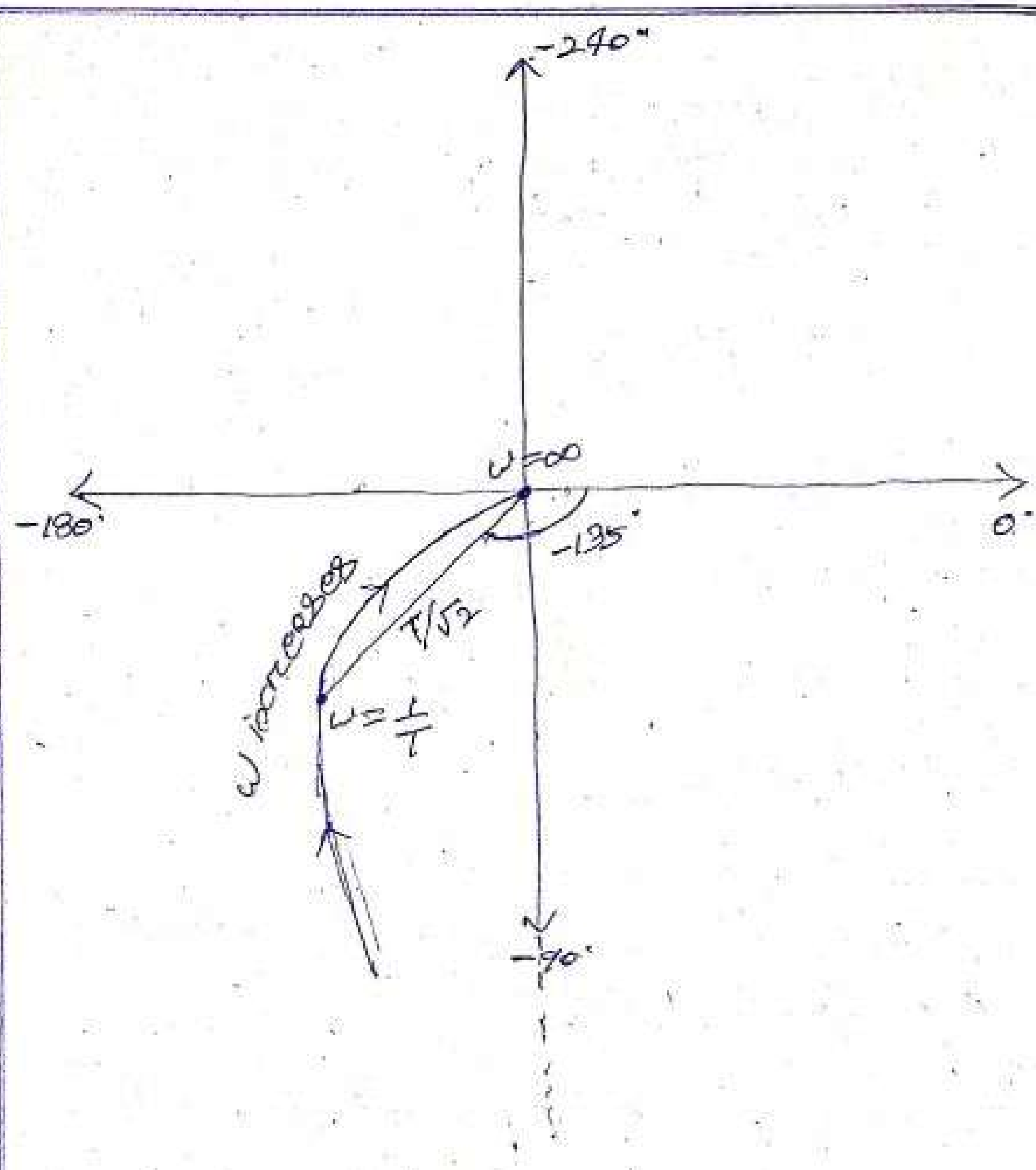

→ The polar plot is drawn by drawing a smooth curve of the tip of the phasor when 'ω' is varied from '0' to '∞' gradually.

EX-3

Draw a polar plot of following transfer function.

$$G(s) = \frac{1}{(1+s)(1+2s)}$$

Ans

$$G(s) = \frac{1}{(1+s)(1+2s)}$$

substituting $s = j\omega$, the transfer function is

$$G(j\omega) = \frac{1}{(1+j\omega)(1+2j\omega)}$$

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$$= \frac{1}{(1+j\omega)(1+2j\omega)}$$

$$= \frac{1}{1+2j\omega+j\omega+2j^2\omega^2}$$

$$= \frac{1}{1+3j\omega-2\omega^2} = \frac{1}{1-2\omega^2+3j\omega}$$

The magnitude and phase angle of the T.F are

$$M = \frac{1}{\sqrt{(1-2\omega^2)^2+(3\omega)^2}} = \frac{1}{\sqrt{1^2-2\cdot1\cdot2\omega^2+(2\omega^2)^2+9\omega^2}}$$

$$= \frac{1}{\sqrt{1-4\omega^2+4\omega^4+9\omega^2}} = \frac{1}{\sqrt{4\omega^4+5\omega^2+1}}$$

$$\phi = -\tan^{-1}(0) - \tan^{-1}\left(\frac{3\omega}{1-2\omega^2}\right)$$

$$= 0 - \tan^{-1}\left(\frac{3\omega}{1-2\omega^2}\right)$$

$$= -\tan^{-1}\left(\frac{3\omega}{1-2\omega^2}\right)$$

By varing 'ω' from '0' to '∞' we can get different value of 'M' & 'φ'.

For

$\omega = 0$, $M = 1$, $\phi = -\tan^{-1}\left(\frac{0}{1-0}\right)$

$$= -\tan^{-1}(0) = 0$$

$\omega = \frac{1}{\sqrt{2}}$, $M = \frac{1}{\sqrt{4\left(\frac{1}{\sqrt{2}}\right)^4+5\left(\frac{1}{\sqrt{2}}\right)^2+1}}$

$$= \frac{1}{\sqrt{4\times\frac{1}{4}+5/2+1}} = \frac{1}{\sqrt{1+5/2+1}}$$

$$= \frac{1}{\sqrt{\frac{2+5+2}{2}}} = \frac{1}{\sqrt{9/2}} = \frac{\sqrt{2}}{3}$$

$$\phi = -\tan^{-1}\left(\frac{3\times\frac{1}{\sqrt{2}}}{1-2\left(\frac{1}{\sqrt{2}}\right)^2}\right) = -\tan^{-1}\left(\frac{3/\sqrt{2}}{1-2/2}\right)$$

$$= -\tan^{-1}\left(\frac{3/\sqrt{2}}{0}\right) = -\tan^{-1}(\infty) = -90$$

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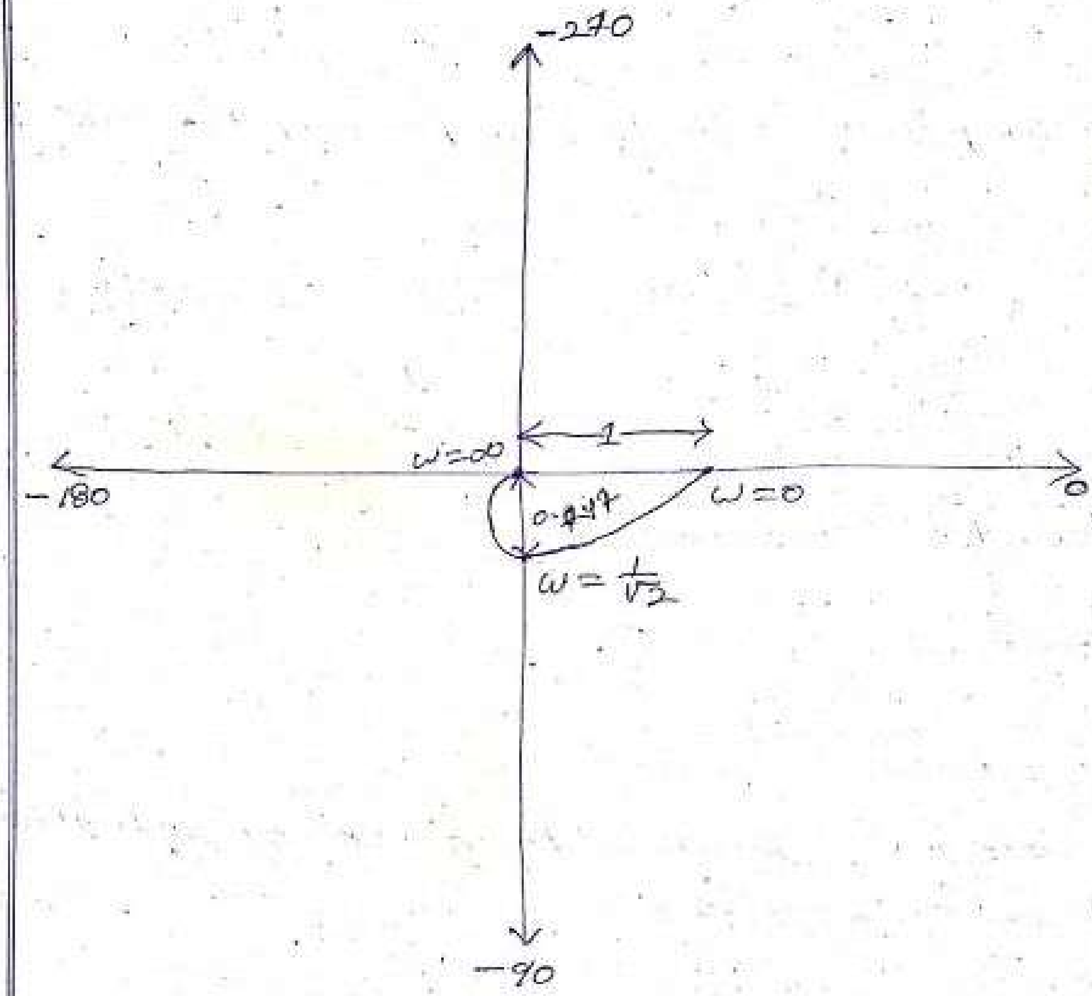
$\omega = \infty$, $M = \frac{1}{\infty} = 0$

$$\phi = -\tan^{-1}(\infty) = -90 \text{ or } -270$$
$$= -270$$



→ The polar plot is drawn by drawing a smooth curve of the tip of the phasor when 'ω' is varied from '0' to '∞' gradually.

EX-4

Draw the polar plot for the T.F

$$G(s) = \frac{1+4s}{s(1+s)(1+2s)}$$

→ By putting $s = j\omega$

$$G(j\omega) = \frac{1+4(j\omega)}{j\omega(1+j\omega)(1+2j\omega)}$$

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$$= \frac{(1+4j\omega)(1-j\omega)(1-2j\omega)}{j\omega(1+j\omega)(1-j\omega)(1+2j\omega)(1-2j\omega)}$$

$$= \frac{(1+4j\omega)(1-2j\omega-j\omega+2j^2\omega^2)}{j\omega(1^2-(j\omega)^2)(1^2-(2j\omega)^2)}$$

$$= \frac{(1+4j\omega)(1-3j\omega-2\omega^2)}{j\omega(1-j^2\omega^2)(1-4j^2\omega^2)}$$

$$= \frac{1-3j\omega-2\omega^2+4j\omega-12j^2\omega^2-8j\omega^3}{j\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{-8j\omega^3+10\omega^2+j\omega+1}{j\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{(1+10\omega^2)+j\omega(-8\omega^2+1)}{j\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{1+10\omega^2}{j\omega(1+\omega^2)(1+4\omega^2)} + \frac{j\omega(-8\omega^2+1)}{j\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{(1-8\omega^2)}{(1+\omega^2)(1+4\omega^2)} - \frac{j(1+10\omega^2)}{\omega(1+\omega^2)(1+4\omega^2)}$$

when, $\omega = 0$

$G(j0) = 1 - j\infty$

when, $\omega = \infty$

$G(j\infty) = 0 - j\times 0$

when the polar plot crosses the imaginary axis the real part of $G(j\omega)$ is equal to zero at that time

so,

$$\frac{(1-8\omega^2)}{(1+\omega^2)(1+4\omega^2)} = 0$$

$\Rightarrow 1 - 8\omega^2 = 0$

$\Rightarrow 1 = 8\omega^2 \Rightarrow \omega^2 = \frac{1}{8} \Rightarrow \omega = \frac{1}{\sqrt{8}}$

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then

$G(j\omega)\Big|_{\omega=\frac{1}{\sqrt{8}}} = 0 + j3.77$

Based on the above information the polar plot can be drawn:

$\omega=0$	$1-j\infty$
$\omega=\frac{1}{\sqrt{8}}$	$0-j3.77$
$\omega=\infty$	$0-j0$

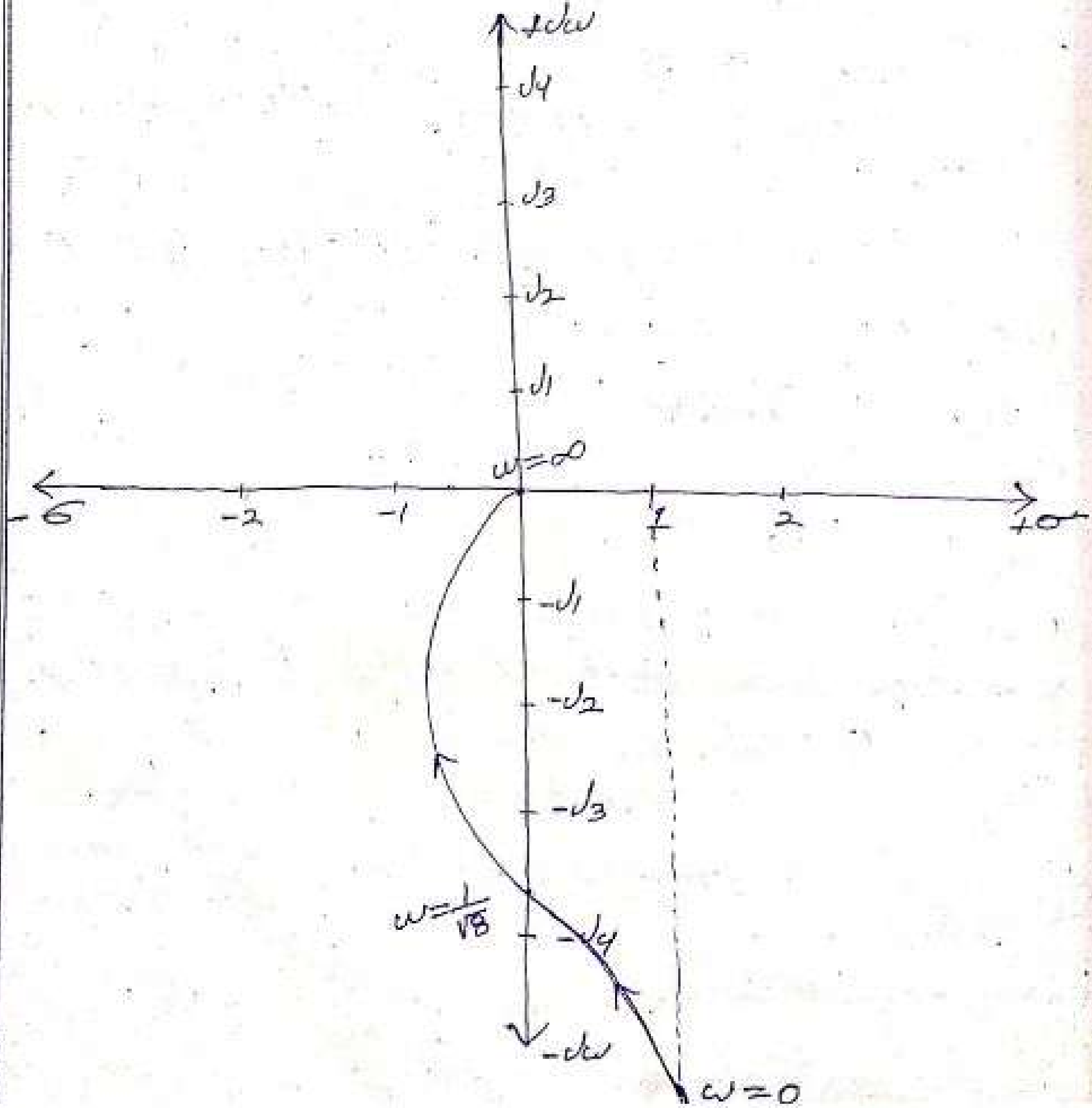

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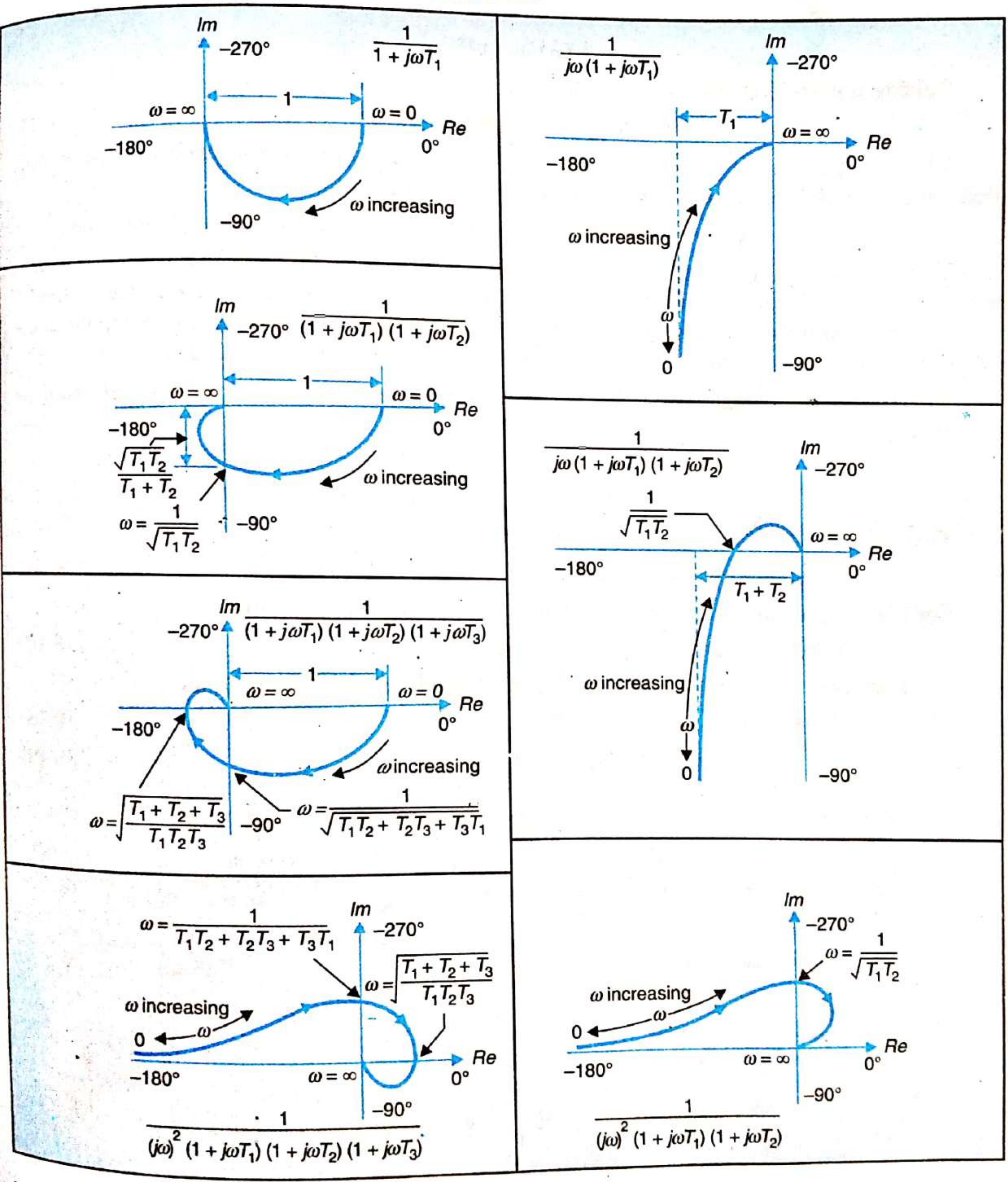
Table 8.1


1 / (1 + jωT1)
Im
–270°
1
ω = ∞
ω = 0
Re
–180°
0°
ω increasing
–90°
1 / jω(1 + jωT1)
T1
ω = ∞
ω increasing
ω
0
1 / (1 + jωT1)(1 + jωT2)
√(T1T2) / (T1 + T2)
ω = 1 / √(T1T2)
1 / jω(1 + jωT1)(1 + jωT2)
1 / √(T1T2)
T1 + T2
1 / (1 + jωT1)(1 + jωT2)(1 + jωT3)
ω = √((T1 + T2 + T3) / (T1T2T3))
ω = 1 / √(T1T2 + T2T3 + T3T1)
ω = 1 / (T1T2 + T2T3 + T3T1)
ω = √((T1 + T2 + T3) / (T1T2T3))
1 / (jω)² (1 + jωT1)(1 + jωT2)(1 + jωT3)
ω = 1 / √(T1T2)
1 / (jω)² (1 + jωT1)(1 + jωT2)

Bode Plot

- Bode plot is a logarithmic plot, which consists of two graphs i.e magnitude plot & phase angle plot of $G(j\omega)$, both plotted against frequency (ω) in logarithmic scale.

Magnitude plot:

- It is the plotting of graph between magnitude of transfer function $20 \log |G(j\omega)|$ and frequency ω in logarithmic scale.

→ Magnitude $20 \log |G(j\omega)|$ unit is decibel (db)

Phase angle plot

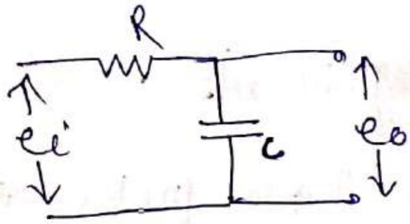
It is the plot between phase angle of transfer function $\phi(\omega)$ and frequency ω in logarithmic scale.

→ Unit of phase angle is degree.

Note! Both the plots are drawn above a common frequency (ω) axis. These graphs are generally drawn over a semilog paper

Consider the transfer function of a RC filter as —

$$G(j\omega) = \frac{1}{1 + j\omega T}$$



In magnitude & phase-angle form it can be written as

$$G(j\omega) = \frac{1}{\sqrt{(1+\omega^2 T^2)}} \angle -\tan^{-1}\omega T$$

$$\left[\because G(j\omega) = |G(j\omega)| \angle G(j\omega) \right]$$

The log magnitude can be given as —

$$20\log|G(j\omega)| = 20\log\left|\frac{1}{\sqrt{1^2+\omega^2T^2}}\right|$$

$$= 20\log\left|(1+\omega^2T^2)^{-1/2}\right|$$

$$= -\frac{1}{2}\times 20\log\left|1+\omega^2T^2\right|$$

$$= -10\log\left|1+\omega^2T^2\right|$$

→ For low frequencies $\omega << 1/T$, the log-magnitude approximated as — $20\log|G(j\omega)| = -10\log|1|$

$$= 0\,db$$

→ for high frequencies $\omega >> 1/T$, the logmagnitude approximated as —

$$20\log|G(j\omega)| = -10\log\left|\omega^2T^2\right|$$

$$= -10\times 2\log|\omega T|$$

$= -20 \log (\omega T)$

$\Rightarrow 20 \log |G(j\omega)| = -20 \log \omega - 20 \log T$

→ The logmagnitude plot of $\frac{1}{(1+j\omega T)}$ can be approximated by two line asymptotes.

one straight line at 0 db for freq $0 < \omega \leq \frac{1}{T}$

another " " with −20 db/decade slope at freq $\frac{1}{T} \leq \omega < \infty$

A unit change in $\log \omega$ means

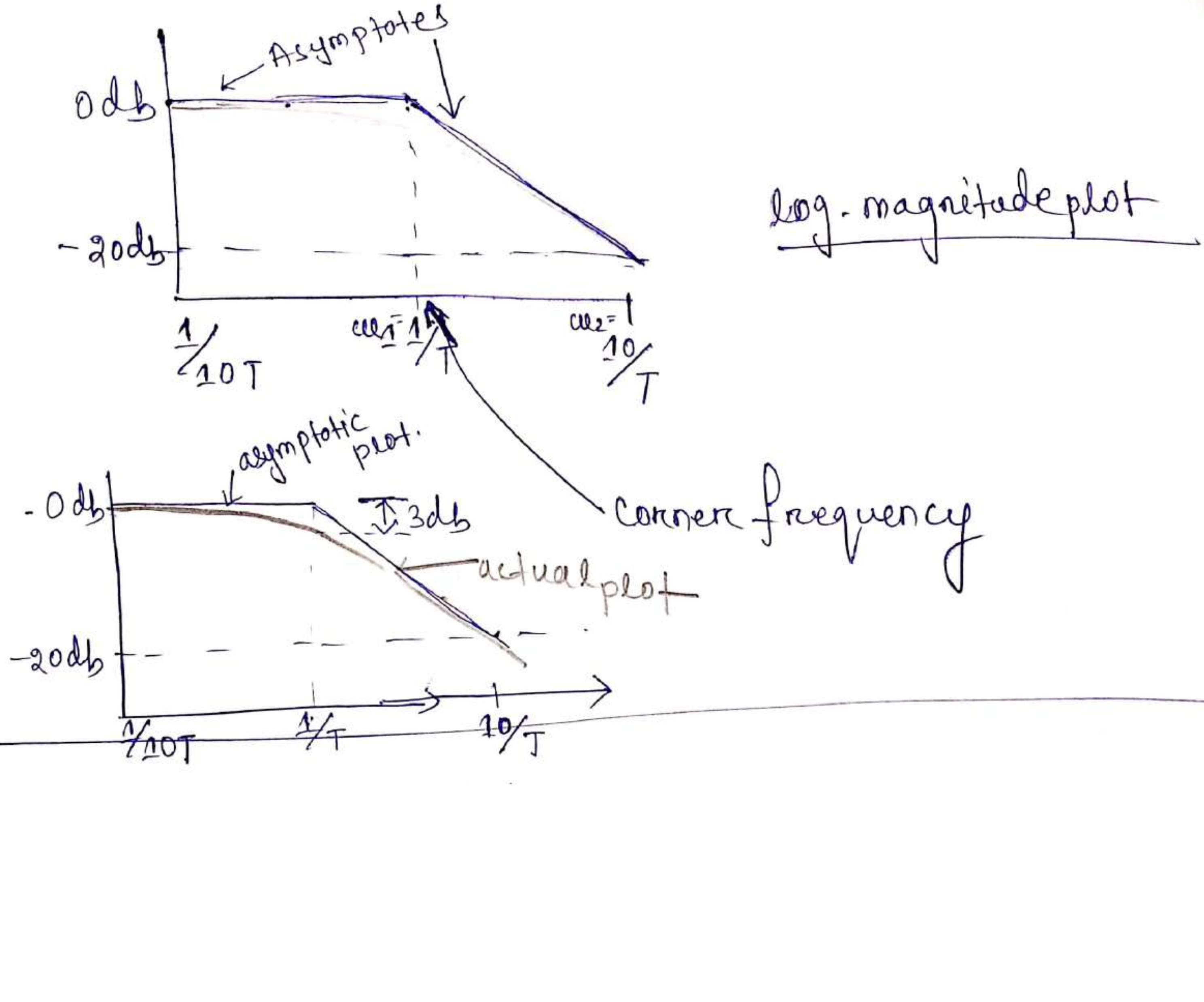
$$\log\left(\frac{\omega_2}{\omega_1}\right) = 1$$

$$\Rightarrow \omega_2 = 10\,\omega_1$$

This range of frequencies is called as decade.

→ The frequency $\omega = 1/T$ at which two asymptotes meet is called as corner frequency or break frequency.

→ The corner frequency divides the plot in to two regions, a low frequency region & a high frequency region.


log. magnitude plot
Asymptotes
0 db
-20db
1/10T
ω1 = 1/T
ω2 = 10/T
Corner frequency
asymptotic plot.
-0 db
3db
actual plot
-20db
1/10T
1/T
10/T

Imp

General Procedure for Constructing Bode Plot

The following steps are required to construct Bode plot for a given $G(j\omega)$.

1. Rewrite the sinusoidal transfer function in the time constant form. —

The Time Constant form for $G(j\omega)$ is given by —

$$G(j\omega) = \frac{K(1+j\omega T_{z_1})(1+j\omega T_{z_2})\ldots\ldots}{(j\omega)^r (1+j\omega T_{P_1})(1+j\omega T_{P_2})\left[1+j2\xi\left(\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2\right]\ldots\ldots}$$

The transfer function $G(j\omega)$ has

real zeros at $-\frac{1}{T_{z_1}}, -\frac{1}{T_{z_2}}$ ......

r no. of pole at origin

real poles at $-\frac{1}{T_{P_1}}, -\frac{1}{T_{P_2}}$ ......

Complex poles at $-\xi\omega_n \pm j\omega_n\sqrt{(1-\xi^2)}$, ...

2. Identify the corner frequencies associated with each factor of the transfer function

3. Asymptotic magnitude plot is drawn at the corner frequencies. This plot consists of straight line segments with slope changing at each corner frequency by —

+20 db/decade for a zero

−20 db/decade for a Pole

+40 db/decade for a complex conjugate zero

−40 db/decade for a complex conjugate Pole

(4) Determine the correction to be applied to the asymptotic plot.

(5) Draw the smooth curve through the corrected points such that it is asymptotic to the line segments. This gives the actual log-magnitude plot.

(6) Draw phase angle curve for each factor & add them algebrically to get the phase plot.

Computation of Gain Margin & Phase Margin

Gain Margin : It is the factor by which the system gain can be increased to drive it to the verge of instability at phase crossover frequency.

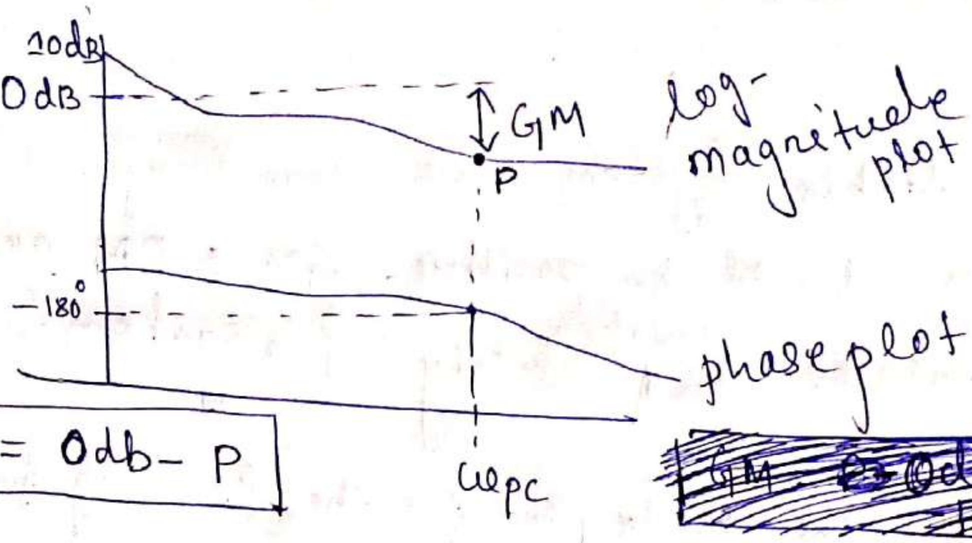
→ Calculation

To find out Gain Margin (GM), phase crossover frequency (ω_{pc}) need to be calculated.

- Phase cross-over frequency (ω_{pc}) :

The frequency at which phase angle is $-180°$ is known as phase crossover frequency. It is ~~calcu~~ determined from phase plot.

→ The difference between 0dB and the magnitude at phase cross over frequency (ω_{pc}) on the magnitude plot is called as Gain Margin.



$GM = 0\,dB - P$

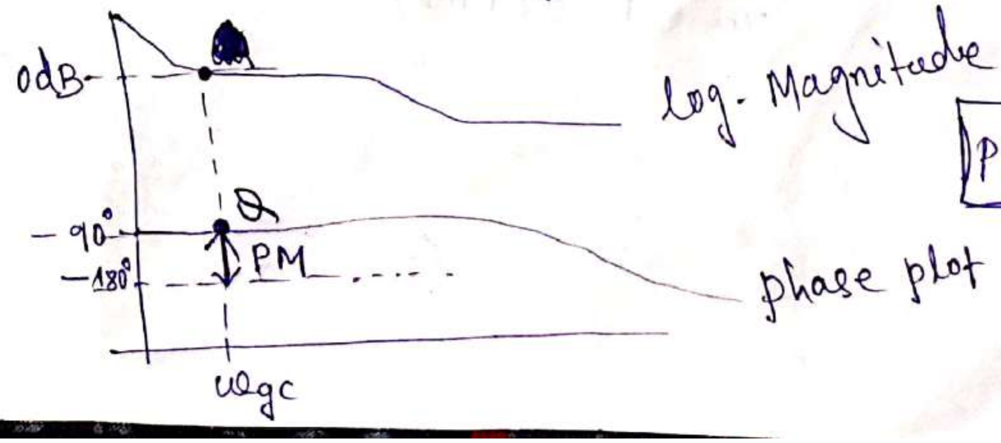
Phase Margin: It is the amount of additional phase lag can be added to bring the system to the verge of instability at gain cross-over frequency.

Calculation:

To findout the Phase Margin (PM), gain crossover frequency (ω_{gc}) need to be calculated.

Gain cross-over frequency (ω_{gc}): The frequency at which log-Magnitude is 0dB is known as gain-cross over frequency. It is determined from log-magnitude plot.

→ The difference between $-180°$ & the phase angle at ω_{gc} (gain cross-over frequency) on the phase plot is called as Phase Margin.



$PM = \theta + 180°$

Note:

For a stable system both Gain Margin & Phase Margin should be positive. GM & PM are used to measure the relative stability of a system.

Q: Obtain the Bode plot for the following function

$$G(s) = \frac{10(s+10)}{s(s+100)}$$

Determine the Gain Margin & Phase Margin of the system. Also, comment on the the stability of the system.

Soln

Given $G(s) = \frac{10(s+10)}{s(s+100)}$

In time-constant form —

$$G(s) = \frac{10\times 10(\frac{s}{10}+1)}{100\times s(\frac{s}{100}+1)}$$

$$= \frac{(1+s/10)}{s(1+s/100)}$$

$$\Rightarrow G(s) = \frac{(1+0.1s)}{s(1+0.01s)}$$

By putting $s = j\omega$,

$$G(j\omega) = \frac{(1+0.1j\omega)}{j\omega(1+0.01j\omega)}$$

Log-Magnitude plot

The characteristics of each factor of the transfer function are given below —

factor	Corner frequency (rad/sec)	slope	change in slope
$1/j\omega$	—	-20 db/decade	—
$(1+0.1j\omega)$	$\omega_{c1} = \frac{1}{0.1} = 10$	$+20$ db/decade	-20db $+20$db $= 0$ db/decade (∵ used in $A\omega_{c2}$)
$\frac{1}{(1+0.01j\omega)}$	$\omega_{c2} = \frac{1}{0.01} = 100$	-20 db/decade	0 db -20 db $= -20$ db/decade (∵ used in $A\omega_h$)

Consider a frequency lower than ω_{c1} i.e

$$\omega_L = 1$$

Consider a frequency higher than ω_{c2} i.e

$$\omega_h = 1000$$

Now the frequencies taken for consideration are —

$\omega_L = 1$, $\omega_{c1} = 10$, $\omega_{c2} = 100$, $\omega_h = 1000$

Magnitude (A) at the corner frequencies

$$A_{\omega_L} = 20\log|1^{st}\text{ factor}| = 20\log\left|\frac{1}{j\omega}\right|$$

$$= -20\log|j\omega|$$

$$= -20\log\omega = -20\log\omega_L$$

$$\Rightarrow A\omega_L = -20\log 1 \quad (\because \omega_L = 1)$$

$\Rightarrow A_{\omega_L} = 0\,db$

$\underline{A_{\omega_{c1}}}$ $(\omega = \omega_{c1} = 10)$

$20 \log|1st \text{ factor}| = 20 \log \left|\frac{1}{j\omega}\right|$

$= -20 \log |j\omega|$

$= -20 \log (\omega)$

$= -20 \log \omega_{c1}$

$= -20 \log 10 \quad (\because \omega_{c1} = 10)$

$= -20\,db$

$\underline{A_{\omega_{c2}}}$ $(\omega = \omega_{c2} = 100)$

$= [\text{Slope from } \omega_{c1} \text{ to } \omega_{c2}] \times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right) + A_{\omega_{c1}}$

$\Rightarrow A_{\omega_{c2}} = 0\,dB \times \log\left(\frac{100}{10}\right) + (-20\,db)$

$= 0 - 20\,db$

$= -20\,db$

$\underline{A_{\omega_h}}$ $(\omega = \omega_h = 1000)$

$A_{\omega_h} = [\text{Slope from } \omega_h \text{ to } \omega_{c2}] \times \log\left(\frac{\omega_h}{\omega_{c2}}\right) + A_{\omega_{c2}}$

$= -20 \times \log\left(\frac{1000}{100}\right) + (-20)$

$\Rightarrow -20 \times \log 10 + (-20)$

$\Rightarrow -20 \times 1 - 20$

$= -40\,db$

$\Rightarrow A_{\omega h} = -40\,db$

So we get —

freq	Gain (in db)
$\omega_L = 1$	0 db
$\omega_{C_1} = 10$	-20 db
$\omega_{C_2} = 100$	-20 db
$\omega_h = 1000$	-40 db

These data are used to plot log. magnitude curve.

Phase plot

$$G(j\omega) = \frac{(1+0.1j\omega)}{j\omega(1+0.01j\omega)}$$

$$\Rightarrow \angle G(j\omega) = \angle(1+0.1j\omega) - \angle(j\omega) - \angle(1+0.01j\omega)$$

$$= \tan^{-1}\left(\frac{0.1\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{0.01\omega}{1}\right)$$

$$= \tan^{-1}(0.1\omega) - \tan^{-1}(\infty) - \tan^{-1}(0.01\omega)$$

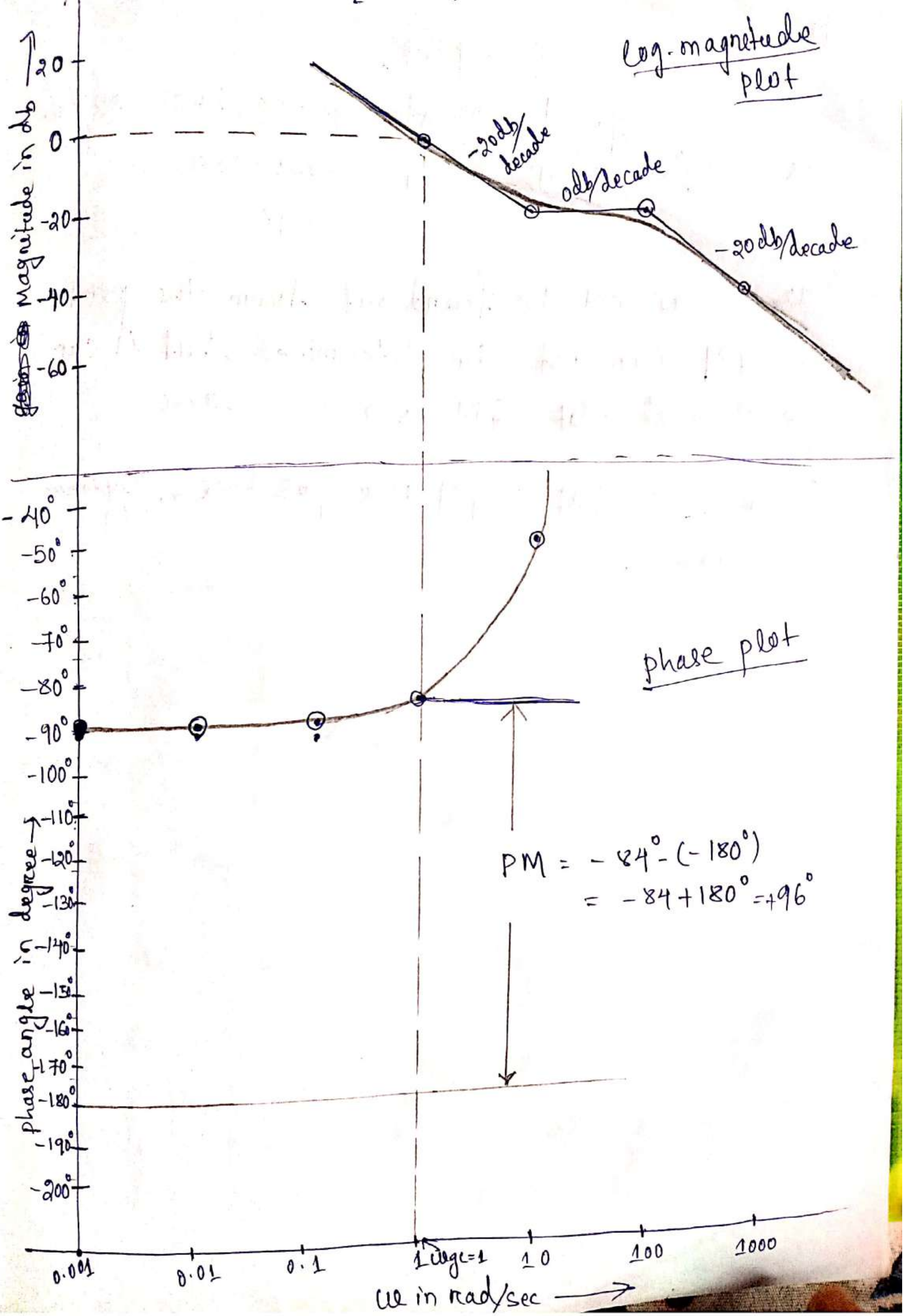
$$= \tan^{-1}(0.1\omega) - 90^\circ - \tan^{-1}(0.01\omega)$$

Finding $\angle G(j\omega)$ for different values of 'ω' we get —

ω	$\angle G(j\omega)$
1	-84°
10	-50°
0.1	-89.48°
0.01	$-89.94 \simeq -90^\circ$
100	-50.71
0.001	$-89.99 \approx -90^\circ$

By using the above data phase plot is drawn

Bode plot


Log-magnitude plot
Gain in db →
20
0
-20
-40
-60
-20db/decade
0db/decade
-20db/decade
phase plot
phase angle in degree →
-40°
-50°
-60°
-70°
-80°
-90°
-100°
-110°
-120°
-130°
-140°
-150°
-160°
-170°
-180°
-190°
-200°
PM = -84° - (-180°)
= -84 + 180° = +96°
0.001
0.01
0.1
1 ωgc=1
10
100
1000
ω in rad/sec →

From the above Bode plot.

$\omega_{gc} = 1$, at ω_{gc} the Phase angle is $-84°$. So the $PM = -84 + 180°$

$= +96°$

ω_{pc} Can not be found out from the plot So GM Can not be determined, but it can be observed that GM is a +ve Value.

Since both PM & GM are positive, System is stable.

Q: Draw the Bode Plot for a unity feedback control system having $G(s) = \frac{400}{s^2(s+2)(s+5)}$

Find the Phase Margin & Gain Margin from the graph.

Soln

Given $G(s) = \frac{400}{s^2(s+2)(s+5)}$

In time-constant form —

$$\Rightarrow G(s) = \frac{400}{2 \times 5 \times s^2 (s/2 + 1)(s/5 + 1)}$$

$$= \frac{40}{s^2(1+s/2)(1+s/5)}$$

$$\Rightarrow G(s) = \frac{40}{s^2(1+0.5s)(1+0.2s)}$$

By Putting $s = j\omega$

$$G(j\omega) = \frac{40}{(j\omega)^2(1+0.5j\omega)(1+0.2j\omega)}$$

Log-Magnitude Plot

The characterstics of each factors of transfer function are given below.

factor	Corner frequency	Slope	change in slope
$\frac{40}{(j\omega)^2}$	—	-20 db/decade $\times 2$ $= -40$ db/decade	—
$\frac{1}{(1+0.5j\omega)}$	$\omega_{c1} = \frac{1}{0.5} = 2$	-20 db/decade	$-40-20 = -60$ db/decade
$\frac{1}{(1+0.2j\omega)}$	$\omega_{c2} = \frac{1}{0.2} = 5$	-20 db/decade	$-60-20 = -80$ db/decade

Considering a frequency lower than ω_{c1}

$$\omega_L = 0.01$$

Considering a frequency higher than ω_{c2}

$$\omega_h = 10$$

Now the frequencies taken for consideration are —

$$\omega_L = 0.01, \ \omega_{c1} = 2, \ \omega_{c2} = 5, \ \omega_h = 10$$

Magnitude (A) at Corner frequencies

at $\omega = \omega_L = 0.01$

$$A_{\omega L} = 20 \log \left| \frac{40}{(j\omega)^2} \right|$$

$$= 20 \log 40 - 20 \log |(j\omega)^2|$$

$= 20 \log 40 - 40 \log |(j\omega)|$ $(\because \omega = 0.01)$

$= 20 \log 40 - 40 \log \omega = 20 \log 40 - 40 \log 0.01$

$= 112.04 \simeq 112 \text{db}$

At $\omega = \omega_{c1} = 2$

$$A_{\omega c_1} = 20 \log \left| \frac{40}{(j\omega)^2} \right|$$

$= 20 \log 40 - 20 \log |(j\omega)^2|$

$= 20 \log 40 - 40 \log \omega$

$= 20 \log 40 - 40 \log 2$ $(\because \text{Put } \omega = 2)$

$= 20 \text{ db}$

At $\omega = \omega_{c_2} = 5$

$$A_{\omega c_2} = [\text{slope from } \omega c_1 \text{ to } \omega c_2] \times \log\left(\frac{\omega c_2}{\omega c_1}\right) + A_{\omega c_1}$$

$= -60 \times \log\left(\frac{5}{2}\right) + 20$

$= -3.87 \text{ db}$

At $\omega = \omega_h = 10$

$$A_{\omega_h} = [\text{slope from } \omega_h \text{ to } \omega c_2] \times \log\left(\frac{\omega_h}{\omega c_2}\right) + A_{\omega c_2}$$

$= -80 \times \log\left(\frac{10}{5}\right) + (-3.87)$

$= -27.95 \simeq -28 \text{ db}$

So we get —

[redacted] frequency (ω)	Gain
$\omega_L = 0.01$	112 db
$\omega_{c1} = 2$	20 db
$\omega_{c2} = 5$	-3.87 db
$\omega_h = 10$	-28 db

By using this data magnitude plot is drawn.

Phase plot

$$\text{Given } G(j\omega) = \frac{40}{(j\omega)^2(1+0.5j\omega)(1+0.2j\omega)}$$

$$\angle G(j\omega) = \angle 40 - \angle(j\omega)^2 - \angle(1+0.5j\omega) - \angle(1+0.2j\omega)$$

$$= \angle 40 - 2\times\angle j\omega - \angle(1+0.5j\omega) - \angle(1+0.2j\omega)$$

$$= \tan^{-1}\left(\frac{0}{40}\right) - 2\times\tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{0.5\omega}{1}\right) - \tan^{-1}\left(\frac{0.2\omega}{1}\right)$$

$$= \tan^{-1}0 - 2\times\tan^{-1}(\infty) - \tan^{-1}(0.5\omega) - \tan^{-1}(0.2\omega)$$

$$= 0 - 2\times 90^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.2\omega)$$

$$\Rightarrow \angle G(j\omega) = -180^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.2\omega)$$

Finding $\angle G(j\omega)$ by substituting different frequency(ω) Values.

ω	$\angle G(j\omega)$
0.01	$\approx -180°$ (= 180.40)
0.1	-184°
1	-218°
2	-246°
10	-322
0.001	$-180.04 \approx -180^\circ$

By using the table data phase plot is drawn.

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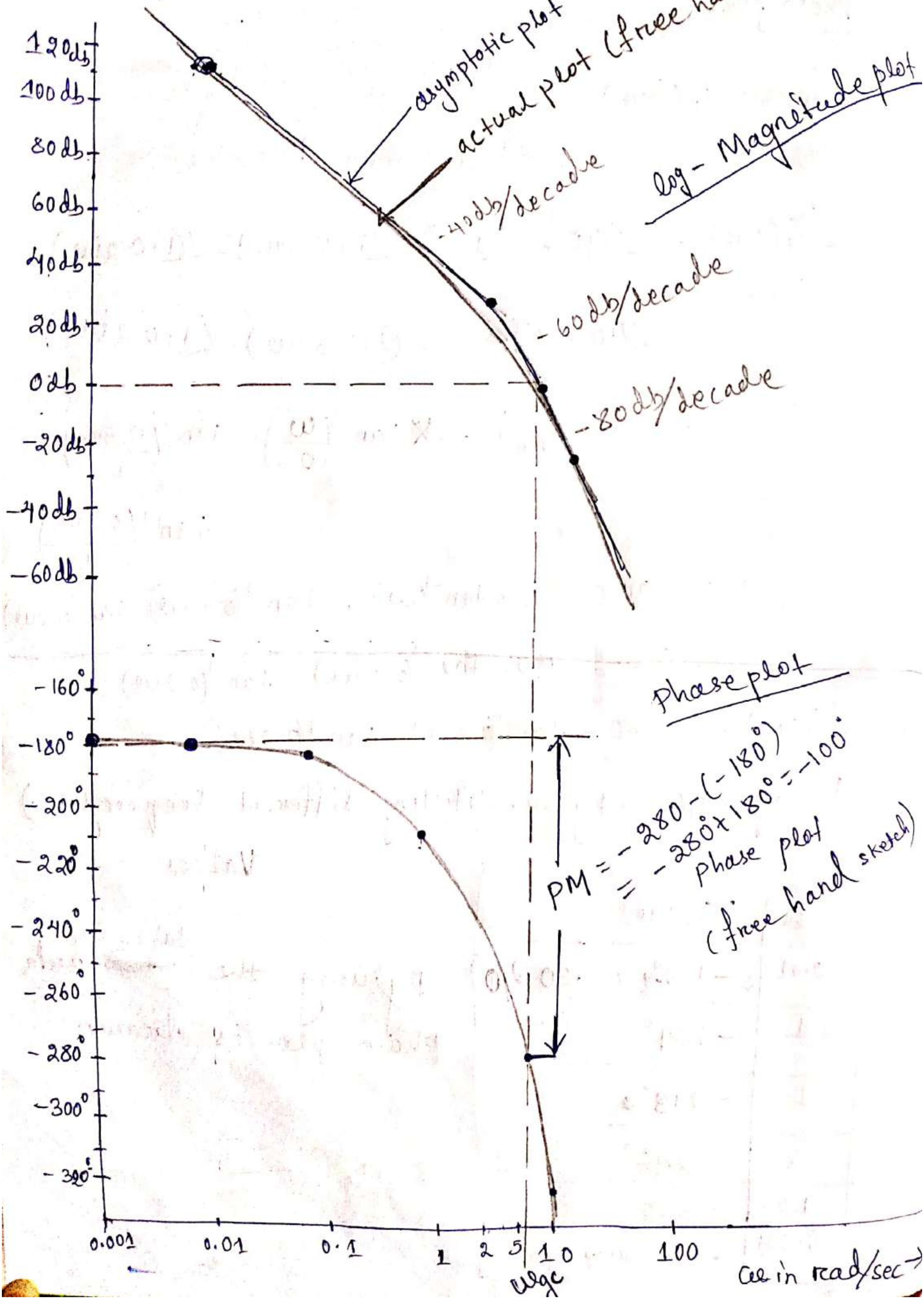
Bode plot



asymptotic plot
actual plot (free hand)
log - Magnitude plot
-40db/decade
-60db/decade
-80db/decade
120db
100db
80db
60db
40db
20db
0db
-20db
-40db
-60db
Phase plot
-160°
-180°
-200°
-220°
-240°
-260°
-280°
-300°
-320°
PM = -280-(-180°)
= -280°+180° = -100°
phase plot
(free hand sketch)
0.001
0.01
0.1
1
2
5
10
100
ωgc
ω in rad/sec→

From the Bode plot,

→ **PM**

$\omega_{gc} \simeq 6$

At ω_{gc}, the phase angle is $\simeq -280^\circ$

So the $PM = -280^\circ - (-180^\circ)$

$= -280^\circ + 180^\circ = -100^\circ$

→ **GM**

Phase crossover frequency can not be determined from the plot. But we can determine phase - crossover frequency $\omega_{pc} \simeq 0$ (very small positive no.)

So we can see the magnitude at ω_p is nearly ∞

So $GM = 0\,db - \infty$

$= -\infty$

Since both PM & GM are negative, the System is unstable.

Imp

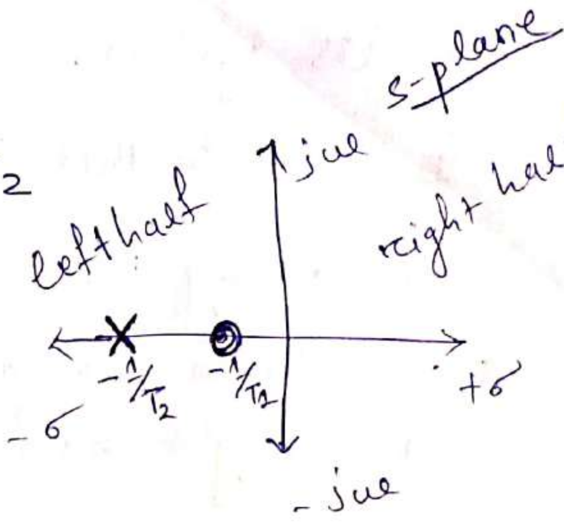
ALL-PASS AND MINIMUM-PHASE SYSTEM

Minimum phase Transfer function: The transfer function in which all poles & zeros are present in the left half of the s-plane.

Ex: $G(j\omega) = \dfrac{1 + j\omega T_1}{1 + j\omega T_2}$

Pole is $-1/T_2$

Zero is $-1/T_1$



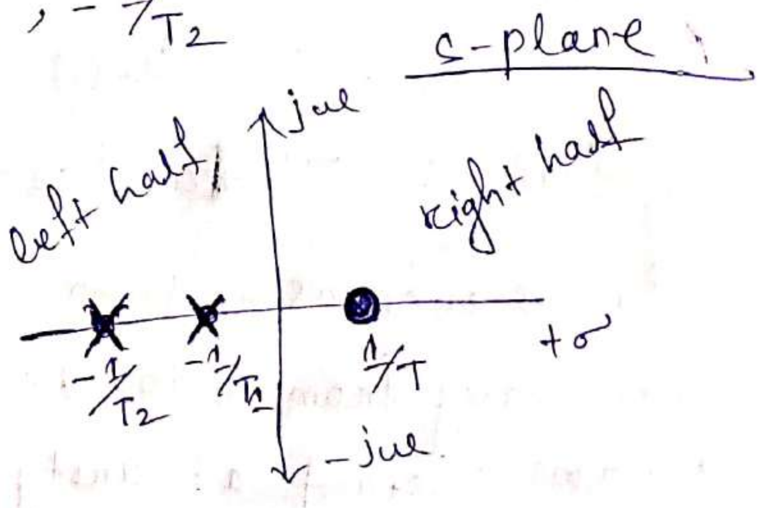
Non-Minimum phase Transfer function

The transfer function in which one or, more zeros present in the right half of the s-plane, then it is called non minimum phase Transfer function.

Ex: $G(j\omega) = \dfrac{(1 - j\omega T)}{(1 + j\omega T_1)(1 + j\omega T_2)}$

Poles are: $-1/T_1$, $-1/T_2$

Zero: $+\dfrac{1}{T}$



All pass System

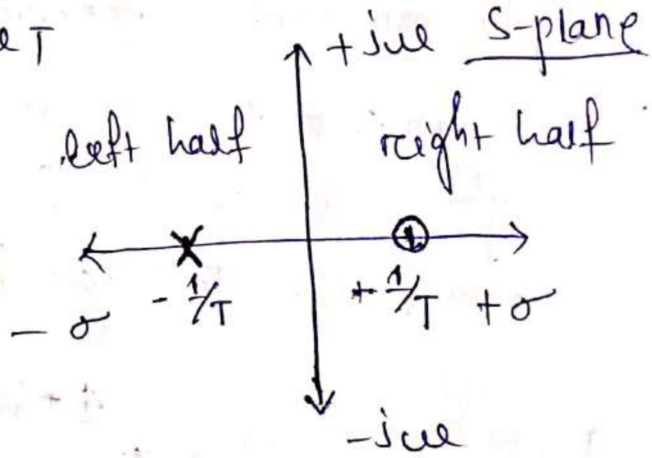
The System Transfer function, which has unit (1) magnitude for all frequencies (ω) is called as all pass system.

→ In this system transfer function, for every pole in the left half, there is a zero in the mirror image position (same position) in the right half of s-plane.

Ex: $G(j\omega) = \frac{1 - j\omega T}{1 + j\omega T}$

Pole : $-1/T$

Zero : $+1/T$



$|G(j\omega)| = 1$

$\angle G(j\omega) = \tan^{-1}\left(\frac{-\omega T}{1}\right) - \tan^{-1}\left(\frac{\omega T}{1}\right)$

$= -\tan^{-1}(\omega T) - \tan^{-1}(\omega T)$

$\Rightarrow \angle G(j\omega) = -2\tan^{-1}(\omega T)$

for ω increased from 0 to ∞ phase angle will vary from $0°$ to $-180°$. But the magnitude remains constant at unity (1).

So the above transfer function shows unit magnitude for all frequencies. Therefore it is an all-pass system.

All pass & Minimum phase systems

Consider a non-minimum phase system, which has poles in the left half of the s-plane and zeros in both left half & right half of the s-plane.

$$\text{Ex:} \quad G(j\omega) = \frac{1 - j\omega T}{(1 + j\omega T_1)(1 + j\omega T_2)}$$

It can be written as —

$$\Rightarrow G(j\omega) = \left[\frac{1 + j\omega T}{(1 + j\omega T_1)(1 + j\omega T_2)}\right] \times \left[\frac{(1 - j\omega T)}{(1 + j\omega T)}\right]$$

$$= G_1(j\omega) \times G_2(j\omega)$$

So $G(j\omega)$ can be written as product of two transfer functions.

$$\rightarrow G_1(j\omega) = \frac{(1 + j\omega T)}{(1 + j\omega T_1)(1 + j\omega T_2)}$$

Since it has no zeros / poles in the right half of s-plane, it is minimum phase transfer function

$$\rightarrow G_2(j\omega) = \frac{(1 - j\omega T)}{(1 + j\omega T)}$$

This is an all pass transfer function.

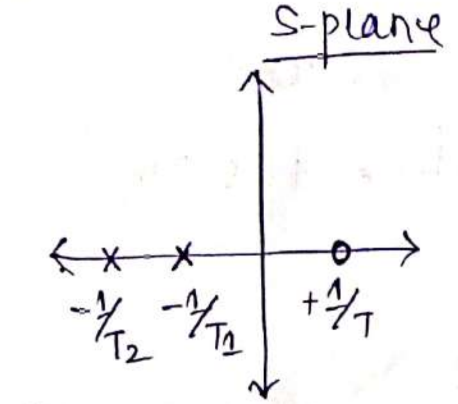


Figure: Pole-zero pattern of $G(j\omega)$

non-minimum phase function

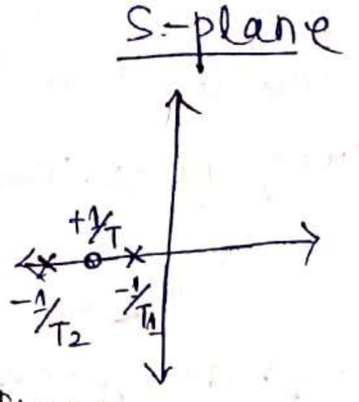


Figure: Pole zero pattern of $G_1(j\omega)$

minimum phase function

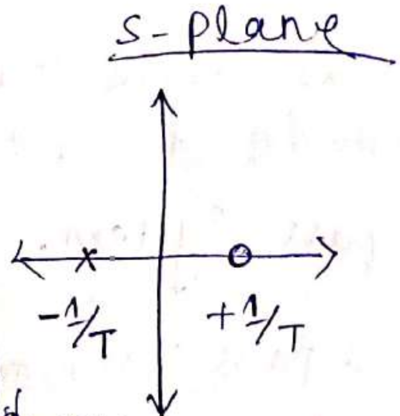


Figure: Pole zero pattern of $G_2(j\omega)$

all pass function

→ The magnitude of $G_1(j\omega)$ is equal to $G(j\omega)$ but their phase are different for various frequencies.

→ So by adding $G_2(j\omega)$ to $G_1(j\omega)$, the phase plot can be changed without affecting the magnitude.

→ So in a non-minimum phase transfer function it is possible to extract all zeros from the right half of the S-plane, by adding one all pass system transfer function in it.

→ Each time when the above process is done the magnitude curve remains same but the phase-lag is reduced.

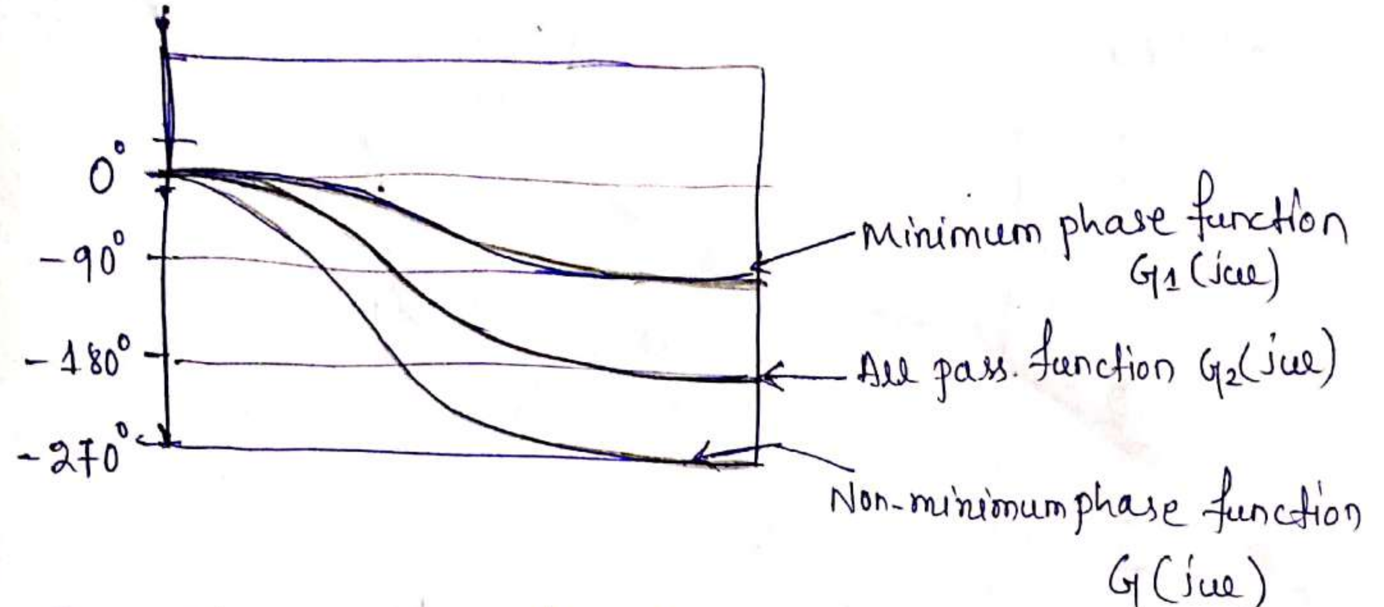


Fig: Phase-Angle characteristics

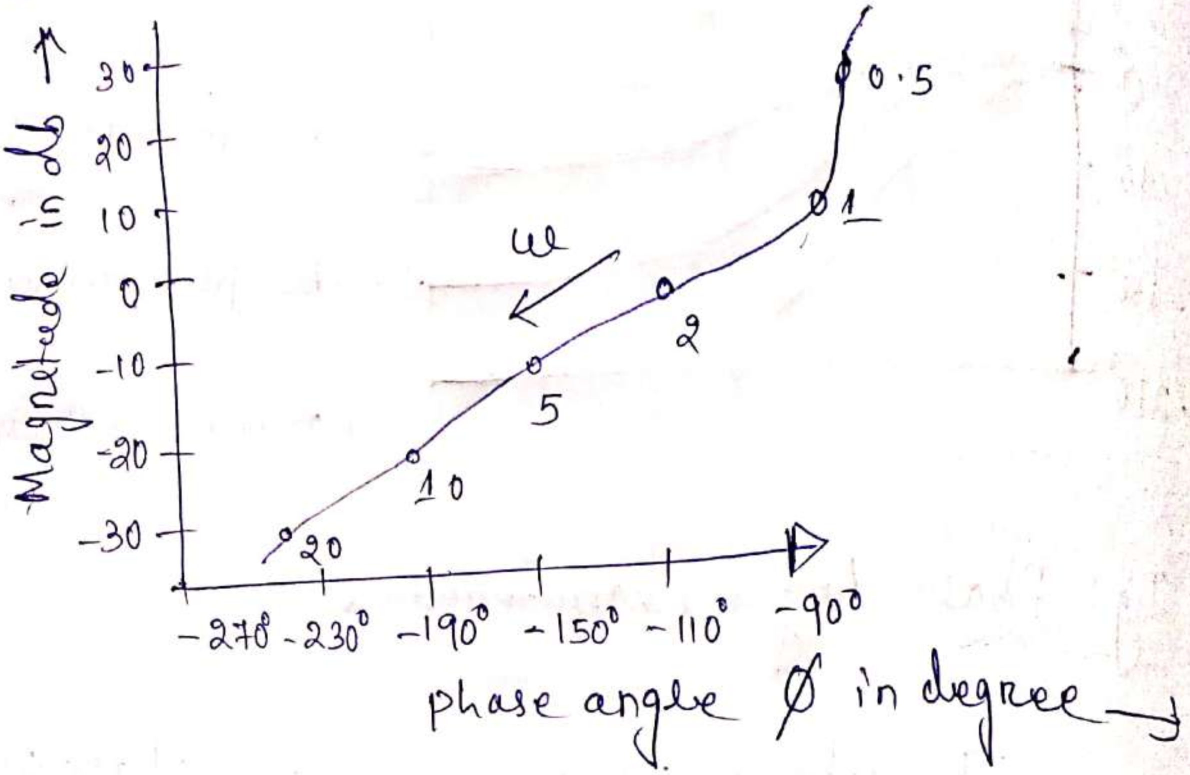
The above figure shows the phase angle characteristics of minimum phase, All-pass & non-minimum phase transfer functions.

Log-Magnitude Versus phase plot

This is a plot between Log-Magnitud and phase angle. which is constructed from bode plot.

-> In this method first the Bode plot is obtained. Then by reading the values of log magnitude and phase angle at different frequencies, various points are indicated on Log-magnitud Vs. phase plot.

-> ~~For~~ Advantage: From this plot the relative stability of closed-loop control system can be determined quickly & compensation can be carriedout easily.



Closed loop frequency Response

- The Study of closed-loop frequency response is very useful as it enables us to predict approximately the time response of feedback systems.
- With the help of correlations, the time response specifications are first converted in to a set of specifications in frequency domains.
- After design & compensation in frequency domain, the frequency response is transformed back to approximate time response.

Usually the specifications in frequency domain are given in the following terms.

1. Resonant Peak (M_r): It is the maximum value of magnitude (M) of the closedloop frequency response.

→ A large resonant Peak corresponds to a large peak overshoot in transient response.

2. Resonant frequency (ω_r): This is the frequency (ω) at which resonant Peak (M_r) occurs.

→ It is related to frequency of oscillation in the Step response in time domain. So it indicates the speed of transient response.

3. Bandwidth: Bandwidth is the range of frequencies for which the system gain is more than -3db.

→ Bandwidth is used to measure the ability of a feedback System to reproduce the input signal & measure noise rejection characteristics.

→ It also indicates the risetime in transient response. for a given damping factor.

A large bandwidth corresponds to Small rise time.

4. Cut-off rate: It is the slope of log-magnitude curve near the cut-off frequency.

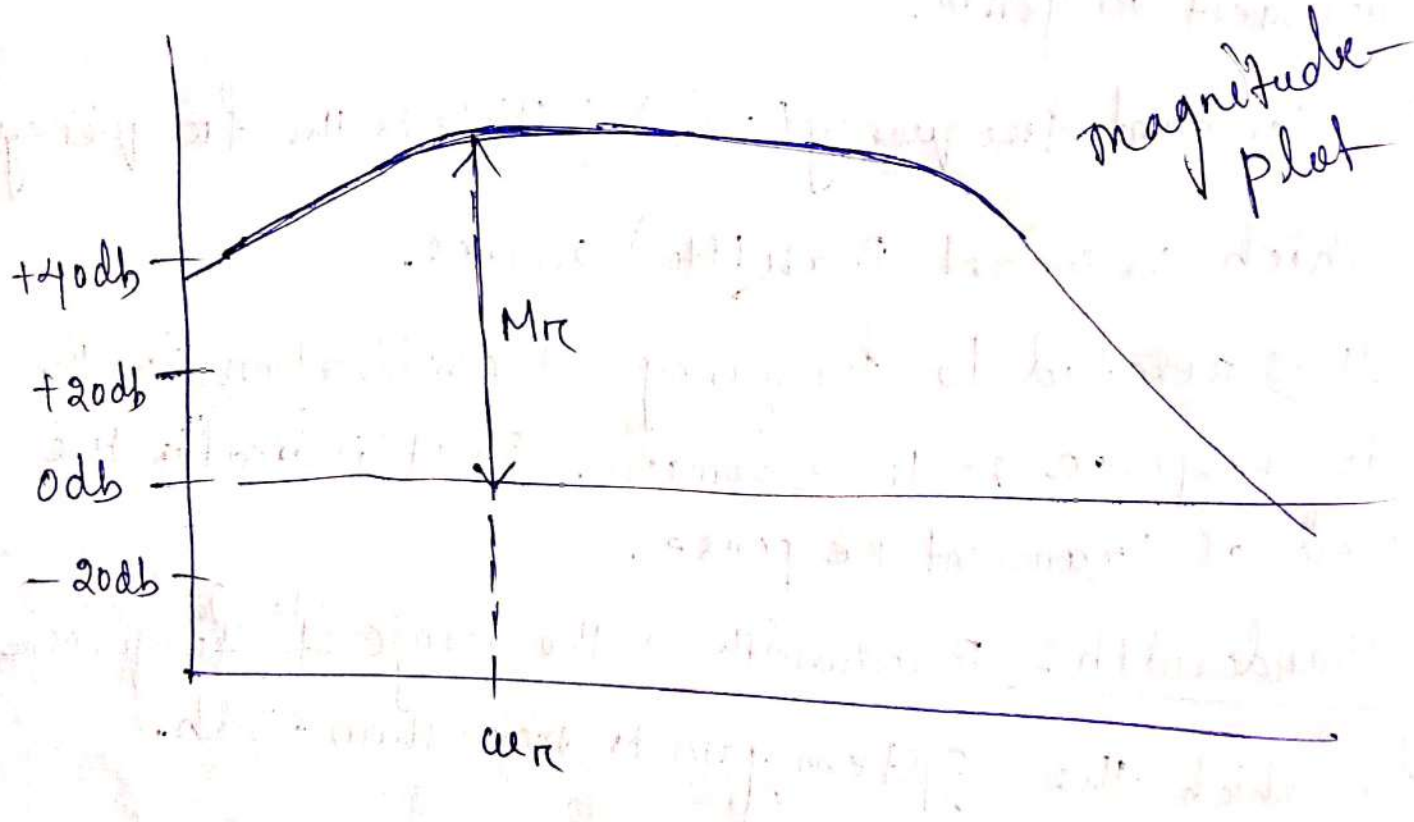
→ It indicates the ability of the system to distinguish the signal from noise.

5. Gain Margin & Phase Margin :

(* It is already defined)

→ These are the measure of relative stability.

→ GM & PM are related to the closeness of the closeloop-poles to the jω-axis.



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$$G(s) = \frac{10}{s(1+0.5s)(1+0.01s)}$$

Draw Bode Plot of the above Transfer function. Also find GM & PM.

Soln?

Given $$G(s) = \frac{10}{s(1+0.5s)(1+0.01s)}$$

It is in time constant form.

$$G(j\omega) = \frac{10}{j\omega(1+0.5j\omega)(1+0.01j\omega)}$$

Log-magnitude Plot

The characteristics of each factor of transfer function are given below.

factors	Corner frequency	Slope	change in slope
$\frac{10}{j\omega}$	—	-20 db/decade	—
$\frac{1}{(1+0.5j\omega)}$	$\omega_{c1} = \frac{1}{0.5} = 2$	-20 db/decade	-20 - 20 = -40 db/decade
$\frac{1}{(1+0.01j\omega)}$	$\omega_{c2} = \frac{1}{0.01} = 100$	-20 db/decade	-40 - 20 = -60 db decade

Scanned by CamScanner

Considering a frequency lower than ω_{c1},

$\omega_l = 0.1$

Considering a frequency higher than ω_{c2},

$\omega_h = 1000$

Magnitude (A) at corner frequencies —

at $\omega = \omega_l = 0.1$

$$A_{\omega l} = 20 \log \left|\left(\frac{10}{j\omega}\right)\right|$$ (first factor is taken)

$$= 20\log 10 - 20 \log \omega$$

$$= 20 \log 10 - 20 \log(0.1)$$

$$= 40\, db$$

at $\omega = \omega_{c1} = 2$

$$A_{\omega c1} = 20 \log \left|\frac{10}{j\omega}\right|$$ (first factor is taken)

$$= 20 \log 10 - 20 \log \omega$$

$$= 20 \log 10 - 20 \log 2$$

$$= 13.97 \simeq 14\, db$$

at $\omega = \omega_{c2} = 100$

$$A_{\omega c2} = [\text{Slope from } \omega_{c1} \text{ to } \omega_{c2}] \times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right) + A_{\omega c1}$$

$$= -40 \times \log\left(\frac{100}{2}\right) + 13.97$$

$$= -53.98 \simeq -54\, db$$

At $\omega = \omega_h = 1000$

$$A_{\omega_h} = [\text{slope from } \omega_{c_2} \text{ to } \omega_h] \times \log\left(\frac{\omega_h}{\omega_{c_2}}\right) + A_{\omega_{c_2}}$$

$$= -60 \times \log\left(\frac{1000}{100}\right) + (-54)$$

$$= -60 \times \log 10 - 54 = -114 \text{ db}$$

So we obtained

ω	Gain/Magnitude
0.1	40 db
2	14 db
100	-54 db
1000	-114 db

using these data log-magnitude plot will be drawn.

Phase plot

We got $G(j\omega) = \dfrac{10}{j\omega\,(1+0.5j\omega)\,(1+0.01j\omega)}$

$$\angle G(j\omega) = \angle 10 - \angle(j\omega) - \angle(1+0.5j\omega) - \angle(1+0.01j\omega)$$

$$= \tan^{-1}\left(\frac{0}{10}\right) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{0.5\omega}{1}\right) - \tan^{-1}\left(\frac{0.01\omega}{1}\right)$$

$$= \tan^{-1}0 - \tan^{-1}\infty - \tan^{-1}(0.5\omega) - \tan^{-1}(0.01\omega)$$

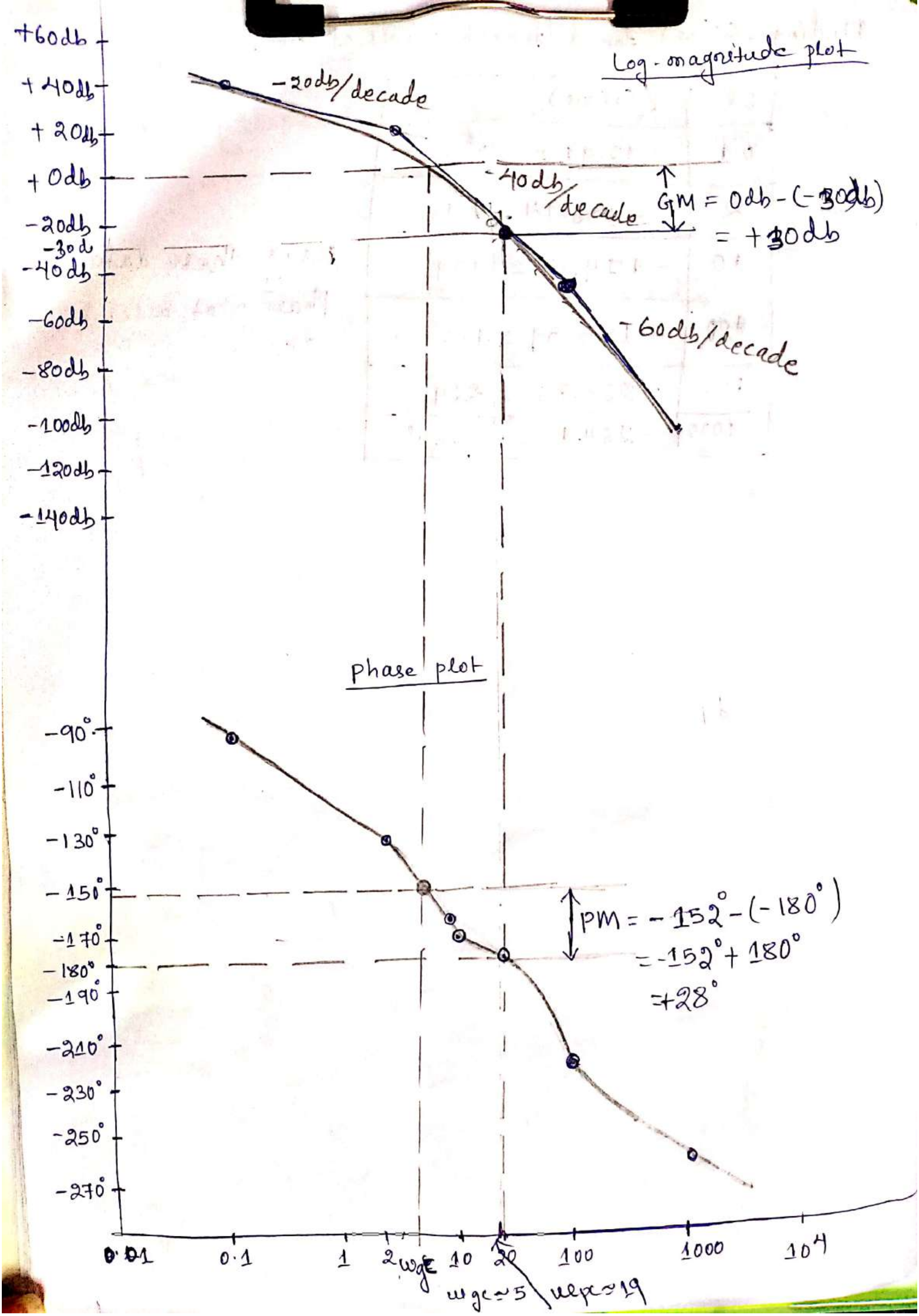
$$= 0^\circ - 90^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.01\omega)$$

$$= -90^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.01\omega)$$

Finding $\angle G(j\omega)$ for different values of 'ω' —

ω	$\angle G(j\omega)$
0.1	$-92.91 \simeq -93^\circ$
2	$-136.14 \simeq -136^\circ$
10	$-174.4 \simeq -174^\circ$
20	$-185.59 \simeq -186^\circ$
100	$-223.85 \simeq -224^\circ$
1000	$-264.1 \simeq -264^\circ$

Using these data Phase plot will be drawn


Log-magnitude plot
+60db
+40db
+20db
+0db
-20db
-30db
-40db
-60db
-80db
-100db
-120db
-140db
-20db/decade
-40db/decade
-60db/decade
GM = 0db - (-30db)
= +30db
Phase plot
-90°
-110°
-130°
-150°
-170°
-180°
-190°
-210°
-230°
-250°
-270°
PM = -152° - (-180°)
= -152° + 180°
= +28°
0.01
0.1
1
2
ωgc
10
20
100
1000
10^4
ωgc ≈ 5
ωpc ≈ 19

From the plot —

Phase Margin :

The value of ω_{gc} from the graph is approximately $\simeq 5$

At ω_{gc}, Phase angle is approx. -152°

So the phase Margin (PM) $= -152^\circ + 180^\circ$

$\Rightarrow$ $\boxed{PM = +28^\circ}$

Gain Margin :

The value of ω_{pc} from the graph is approx. $\simeq 19$

At ω_{pc}, Gain is approx. $-30\,db$.

So Gain Margin (GM) $= 0\,db - (-30\,db)$

$\boxed{GM = +30\,db}$

Since both GM & PM are positive, the system is relatively stable.

Ch-5 : NYQUIST PLOT

Principle of Argument

Let us consider a function $q(s)$ which can be expressed as quotient of two polynomials.

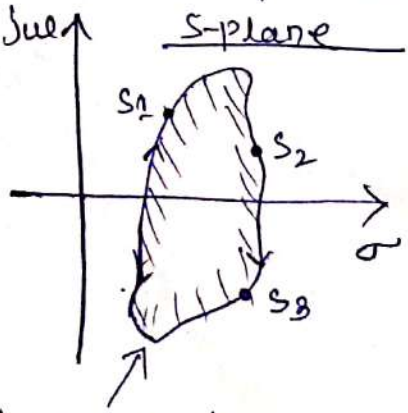
$$q(s) = \frac{(s-\alpha_1)(s-\alpha_2)\cdots(s-\alpha_m)}{(s-\beta_1)(s-\beta_2)\cdots(s-\beta_n)} \quad \text{— eq(i)}$$

→ Each polynomial is assumed to be known in the form of product of linear factors as shown above.

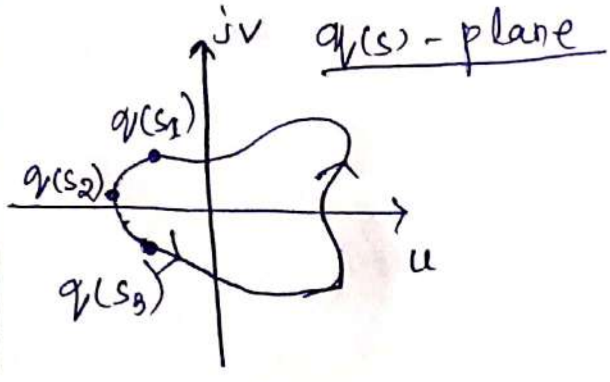
S is a complex variable, represented by — $s = \sigma + j\omega$ on the complex s-plane.

Since s is a complex variable, $q(s)$ is also complex & can be represented by $q(s) = u + jv$

→ So from eq(i), we can find that, for every point s in the s-plane, we can find $q(s)$ point in the $q(s)$-plane.



Arbitarily chosen s-plane contour



Corresponding q(s)- plane contour.

→ Since any no. of points of the s-plane, can be mapped into the q(s)-plane.

So for a contour ~~of~~ in the s-plane, there corresponds a contour in the q(s)-plane as shown in the above figure.

Principle of argument:

If there are 'P' poles & 'Z' zeros of q(s) enclosed by the s-plane contour, then the corresponding q(s)-plane contour must encircle the origin

- P times in the counter-clockwise direction.
- Z times in the clockwise direction.

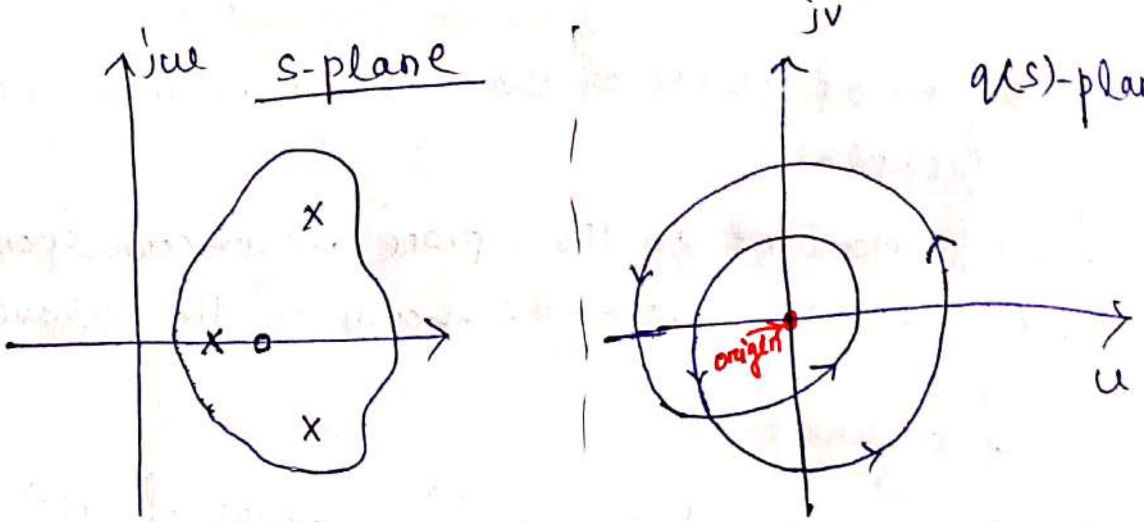
resulting in a net encirclement of the origin (P-Z) times in the counter-clockwise direction.

This relationship between the enclosure of poles and zeros of q(s) by the s-plane & the encirclement of the origin by q(s)-plane ~~count~~ contour is commonly known as principle of argument.

Ex: Let 1 zero & 3 poles enclosed by s-plane contour, then the net encirclement of the origin by the q(s)-plane contour is (3-1) = 2 counter clockwise revolutions.

2 counter clockwise revolution = $2 \times 2\pi$
$= 4\pi$ rad.

Diagramatically it is shown as —

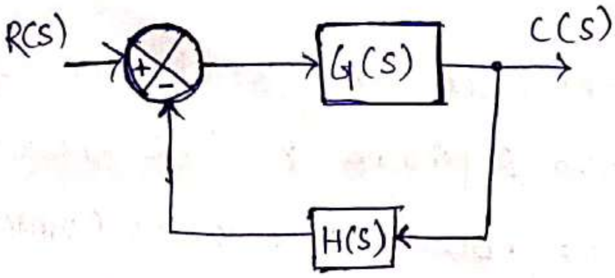


3 poles and 1 zero enclosed by s-plane contour

two counter-clockwise revolutions around origin

Fig: Mapping of the s-plane contour with q(s) plane

NYQUIST STABILITY CRITERION



Consider the above single loop feedback system.

The characteristic equation of the system is

$$q(s) = 1 + G(s)H(s) = 0 \quad \text{--- (1)}$$

→ We know the standard polezero form of the open loop transfer function is –

$$G(s)H(s) = K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)};$$

where $m \leq n$

Substing the value of $G(s)H(s)$ in eq(i) —

$$q(s) = 1 + K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)} = 0$$

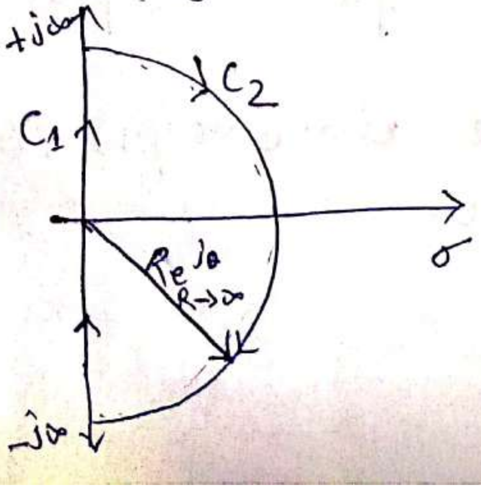
$$\Rightarrow \frac{(s+P_1)(s+P_2)\cdots(s+P_n)+K(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)} = 0$$

$$\Rightarrow \frac{(s+z_1')(s+z_2')\cdots(s+z_m')}{(s+P_1)(s+P_2)\cdots(s+P_n)} = 0$$

So from the above eqn $-z_1', -z_2' \cdots -z_m'$ are the zeros & $-P_1, -P_2 \cdots -P_n$ are the poles of $q(s)$.

For a stable system, the zeros of $q(s)$ must lie in the left half of the s-plane.

→ To investigate (check) the presence of any zero in the right half of $q(s)$ plane, let us choose a contour which completely encloses this right half of the s-plane.



This contour in the s-plane is called Nyquist contour.

→ The Nyquist contour comprises of two segments C_1 & C_2.

C_1 : It is an infinite linesegment along $j\omega$ axis

C_2 : It is an infinite radius arc.

→ Along C_1, s varies from $-j\infty$ to $+j\infty$, $s = j\omega$

→ Along C_2, $s = Re^{j\theta}$, $R \to \infty$, where θ varies from $+\pi/2$ to 0 to $-\pi/2$

The Nyquist contour encloses all the right half s-plane zeros & poles of $q(s) = 1 + G(s)H(s)$

→ We know the closed contour traverse the origin for $\boxed{N = P - Z}$ times in counter-clockwise direction. — eq(ii)

→ In order for the system to be stable, there should not be any zeros in the right half s-plane i.e $Z = 0$.

Putting this in the above eq(ii), we get

$N = P - 0$

$\Rightarrow \boxed{N = P}$

So for a closed loop system to be stable, the number of counter-clockwise encirclement of the origin of the q(s) plane contour should be equal to number of right half s-plane poles.

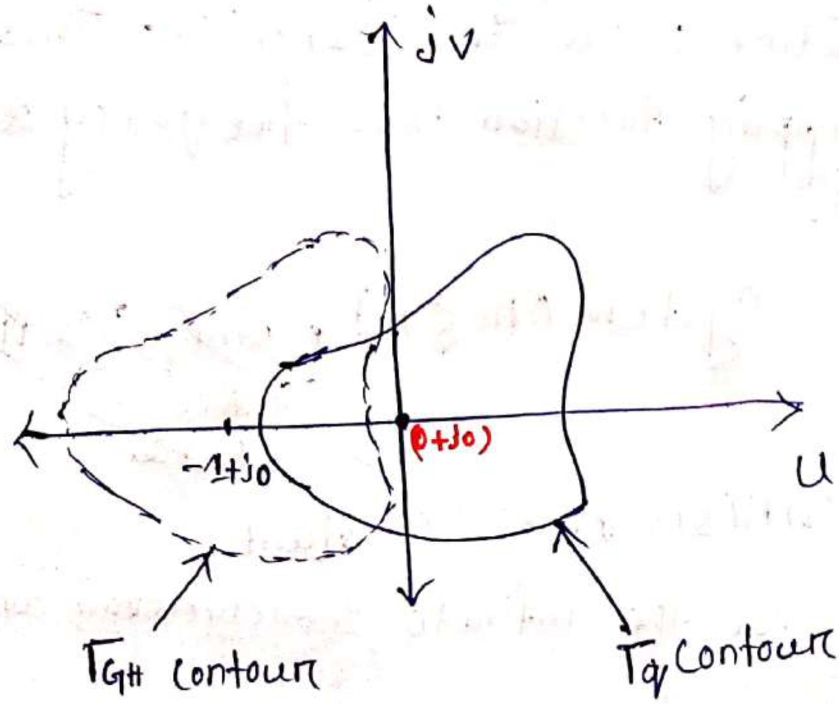
We can write,

$$G(s)H(s) = [1+G(s)H(s)] - 1$$

Let the contour of above G(s)H(s) is Γ_{GH} (open loop)

Γ_q is the Nyquist contour of $q(s) = 1+G(s)H(s)$ (closed loop)

The encirclement of the origin by Γ_q contour is equivalent to the encirclement of the point (−1+j0) by the Γ_{GH} contour as shown in the below figure.



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So the Nyquist stability criterion is stated as –

" If the contour Γ_{GH} of the openloop transfer function $G(s)H(s)$ corresponding to Nyquist contour in the s-plane, encircles the point $(-1+j0)$ in the counter-clockwise direction as many times as the number of right half s-plane poles of $G(s)H(s)$, then the closed-loop system is stable."

<u>Mapping of the Nyquist contour into the Contour Γ_{GH} is carried out as below :</u>

1. The mapping of the imaginary axis is carried out by substituting $s = j\omega$ in $G(s)H(s)$. This converts the mapping function into frequency domain $G(j\omega)H(j\omega)$.

2. In physical system $(m \leq n)$, ~~$\lim_{\substack{s=Re^{j\theta} \\ R\to\infty}} G(s)H(s)$~~:

$$\lim_{\substack{s=Re^{j\theta} \\ R\to\infty}} G(s)H(s) = \text{real Constant.}$$

for the infinite semicircular arc.

→ The complete contour of Γ_{GH} is thus the polar plot of $G(j\omega)H(j\omega)$ with varying ω from $-\infty$ to $+\infty$. This is called Nyquist plot / locus plot of $G(s)H(s)$.

→ Nyquist plot is symmetrical about the real axis since –
$G(j\omega)H(j\omega) = G(-j\omega)H(-j\omega)$

Open-loop Poles on jω-Axis

If G(s)H(s) or, 1+G(s)H(s) has any open loop poles on the jω-axis, the Nyquist contour will be different, as the s-plane contour should not pass through a singularity of 1+G(s)H(s).

-> So the Nyquist is modified to bypass any jω-axis poles.

-> This is done by indenting the nyquist contour around jω-axis poles along a semicircle of radius ε, where $\epsilon \to 0$

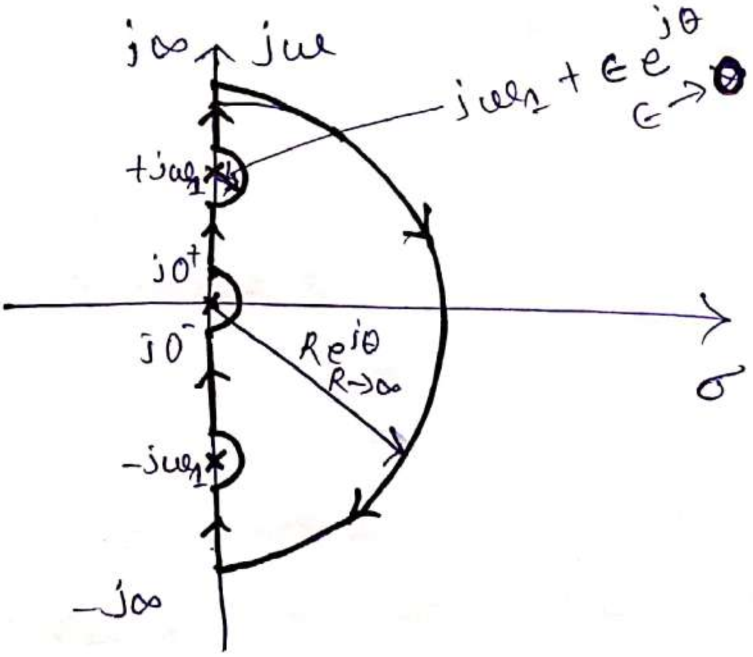


Fig: Indented Nyquist Contour for jω-axis open-loop poles.

-> The radius of the small contour around the open loop poles on jω-axis is given by —

$$j\omega_1 + \epsilon e^{j\theta}$$
$$\epsilon \to 0$$

where $j\omega_1$ is the pole on the jω-axis

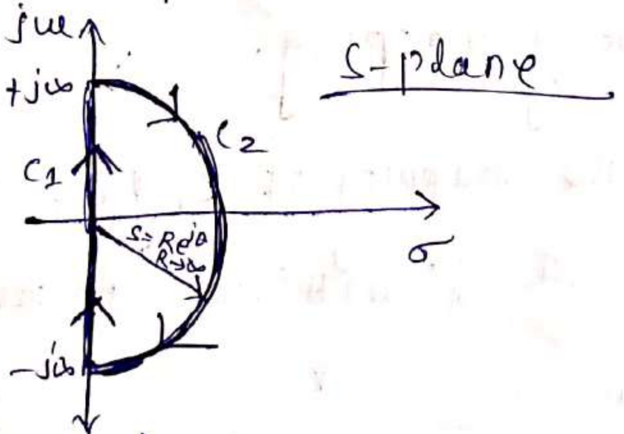
Ex consider a feedback system open-loop transfer function is given by $G(s)H(s) = \frac{K}{(T_1 s+1)(T_2 s+1)}$

Find whether the system is stable or, not by using Nyquist stability criterion.

Solu^n Given $G(s)H(s) = \frac{K}{(T_1 s+1)(T_2 s+1)}$

$$\Rightarrow G(j\omega)H(j\omega) = \frac{K}{(T_1 j\omega+1)(T_2 j\omega+1)}$$

There is no pole at the origin. So the Nyquist contour in the s-plane is



The mapping of the s-plane Nyquist contour to the q(s) plane

① The infinite semicircular arc C_2 is represent by $s = \lim_{R\to\infty} Re^{j\theta}$ (θ varies from $+90°$ to $-90°$).

By putting the value of s in G(s) H(s) we get —

$$\lim_{R\to\infty} \frac{K}{(T_1 Re^{j\theta}+1)(T_2 Re^{j\theta}+1)}$$

$$= \lim_{R\to\infty} \frac{K}{T_1T_2R^2e^{j2\theta} + T_1Re^{j\theta} + T_2Re^{j\theta} + 1}$$

$$\Rightarrow 0\, e^{j2\theta}$$

So the magnitude part will becom 0 if $R\to\infty$ & the θ will vary from $+90^\circ$ to -90°.

→ Since magnitude is 0, the semicircular contour will shrink to a point at the origin during mapping.

(2). To find the mapping of C_1 line segment, the polar plot of $G(j\omega)H(j\omega)$ is required.

$$G(j\omega)H(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{K(1-j\omega T_1)(1-j\omega T_2)}{(1+j\omega T_1)(1-j\omega T_1)(1+j\omega T_2)(1-j\omega T_2)}$$

$$= \frac{K(1-j\omega T_2-j\omega T_1+j^2\omega^2T_1T_2)}{\{1^2-(j\omega T_1)^2\}\{1^2-(j\omega T_2)^2\}}$$

$$= \frac{K(1 - j\omega T_1 - j\omega T_2 - \omega^2 T_1 T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

$$= \frac{K[(1-\omega^2 T_1 T_2) - j\omega(T_1+T_2)]}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

$$= \frac{K(1-\omega^2 T_1 T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)} - \frac{jK\omega(T_1+T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

Varying $\omega = 0$ to ∞

At $\omega = 0$,

$$G(j\omega)H(j\omega) = \frac{K(1-0)}{(1+0)(1+0)} - \frac{jK\times 0(T_1+T_2)}{(1+0)(1+0)}$$

$$= K - j0$$

At $\omega = \infty$,

$$G(j\omega)H(j\omega) = 0 - j0$$

When the polar plot crosses imaginary axis real part of $G(j\omega)H(j\omega) = 0$, so

$$\frac{K(1-\omega^2 T_1 T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)} = 0$$

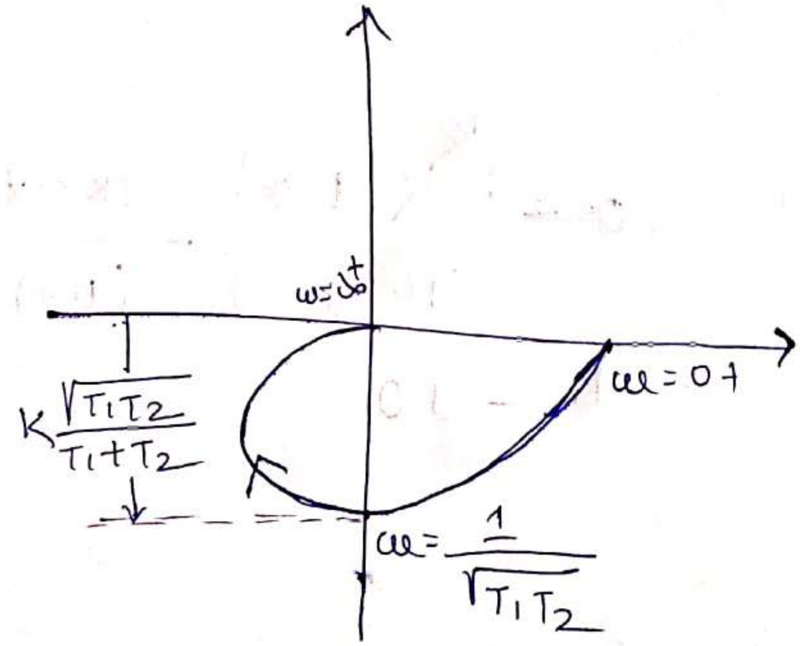
$$\Rightarrow K(1-\omega^2 T_1 T_2) = 0$$

$\Rightarrow \quad 1 - \omega^2 T_1 T_2 = 0$

$\Rightarrow \quad \omega^2 = \frac{1}{T_1 T_2}$

$\Rightarrow \quad \omega = \frac{1}{\sqrt{T_1 T_2}}$ B

So the polar plot is —



By putting $\omega = \frac{1}{\sqrt{T_1T_2}}$ in imaginary part of $G(j\omega)H(j\omega)$

we get, Im part $= \frac{K\omega(T_1+T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$

$$= \frac{K \times \frac{1}{\sqrt{T_1T_2}}(T_1+T_2)}{\left(1+\frac{1}{T_1T_2}\times T_1^2\right)\left(1+\frac{1}{T_1T_2}\times T_2^2\right)}$$

$$= \frac{K \times \frac{1}{\sqrt{T_1T_2}}(T_1+T_2)}{(1+T_1/T_2)(1+T_2/T_1)}$$

$$= K \frac{\sqrt{T_1 T_2}}{T_1 + T_2}$$

(3) The complete Nyquist contour mapping is symmetrical about the real axis. So the mirror image of polar plot is drawn.

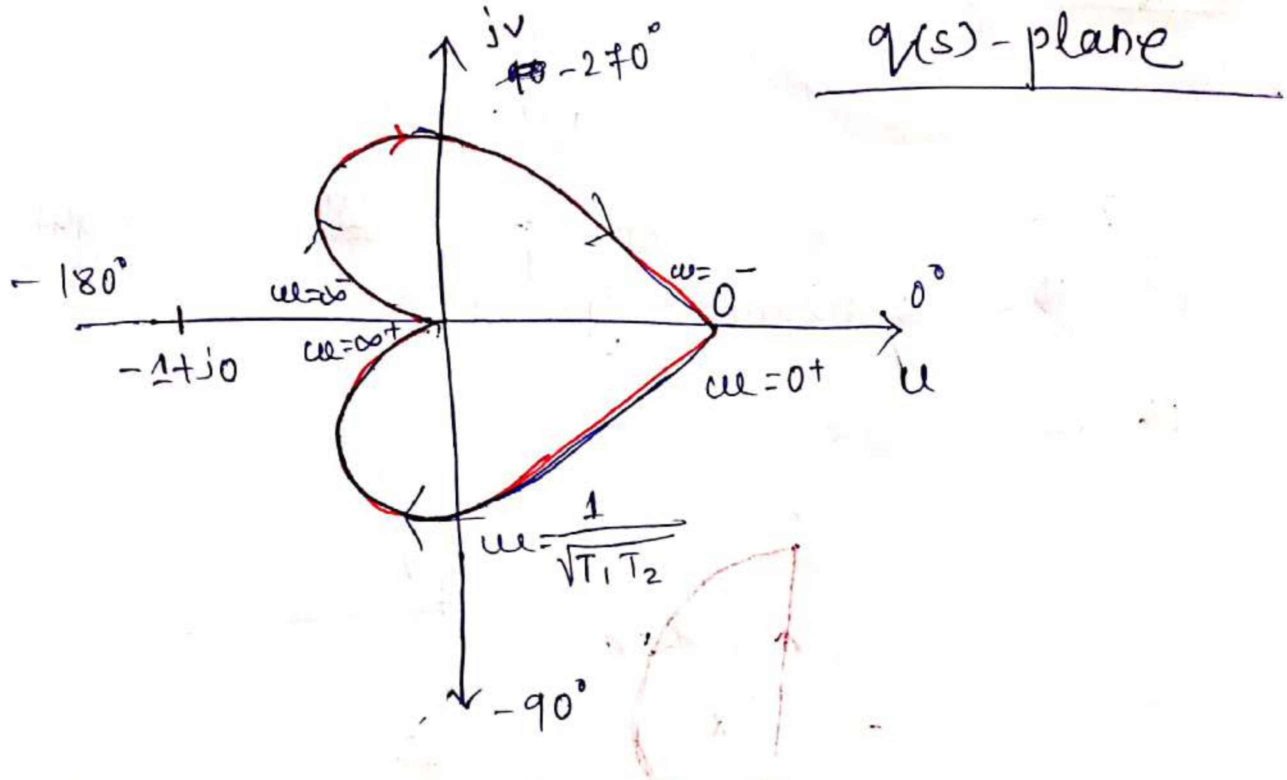


Fig: Nyquist plot

Since the above Nyquist ~~contour~~ contour encircles the $(-1+j0)$ point 0 times. So $N = 0$.

From the $G(s)H(s) = \frac{K}{(T_1 s+1)(T_2 s+1)}$

There is no pole in the right half of s-plane.

So $P = 0$

Since $N = P$

The System is Stable.

Q: Given $G(s)H(s) = \frac{s+2}{(s+1)(s-1)}$

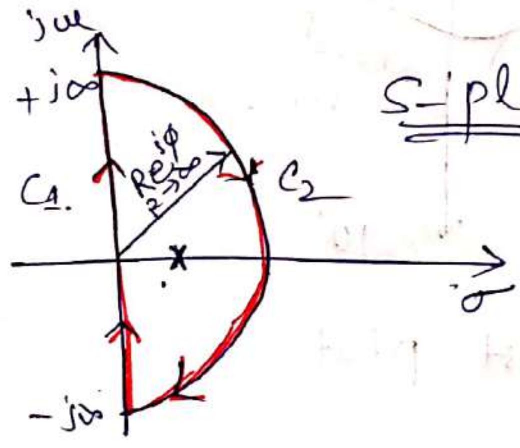
By using Nyquist stability criteria, find the stability of the system.

Ans

$$\text{Given} = \frac{s+2}{(s+1)(s-1)}$$

Since there is one pole in the right half of the s-plane, so $P = 1$.

Nyquist contour



Mapping of s-plane contour to q(s) plane

(1) For C_2 semicircle, the mapping will be a point at the origin.

(2) For C_1 line segment, polar plot is reqd. to draw the mapping contour.

Given $G(s)H(s) = \frac{(s+2)}{(s+1)(s-1)}$

$$G(j\omega)H(j\omega) = \frac{(j\omega+2)}{(j\omega+1)(j\omega-1)}$$

$$= \frac{j\omega+2}{\{(j\omega)^2 - 1^2\}}$$

$$= \frac{2+j\omega}{-1 + j^2\omega^2}$$

$$= \frac{2+j\omega}{-1 - \omega^2} = -\frac{(2+j\omega)}{(1+\omega^2)}$$

$$= -\frac{2}{(1+\omega^2)} - j\frac{\omega}{(1+\omega^2)}$$

At $\omega = 0$, $G(j\omega)H(j\omega) = -\frac{2}{1+0} + j\frac{0}{1+0}$

$= -2 - j0$ or, $-2 + j0$

At $\omega = \infty$ $G(j\omega)H(j\omega) = 0 - j0$ or, $0 + j0$

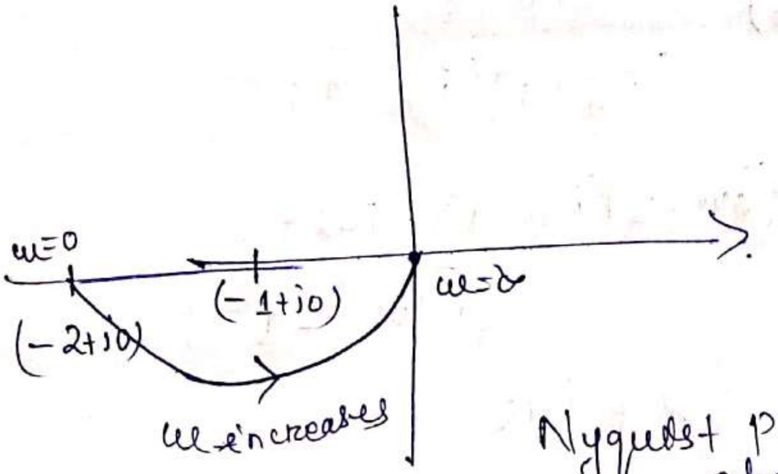
When polar plot crosses imaginary axis real part is 0.

So $-\frac{2}{1+\omega^2} = 0 \Rightarrow \omega = \infty$

So Polar plot only crosses imaginary axis at $\omega = \infty$

Similarly it only crosses the real axis at $\omega = 0$

So the Polar plot is ———



Nyquist plot is Symmetrical about the real axis.

③ So the Nyquist plot is ———

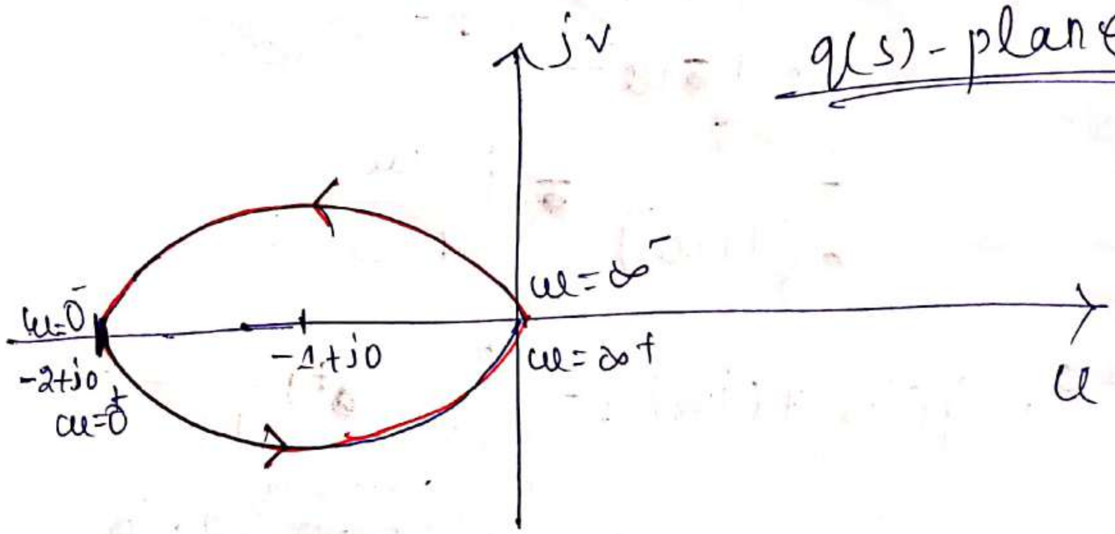


Fig: Nyquist plot.

So the plot encircles (−1+j0) point 1 time Counter clock wise.

So N = 1.

Since $N = P = 1$

The closed loop System is stable.

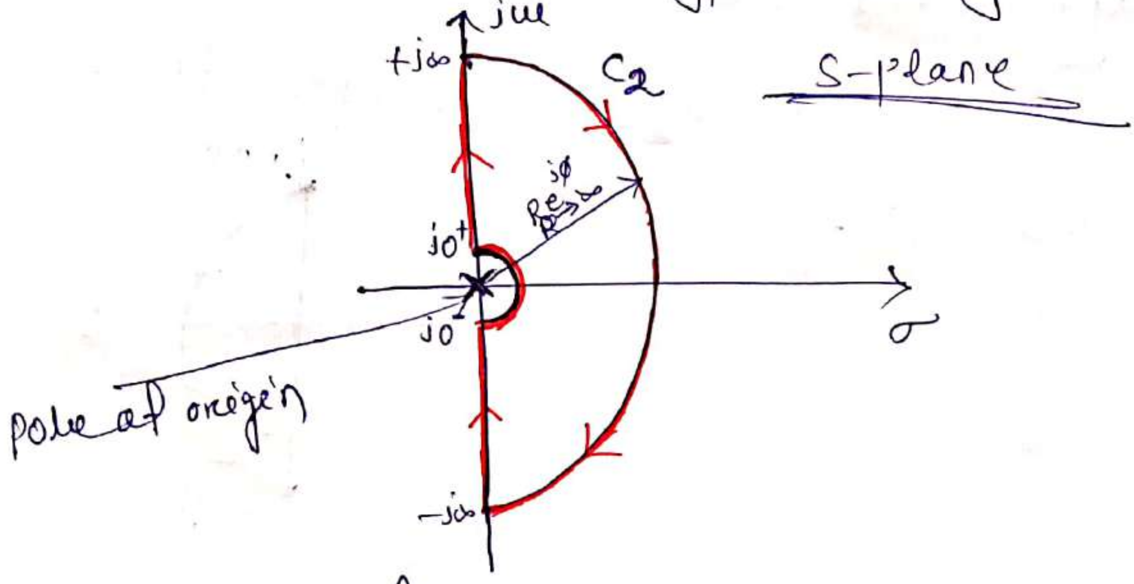
Q: $G(s)H(s) = \dfrac{K}{s(Ts+1)}$

Analyse the stability of above open loop T.F. by using Nyquist Stability criterion.

Solution

Given, $G(s)H(s) = \dfrac{K}{s(Ts+1)}$

In this case there is one pole at origin $(s=0)$. So the Nyquist contour must bypass the origin.



The mapping of the Nyquist contour in $q(s)$-plane is carried out as follows:

1. The infinite semicircular arc C_2, represented by $s = Re^{j\theta}_{R \to \infty}$ (θ varies from $+90^\circ$ to -90°). is mapped in to a point at the origin.

2. The semicircular indent around the origin is represented by $s = \epsilon e^{j\theta}_{\epsilon \to 0}$ (θ varies from -90° to $+90^\circ$)

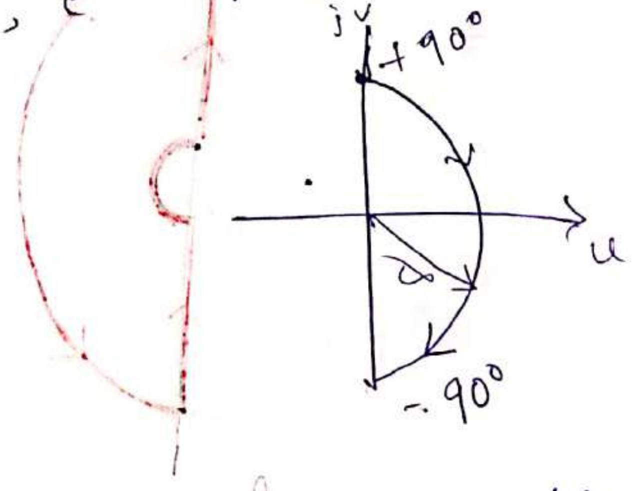
So by putting the value of $s = \epsilon e^{j\theta}$, $\epsilon \to 0$ in $G(s)H(s)$ it maps into —

$$\lim_{\epsilon \to 0} \frac{K}{(\epsilon e^{j\theta})(1+T\epsilon e^{j\theta})}$$

$$\Rightarrow \lim_{\epsilon \to 0} \frac{K}{\epsilon} e^{-j\theta}$$

$$\Rightarrow \infty e^{-j\theta}$$

K/ϵ approaches to ∞ as $\epsilon \to 0$ & θ varies from -90° to $+90^\circ$ So, $e^{-j\theta}$ varies from $+90^\circ$ to -90°.



3. The mapping of of positive imaginary axis ($\omega = 0^+$ to ∞^+) is obtained by drawing the polar plot for $\frac{K}{s(Ts+1)}$

$$G(j\omega)H(j\omega) = \frac{K}{j\omega\,(1+Tj\omega)}$$

$$= \frac{K\,(1-Tj\omega)(-j\omega)}{(j\omega)(-j\omega)\,(1+Tj\omega)(1-Tj\omega)}$$

$$= \frac{K(-j\omega + Tj^2\omega^2)}{(-j^2\omega^2)\{1^2 - (Tj\omega)^2\}}$$

$$= \frac{K(-j\omega - T\omega^2)}{\omega^2(1+T\omega^2)}$$

$$= \frac{-K(T\omega^2 + j\omega)}{\omega^2(1+T\omega^2)}$$

$$= \frac{-KT\omega^2}{\omega^2(1+T\omega^2)} - j\frac{K\omega}{\omega^2(1+T\omega^2)}$$

$$= \frac{-KT}{(1+T\omega^2)} - j\frac{K}{\omega(1+T\omega^2)}$$

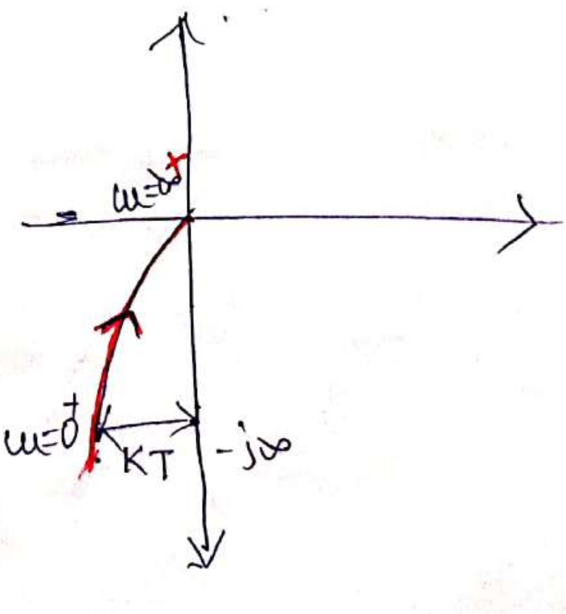
At $\omega = 0$

$G(j\omega)H(j\omega) = -KT - j\infty$

At, $\omega = \infty$

$G(j\omega)H(j\omega) = 0 - j0$

So the polar plot is -



4. The complete Nyquist contour in q(s)-plane is given by ______

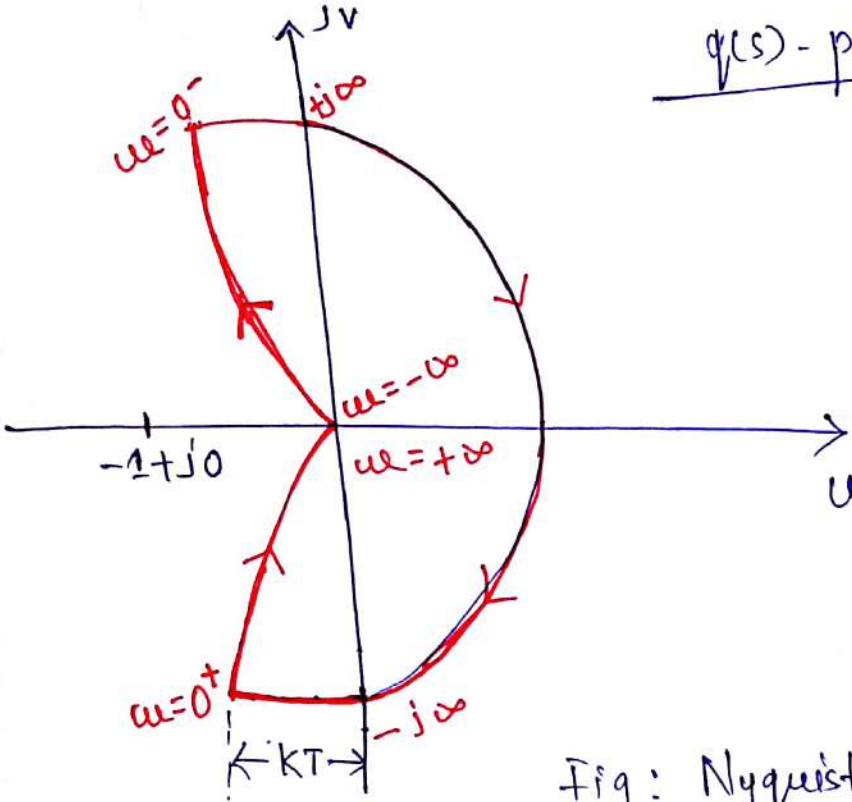


Fig: Nyquist contour plot.

In the above plot, there is no encirclement around the point $(-1+j0)$. So $N = 0$

From the $G(s) = \frac{K}{S(Ts+1)}$, there is no poles in the right side of the s-plane. so $P = 0$.

Since $\boxed{N = P = 0}$

The system is stable. Ans

Q: Apply Nyquist stability criterion to the system with loop Transfer Function given by —

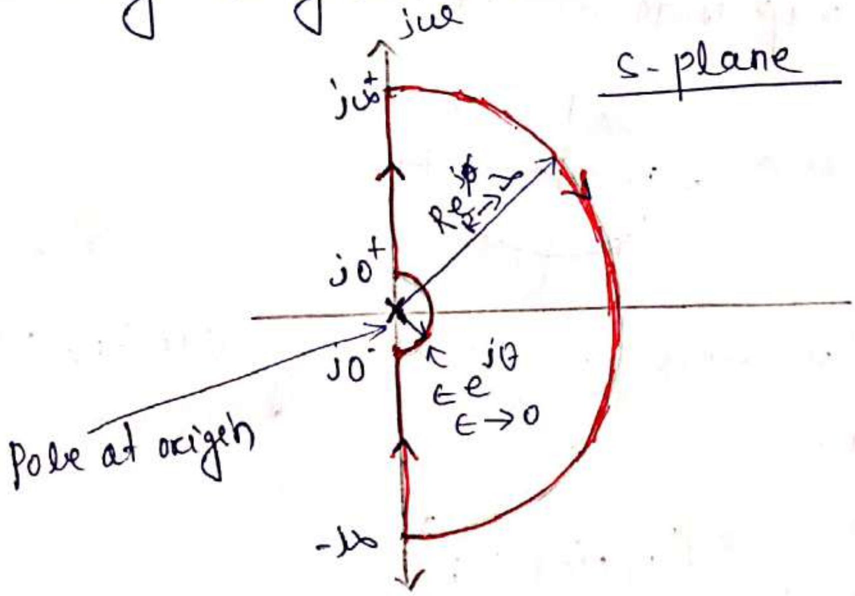
$$G(s)H(s) = \frac{(4s+1)}{s^2(s+1)(2s+1)}$$

and ascertain its stability. [Prev Yr question 2014 (S)(New)]

Soln:

Given $G(s)H(s) = \dfrac{(4s+1)}{s^2(s+1)(2s+1)}$

This system has two poles at the origin. So the Nyquist contour is therefore indented to bypass the origin as given below.



The mapping of Nyquist contour in $G(s)$-plane is carried out as follows:

1. The infinite semicircle of the Nyquist contour represented by $s = \lim_{R\to\infty} Re^{j\phi}$ (ϕ varies from $+90°$ through $0°$ to $+90°$) is mapped into a point at the origin on $G(s)$-plane.

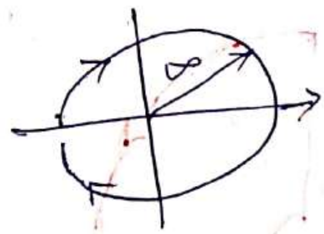
2. The semicircular indent around the origin is represented by $s = \lim_{\epsilon \to 0} \epsilon e^{j\theta}$ (where θ varies from -90° to $+90^\circ$, through 0°) is mapped into —

$$\lim_{\epsilon \to 0} \frac{(4\epsilon e^{j\theta} + 1)}{(\epsilon e^{j\theta})^2 (\epsilon e^{j\theta} + 1)(2\epsilon e^{j\theta} + 1)}$$ [putting $s = \lim_{\epsilon \to 0} \epsilon e^{j\theta}$ in $G(s)$]

$$= \lim_{\epsilon \to 0} \frac{1}{\epsilon^2 e^{2j\theta} \times 1 \times 1} = \lim_{\epsilon \to 0} \frac{1}{\epsilon^2} e^{-2j\theta}$$

$= \infty \angle 180^\circ \to \angle 0^\circ \to \angle -180^\circ$

So it will map into an infinite circle from 180° to -180°.



3. Along the $j\omega$-axis, the mapping can be drawn from polar plot.

We know,

$$G(j\omega)H(j\omega) = \frac{(4 \times j\omega + 1)}{(j\omega)^2 (j\omega + 1)(2j\omega + 1)}$$

$$\Rightarrow G(j\omega)H(j\omega) = \frac{(4j\omega + 1)(j\omega - 1)(2j\omega - 1)}{-\omega^2 (j\omega + 1)(j\omega - 1)(2j\omega + 1)(2j\omega - 1)}$$

$$= \frac{(4j\omega + 1)(2j^2\omega^2 - j\omega - 2j\omega + 1)}{-\omega^2 \{(j\omega)^2 - 1^2\}\{(2j\omega)^2 - 1^2\}}$$

$$= \frac{(4j\omega+1)(-2\omega^2-3j\omega+1)}{-\omega^2(-\omega^2-1)(-4\omega^2-1)}$$

$$= \frac{-8j\omega^3-12j^2\omega^2+4j\omega-2\omega^2-3j\omega+1}{(\omega^4+\omega^2)(-4\omega^2-1)}$$

$$= \frac{-8j\omega^3+12\omega^2+j\omega-2\omega^2+1}{-4\omega^6-\omega^4-4\omega^4-\omega^2}$$

$$= \frac{+10\omega^2+1-8j\omega^3+j\omega}{-4\omega^6-5\omega^4-\omega^2}$$

$$= -\frac{10\omega^2+1}{4\omega^6+5\omega^4+\omega^2}+j\,\frac{8\omega^3-\omega}{4\omega^6+5\omega^4+\omega^2}$$

$$= -\frac{10\omega^2+1}{4\omega^6+5\omega^4+\omega^2}+j\,\frac{8\omega^2-1}{4\omega^5+5\omega^3+\omega}$$

At $\omega = 0$,

$$G(j\omega)H(j\omega) = -\frac{1}{0}+j\,\frac{(-1)}{0}$$

$$= -\infty - j\infty$$

At $\omega = \infty$, $G(j\omega)H(j\omega) = 0 - j0$

~~When the plot crosses~~

When $G(j\omega)H(j\omega)$ locus intersects the real axis, at that point the imaginary part is equal to zero.

So $\frac{8\omega^2 - 1}{4\omega^5 + 5\omega^3 + \omega} = 0$

$\Rightarrow 8\omega^2 - 1 = 0 \Rightarrow 8\omega^2 = 1$

$\Rightarrow \omega^2 = 1/8$

$\Rightarrow \omega = \sqrt{1/8} = \frac{1}{2\sqrt{2}}$

By putting this value of ω, in real part of $G(j\omega)H(j\omega)$ we can get the real axis intersection point.

At $\omega = \frac{1}{2\sqrt{2}}$, value of real part is —

$$-\frac{(10\omega^2 + 1)}{4\omega^6 + 5\omega^4 + \omega^2}$$

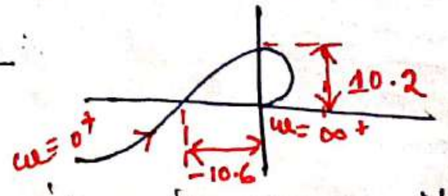
$$= -\left[\frac{10\times\left(\frac{1}{2\sqrt{2}}\right)^2 + 1}{4\times\left(\frac{1}{2\sqrt{2}}\right)^6 + 5\times\left(\frac{1}{2\sqrt{2}}\right)^4 + \left(\frac{1}{2\sqrt{2}}\right)^2}\right]$$

$$= -\left[\frac{10\times\frac{1}{8} + 1}{4\times\frac{1}{512} + 5\times\frac{1}{64} + \frac{1}{8}}\right]$$

$$= -\frac{10/8 + 1}{4/512 + 5/64 + 1/8} = -\left[\frac{18/8}{\frac{4+40+64}{512}}\right]$$

$$= -\frac{18}{8}\times\frac{512}{108}$$

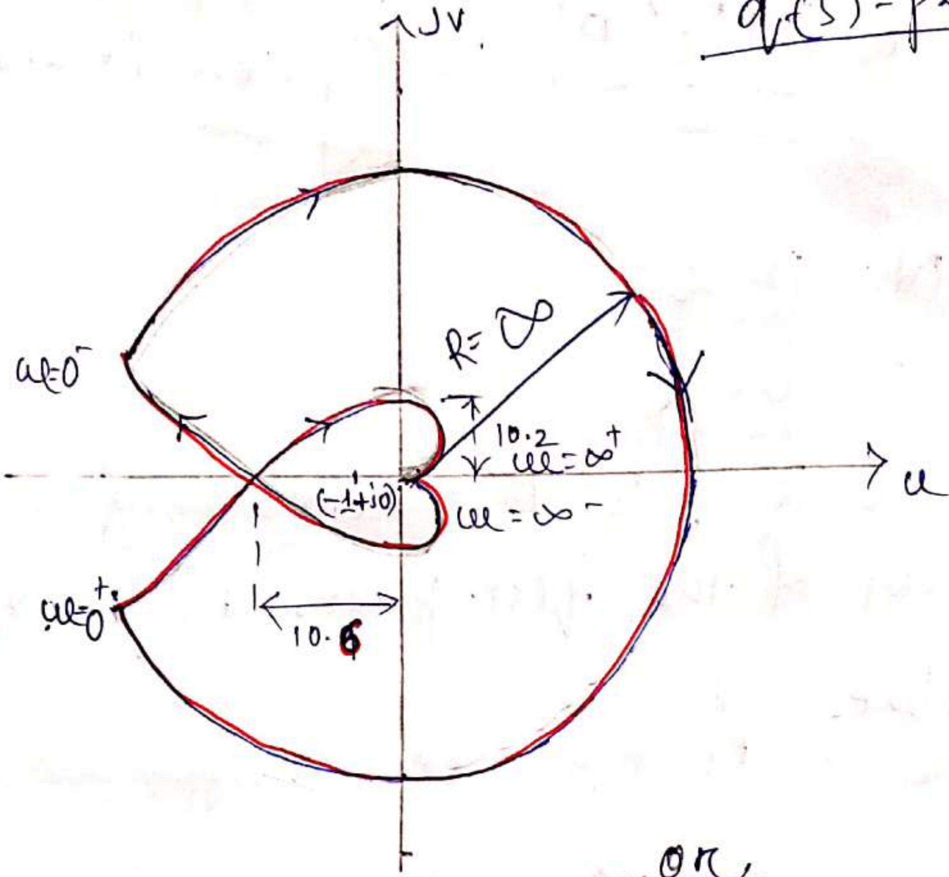
$$= -10.6$$



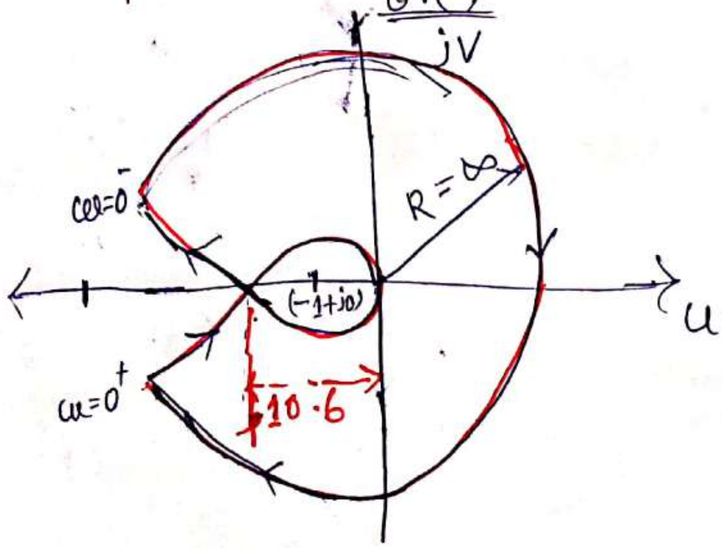
Similarly we can find the it will cross the imaginary axis at $10.2j$, by equating real part to zero.

4. Since Nyquist plot is symmetrical about the real axis
So we have to draw the mirror image of polar plot

The Nyquist contour mapping is —



Or,



from the Nyquist Contour.

$(-1+j0)$ point is encircles twice in clockwise direction. So $= N = -2$

But $P = 0$, from the given transfer function

Since $\boxed{N \neq P}$, so System is unstable.

or,

$N = P - Z$

$\Rightarrow -2 = 0 - Z$

$\Rightarrow Z = 2$, So two zeros lies on the right half of the $q(s)$-plane. Hence system is unstable.

Nyquist Stability Criterion Applied to Inverse Polar Plots

→ Polar plot of $\frac{1}{G(j\omega)H(j\omega)}$ is called as inverse polar plot of $G(j\omega)H(j\omega)$.

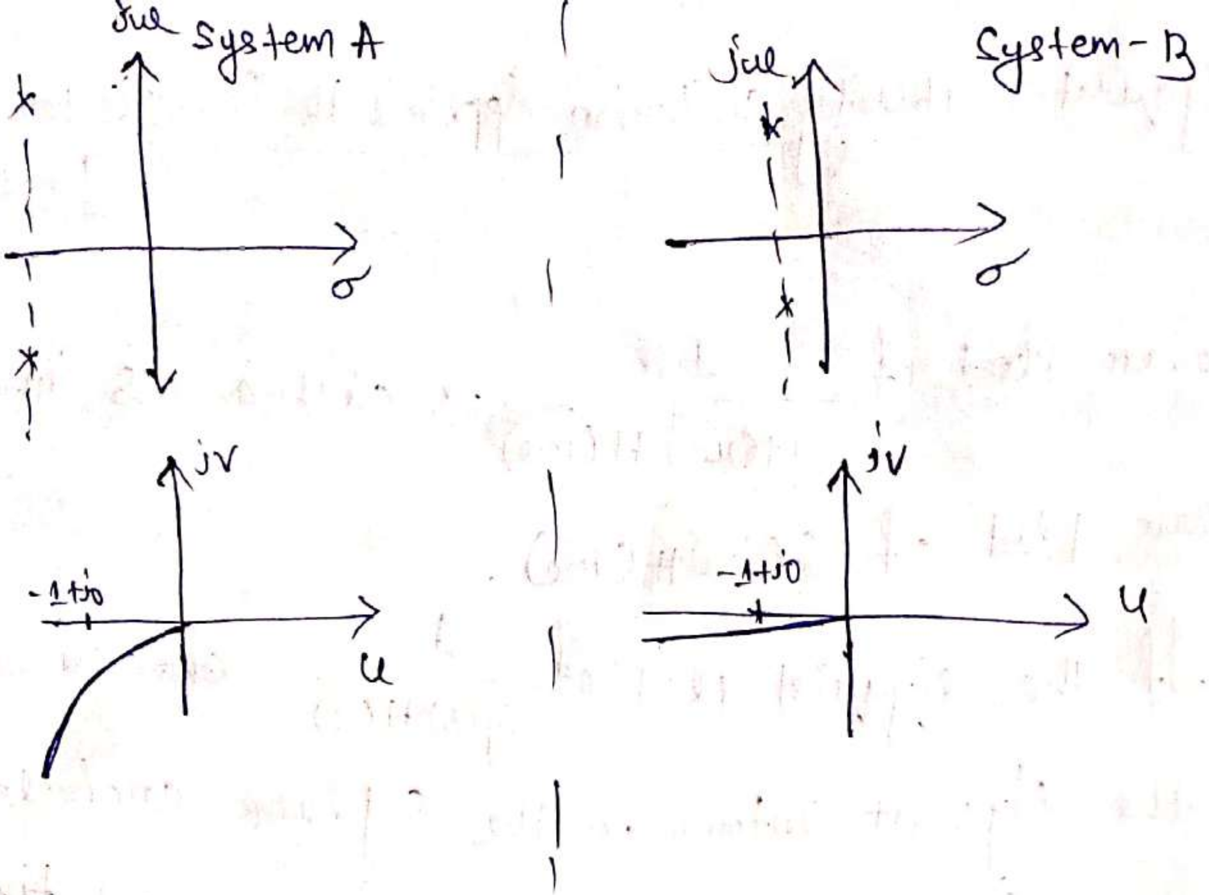
→ If the Nyquist plot of $\frac{1}{G(s)H(s)}$, corresponding to the Nyquist contour in the s-plane, encircles the poin $(-1+j0)$ counter clockwise as many times as the number of right half s-plane poles of $\frac{1}{G(s)H(s)}$, then the closed loop system is stable.

This is the Nyquist stability criterion for inverse polar plot.

Assessment of Relative stability using Nyquist Criterion

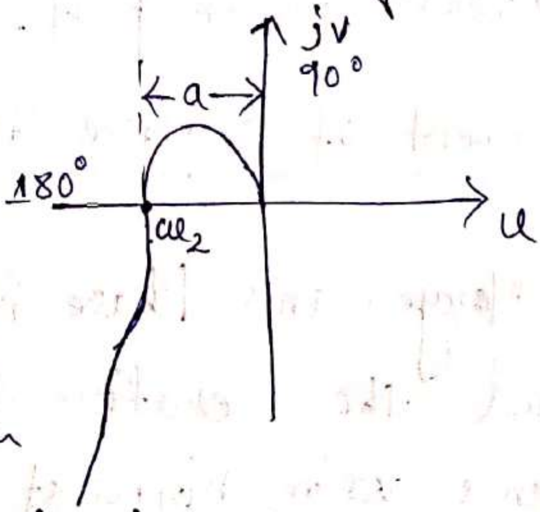
Gain Margin and Phase Margin factors can be used to find the relative stability between two systems in Nyquist plot.

→ Relative stability is the stability of a system as compared another system stability.



System-A is more stable than System-B from the above plots.

__Gain Margin__: It is the factor by which system gain can be increased to drive it to the verge of instability.



In the figure at $\omega = \omega_2$,

$\angle G(j\omega) H(j\omega) = 180°$ and

the $|G(j\omega) H(j\omega)| = a$.

If the Gain of the System is increased by a factor $(1/a)$ then the gain of the System will be $a \times 1/a = 1$, and the plot will pass through $(-1+j0)$ to drive it to the verge of instability.

→ So. Gain Margin (GM) is defined as the reciprocal of the gain at the frequency (ω) at which phase angle is 180°.

→ The frequency where phase angle is 180° is known as phase-crossover frequency.

In the figure, $GM = \frac{1}{a}$

where $a = \left| G(j\omega) H(j\omega) \right|_{\omega = \omega_2}$

<u>Phase Margin</u>: It is defined as the additional phase-lag at the gain cross-over frequency required to bring the system to the verge of instability.

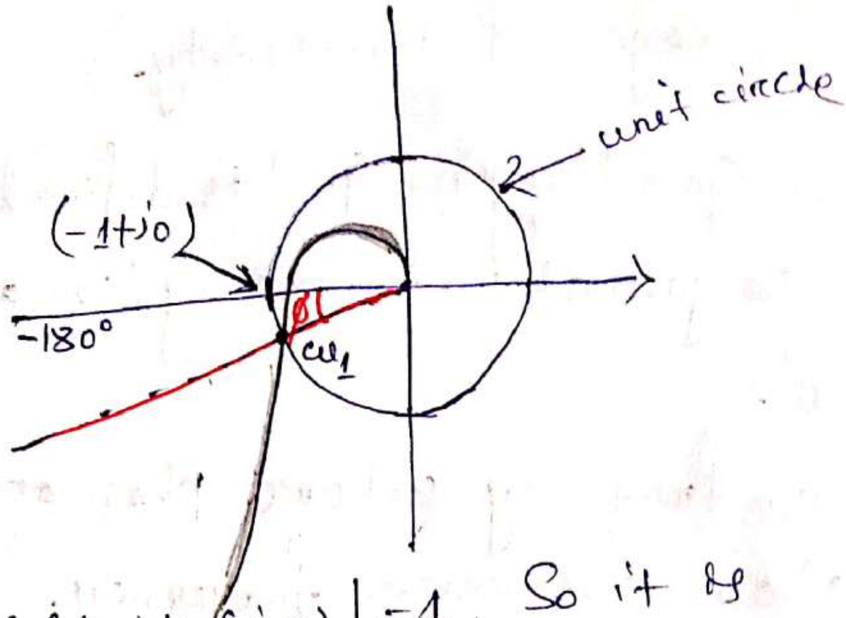
→ It is the additional phase lag introduced at the gain crossover frequency to make the phase angle

$\angle G(j\omega) H(j\omega) = -180^\circ$

→ Gain crossover frequency is the frequency at which magnitude $\left| G(j\omega) H(j\omega) \right| = 1$.

→ It is determined by intersection of $G(j\omega) H(j\omega)$-plot by an unit circle centred at the origin.

In the figure unit circle intersects the plot at $\omega = \omega_1$.



→ At $\omega = \omega_1$, $|G(j\omega)H(j\omega)| = 1$. So it is known as gain crossover frequency.

→ At ω_1, phase lag is ø.

→ Phase margin is always measured positively in counter clockwise direction.

The value of $\text{phase Margin} = \angle G(j\omega)H(j\omega)\big|_{\omega=\omega_1} + 180°$

Stability

If both GM & PM are positive then the system is stable.

→ A large GM & PM indicates a very stable feedback system but usually it becomes sluggish (slow).

→ A small GM & PM indicates highly oscillatory & fast system.

Constant M- circles & N- circles

The value of Resonant Peak (M_r) and resonant frequency (ω_r) are directly can be determined from graphical techniques.

-> These graphical technique require constant M-circles and constant N- circles.

Constant M- circle :

Consider any point $G(j\omega) = x + jy$, on the polar plot of $G(j\omega)$.

The transfer function of closed loop frequency response is —

$$T(s) = \frac{C(s)}{R(s)}$$

$$\Rightarrow T(j\omega) = \frac{C(j\omega)}{R(j\omega)}$$

$$= \frac{G(j\omega)}{1+G(j\omega)} \quad (\because \text{if } H(j\omega) = 1)$$

$$= \frac{x+jy}{1+x+jy} \quad \text{——— (i)}$$

$$= Me^{j\alpha}$$

From the above M is the magnitude, which is given by —

$$M = \left| \frac{x+jy}{1+x+jy} \right| = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$$

$$\Rightarrow M^2 = \frac{x^2+y^2}{(1+x)^2+y^2}$$

~~Rearrange~~ $\Rightarrow M^2\left[(1+x)^2+y^2\right] = x^2+y^2$

Rearranging the above equation

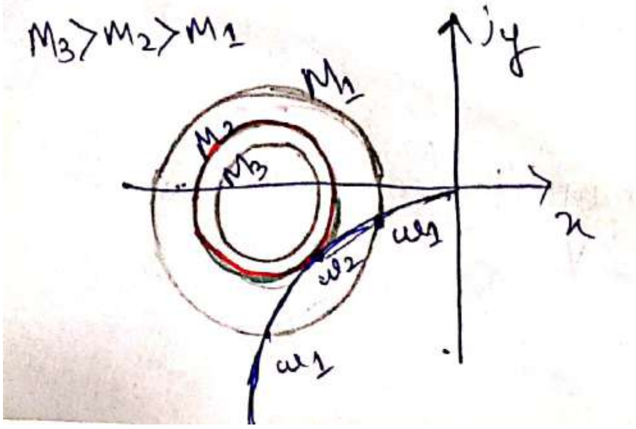
$$y^2 + \left[x + \frac{M^2}{M^2-1}\right]^2 = \frac{M^2}{(M^2-1)^2}$$

The above equation is equivalent to equation of a circle with centre at —

$$x_0 = -\frac{M^2}{M^2-1}, \quad y_0 = 0 \quad \text{———— (ii)}$$

and radius $r_0 = \frac{M}{M^2-1}$ ———— (iii)

using these above two equations (ii) & (iii), constant M circles for various value of M can be drawn.



In this figure 3 no. of constant M-circles are plotted.

It is observed that M_2-circle is tangent to $G(j\omega)$-plot. Therefore maximum value of M is M_2.

i.e $M_r = M_2$ & $\omega_r = \omega_2$.

Constant N-circles :

From eq(i), the phase angle of $T(j\omega)$ is given by -

$$\angle T(j\omega) = \alpha = \angle \frac{x+jy}{1+x+jy}$$

$$\Rightarrow \alpha = \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x}$$

$$\Rightarrow \alpha = \tan^{-1}\frac{y}{x^2+x+y^2}$$

$$\Rightarrow \tan\alpha = \frac{y}{x^2+x+y^2}$$

Let $\tan\alpha = \frac{y}{x^2+x+y^2} = N$

For constant value of α, $N = \tan\alpha$ is also constant.

Rearranging the equation $\frac{y}{x^2+x+y^2} = N$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2 + \left(y-\frac{1}{2N}\right)^2 = \frac{N^2+1}{4N^2} \quad \text{---(iv)}$$

The above equation is equivalent to equation of a circle with centre at ——

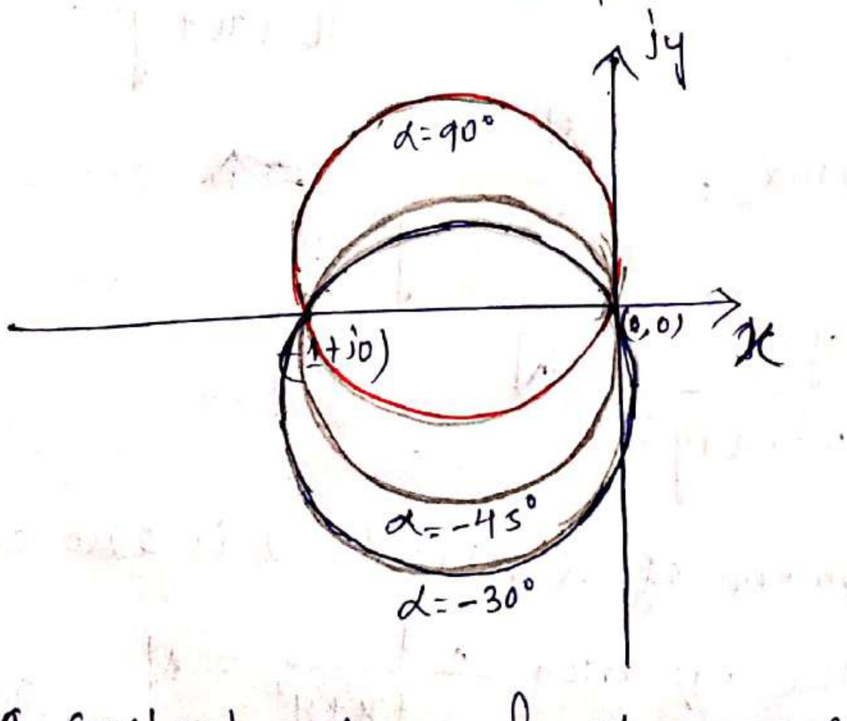
$$x_0 = -\frac{1}{2}, \quad y_0 = \frac{1}{2N}$$

and radius, $r_0 = \frac{1}{2N}(N^2+1)^{1/2}$

$$\left[\because r_0^2 = \frac{N^2+1}{4N^2}\right]$$

For different values of α, a family of N-circles can be constructed.

→ Since eq (iv) is satisfied for $x=0, y=0$ & $x=-1, y=0$, all the constant N-circles pass through the origin $(0+j0)$ and $(-1+j0)$ points regardless of the value of N.



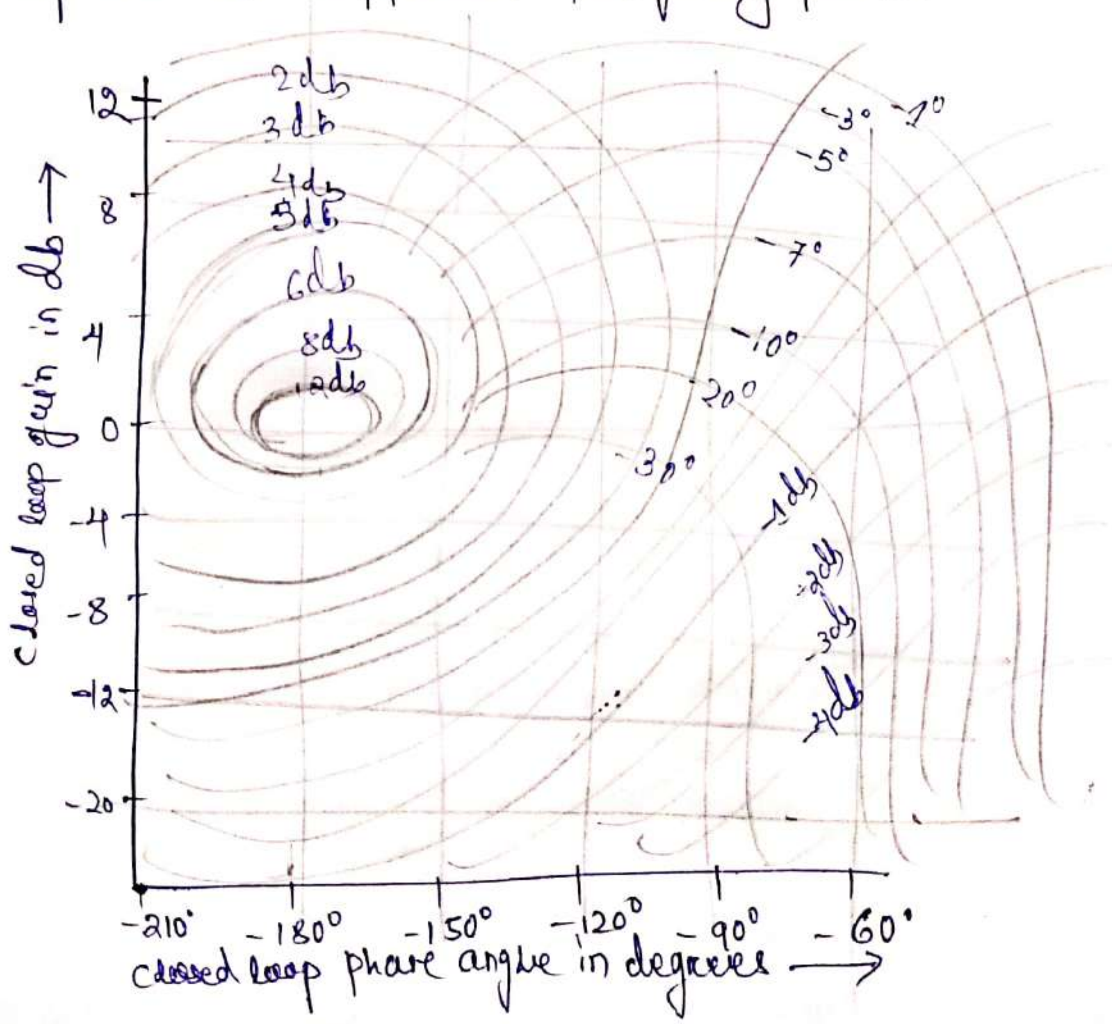
For a constant value of phase angle α, N value is constant

The Nichols Chart

-> Plotting of constant-M & constant-N contours on the Bode plot is called Nichols chart.

-> N.B. Nichols transformed the constant-M & constant N-contours (circles) to log-magnitude and phase angle co-ordinates, and resulted chart is known as Nichols chart.

-> The Intersections of the log-magnitude versus phase angle plot & constant-M & constant-α contours give the magnitude M & phase angle α of the closed loop frequency response at different frequency points.



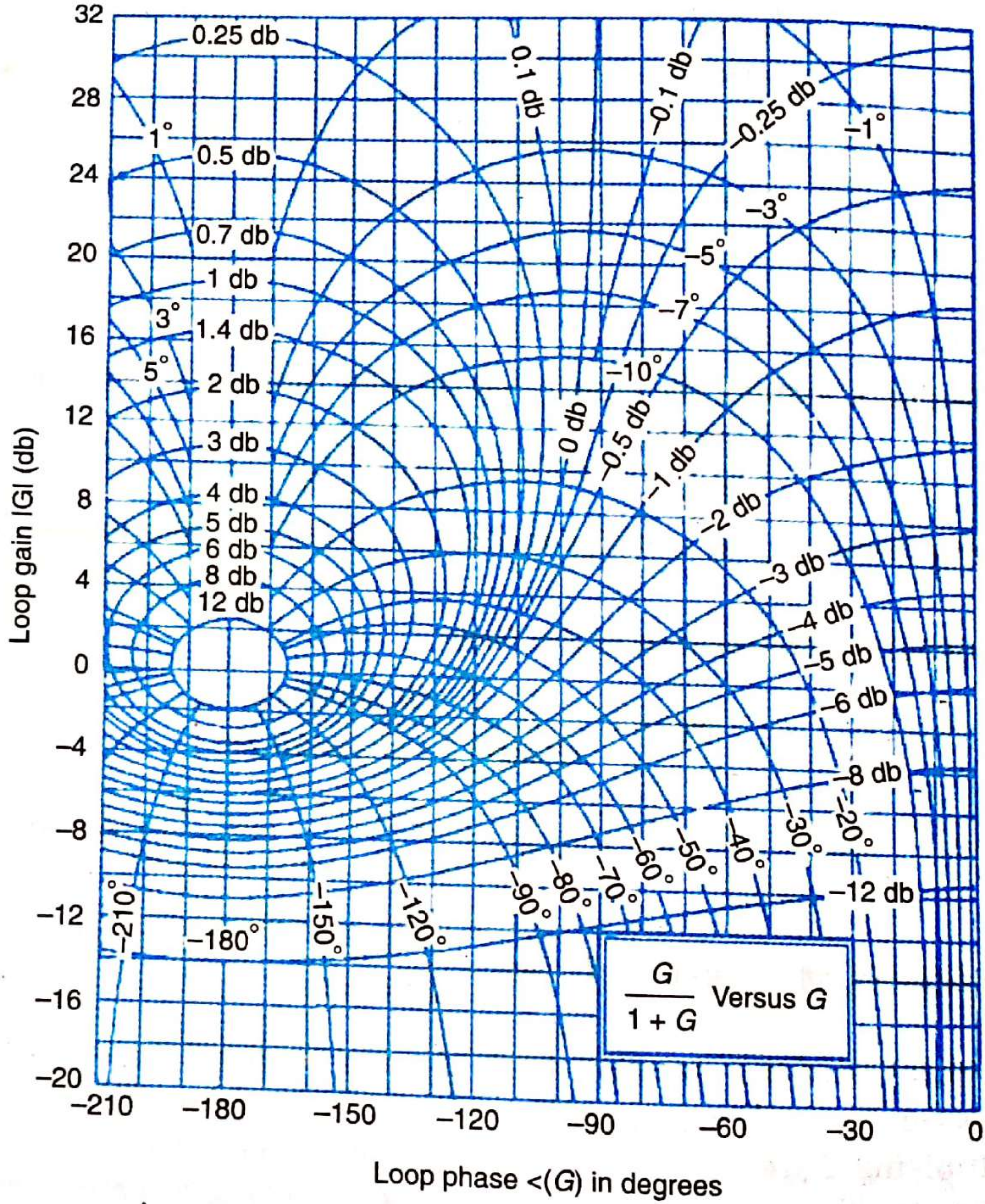


Fig. 9.37. The Nichols chart.