

4.

Working Stress method

- 1.1 Objectives of design & detailing. State the different methods of design of concrete structures.
- 1.2 Introduction to reinforced concrete, R.C. sections their behaviour, grades of concrete & steel. Permissible stresses, assumptions in WSM.
- 1.3 Flexural design & analysis of single reinforced sections from first principles.
- 1.4 Concept of underreinforced, over reinforced & balanced sections.
- 1.5 Advantages & disadvantages of WSM, reasons for its obsolescence.

Structural Design

Structural design is a methodical investigation to get the economical specification of a structure or a structural element to carry the predicted load safely. With the application of structural design we can obtain required size, grade, reinforcement etc of structural members to withstand the internal forces calculated from the structural analysis.

* Structural design is a part of structural engineering which is a sub-division of civil Engg. Structural engineers are trained to design the structural elements accurately & supervise during construction to ensure the proper implementation of the structural design.

Objectives of design & detailing

The objectives of structural design is to design the structure for stability, strength & serviceability. It must also be economical & aesthetic.

* The design of a structure must satisfy 3 basic requirements

- (1) Stability to prevent overturning, sliding or buckling of the structure or parts of it, under the action of loads.

(2) Strength to resist safely the stresses induced by ^{the} loads in the various structural members.

(3) Serviceability to ensure the satisfactory performance under service load conditions. which implies providing adequate stiffness & reinforcement to contain deflections, crack-widths & vibrations within acceptable limits & also providing impermeability & durability.

* There are 2 other considerations that a sensible designer ought to bear in mind, viz., economy & aesthetics. One can always design a massive structure, which has more than adequate stability, strength & durability serviceability, but the ensuing cost of the structure may be exorbitant & the end product far from aesthetic.

Different methods of design of concrete structures

There are 4 philosophies for the design of reinforced concrete, prestressed ^{covered} as well as steel structures.

- Working stress design
- Ultimate load design
- Limit state design
- Performance based design.

Introduction to reinforced concrete

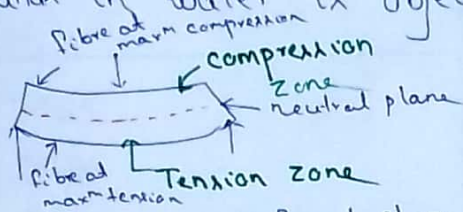
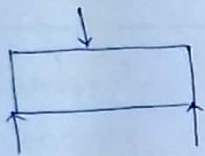
* Concrete being extremely weak in tension, has the tendency to crack when tensile stresses are induced in any part of the structure.

To arrest these cracks, reinforcement is provided in the tension zone.

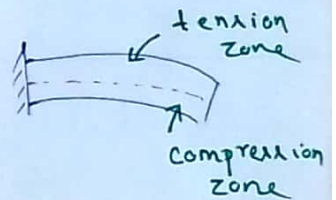
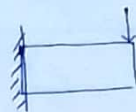
* It may, however, be noted that reinforcement does not avoid cracks but it simply arrests further widening of cracks. So by providing tensile reinforcement at a proper place, the microcracks do develop but generally they are small in width that ^{these} are not visible to the naked eye. Such a concrete which has reinforcement in tension zone, is known as "Reinforced Cement Concrete". Concrete develops in

fact, a very good bond with steel.

* The ratio betⁿ the amounts of fine & coarse aggregates necessarily depends on the grading & other characteristics of the materials in order that the volume of sand is sufficient to fill the voids in the coarse aggregate to produce a dense concrete. Water used for mixing & curing should be clean & free from injurious amounts of oils, acids, alkalis, salts, sugar, organic materials or other substances that may be deleterious to concrete or steel. Potable water is generally considered satisfactory for mixing concrete. Mixing or curing of concrete with sea water is not recommended because of the presence of the harmful salts in sea water. The presence of ferric acid or iron compounds in water is objectionable.

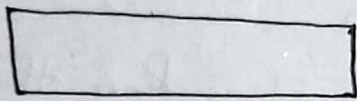


Deformed shape of beam

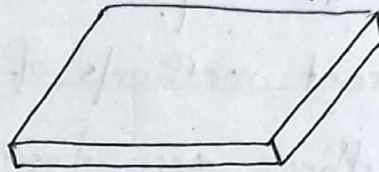


Different R.C. Sections

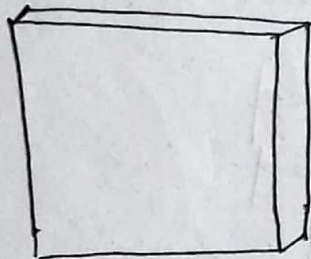
A reinforced concrete flexural member may be a beam, slab, wall or component of a foundation.



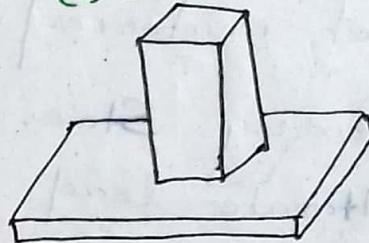
(a) Beam



(b) Slab

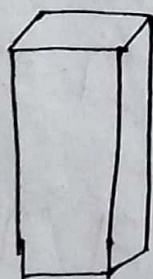


(c) wall



(d) Foundation.

A reinforced concrete compression member can be a column, which can be made rectangular, square or circular.



(a) Rectangular column



(b) Square column



(c) Circular Column.

Grade of concrete and Steel

Concrete is graded according to its compressive strength. The various grades

Grades of Concrete:

Concrete is graded according to its compressive strength. The various grades of concrete as stipulated in IS:456-2000 are shown in table below.

Group	Designation	Characteristic compressive strength of 150mm cube at 28 days (N/mm^2)
Ordinary Concrete	M10	10
	M15	15
	M20	20
Standard Concrete	M25	25
	M30	30
	M35	35
	M40	40
	M45	45
	M50	50
	M55	55
High Strength Concrete	M60	60
	M65	65
	M70	70
	M75	75
	M80	80

* In the classification of a concrete mix, letter M refers to the mix & the number to the specified characteristic compressive strength of 150mm cubes at 28 days expressed in N/mm^2 or MPa.

* The characteristic strength is defined as the strength of material below which not more than 5% of the test results are expected to fall.

* Concrete of compressive strength lower than 20 MPa may be used for lean in reinforced concrete work bases & simple foundations for masonry walls. Grades lower than M20 should not be used in reinforced concrete works, lower than M40 should not be used in pre-tensioned prestressed concrete, and lower than M30 should not be used in post-tensioned prestressed concrete.

* For high strength concrete, the design parameters such as stress strain curve & stress block parameters given in IS 456:2000 code may have to be obtained from specialized literature.

* The mix proportions shall be selected to ensure the workability of the fresh concrete & when concrete is hardened, it shall have the required strength, durability & surface finish.

* The determination of the proportions of cement, aggregates & water to attain the required strengths shall be made as follows.

(a) by designing the concrete mix; such concrete shall be called 'Design mix concrete.' or

(b) by adopting nominal concrete mix; such concrete shall be called 'Nominal mix concrete.'

* Design mix concrete is preferred to nominal mix. If design mix concrete cannot be used for any reason on the work for grades of M20 or lower, nominal mixes may be used with the permission of engineer-in-charge, which however, is likely to involve a higher cement content.

(1) Design mix Concrete

The proportion of ingredients of concrete to obtain the desired mix can be found out by laboratory method of mix design. Design mix concrete is treated in IS:10262 & may be preferred.

a) Nominal mix concrete.

When the concrete is produced by taking standard arbitrary proportions of concrete ingredients, it is known as nominal mix concrete. Some nominal mix proportions are:-

M5 - 1:5:10

M7.5 - 1:4:8

M10 - 1:3:6

M15 - 1:2:4

M20 - 1:1.5:3

M25 - 1:1:2

M30 - 1:0.75:1.5

M35 - 1:0.5:1

M40 - 1:0.25:0.5

There is no guarantee that such mixes will give the desired strength, will be economical. Hence nominal mix may be used for grades of M20 or lower, if design mix cannot be used for some reason.

Grades of Steel

Steel as reinforcement:

In reinforced concrete, concrete being weak in tension, steel bars are used to carry the tension. Steel bars are also used to carry compression in beams and columns. For any material to be used as reinforcement for concrete, it should possess the following properties.

- 1) It should possess high tensile strength.
- 2) It should be able to develop a good bond with concrete.
- 3) It should possess a high modulus of elasticity.
- 4) It should have the same temp. coefficient of expansion & contraction as concrete to avoid the development of thermal stresses.
- 5) It should be easily available.

Types of reinforcement

Different types of reinforcement used are

1> mild steel

2> Medium tensile steel

3> hot rolled deformed bars

4> high yield strength deformed (HYSD) steel

5> hard drawn steel ^{wire} fabric.

→ HYSD bars are available as CTD (cold twisted Deformed) bars & TMT (Thermo mechanically treated) bars.

Mild Steel bars (Fe250)

Mild steel bars have been widely used as reinforcement in construction industry. The yield strength of mild steel is 250 N/mm^2 . Among all kinds of steel, mild steel is the most ductile. A typical stress strain curve for mild steel is shown in fig.

* In elastic range, the strain is directly proportional to stress. If the load is increased, the stress increases & proportionally the strain increases. At one particular point, the stress remains constant with significant increase in strain. This is known as yielding of steel & the stress is known as yield stress which can be measured by drop of beam method on universal testing m/c using standard tensile test specimen.

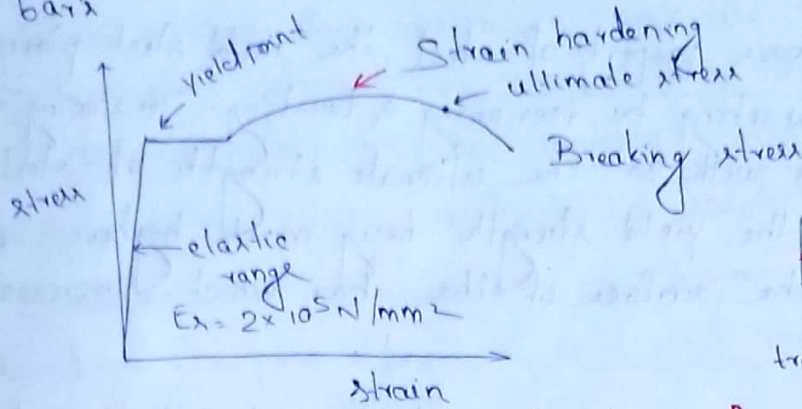
* With a further increase of load, the stress increases slowly upto ultimate stress while strain increases rapidly. This is known as strain hardening. Beyond the ultimate load, the deformations are due to the creaking & at a reduced load upto failure.

* Mild steel, when used as reinforcement, can yield on over loading. During yielding, extensive cracking in concrete takes place before it fails. This can give sufficient warning before failure. Now-a-days, because of the extensive use of high yield strength deformed bars, the use of mild steel is limited; however, it is ^{sometimes} used for secondary reinforcement.

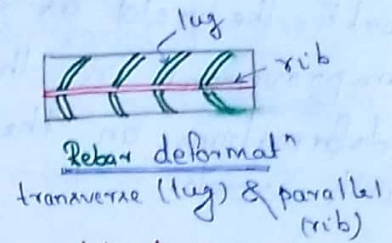
* mild steel can be plain or deformed to increase the capacity of bond. The deformed bar is one which is provided with lugs, ribs or deformations on the surface of the bar to minimize the slippage of the bar in concrete.

4) High Yield Strength Deformed bars:

HYSD bars



longitudinal rib
transverse rib



Stress-strain relationships for mild steel.

(4) High Yield Strength Deformed bars.

HYSD bars are available in two varieties.

(a) Cold Twisted Deformed [CTD] bars

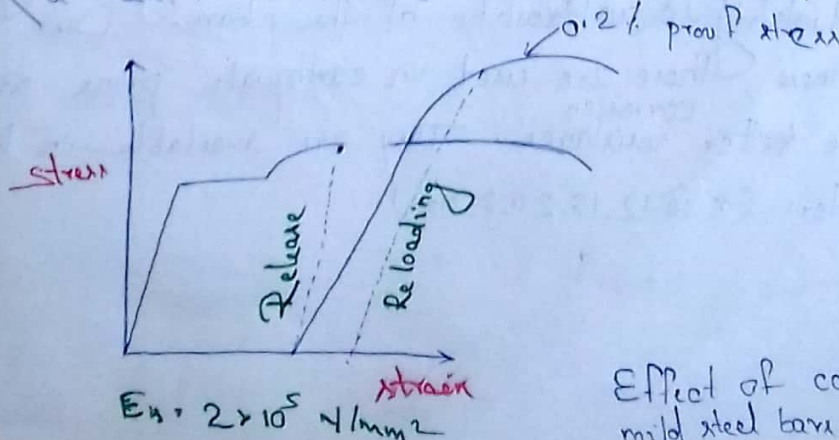
(b) Thermo Mechanically Treated (TMT) bars

(a) CTD bars

upto the late 1960s, only mild steel was used as reinforcement in India. There was a shortage of steel which made it costly. This created a demand to reduce the quantity of steel used.

* The quantity of steel can only be reduced by using the reinforcement of quality marked by a higher strength. Based on experiments & prevailing practice in foreign countries, the production of high strength ribbed bars was started.

* It was noticed that if a mild steel bar is strained beyond yield point by tensioning & twisting & then unloaded, the yield plateau is eliminated & a new stress-strain curve is obtained as shown in fig.



Effect of cold working on mild steel bars.

* As the high yield strength does not possess the definite yield point, 0.2% proof stress is used as a yield stress.

* Based on the above property of steel, the mild steel plain bars are subjected to cold working by tensioning & twisting. Tensioning raises both the yield as well as the ultimate strength of steel, the improvement in the yield strength being much higher. Twisting produces deformations on the surface of the bar which increases its bond with concrete.

* In India, the CTD bars are available under the trade names of TOR steel, listrong, Ticon, GIRP etc. These bars are available in 3 grades, Fe 415, Fe 500, Fe 550. Where Fe denotes Ferrous material & the number, the yield strength of steel in N/mm^2 .

Thermo-mechanically treated (TMT) bars

The TMT bars are manufactured by subjecting the hot rolled MS bars to a sophisticated & controlled cooling process which converts the outer surface of the bar into hardened structure.

* The subsequent cooling allow the hot inner core to soften the outer surface through thermal exchange resulting into tempered martensite in the peripheral zone & a fine grained ferrite-pearlite at the central zone.

* This process increases the yield strength of the bar without losing its yielding property. Thus, these bars possess definite yield point. In addition, the bond strength is also increased due to such treatment.

* TMT bars have high strength combined with high elongation. This increases the ductility & weldability of the bars. Due to higher ductility & toughness, they are used in earthquake prone areas. Also these bars have better ^{corrosion} resistance. They are available in limited variety of diameters (8, 10, 12, 16, 20, 25 mm).

Permissible stresses (B.2, Table 21 & 22)

In working stress method, the stresses in materials are not exceeded beyond their permissible values. The permissible stresses are found by using suitable factors of safety to the material strengths; eg, for concrete in compression in bending, a factor of safety equal to 3 is considered on characteristic of concrete & a factor of safety equal to 1.8 is considered on the yield strength of mild steel reinforcement in tension due to bending.

* From table 21, it can be observed that for a given grade of concrete $\sigma_{cc} < \sigma_{cbc}$, i.e. a greater factor of safety is adopted for direct stress than for a bending stress. When a c/s is subjected to bending stress, the stress induced on it is variable, being max^m at extreme fibre & zero at neutral axis.

* When the max^m stress induced exceeds the permissible value, the extreme fibre will not fail actually but will transfer the additional force to the inner fibre which has a lower stress.

* When the sectⁿ is subjected to a direct stress, all points of the sectⁿ have uniform stress having no scope for such a transfer of the force. It is for this reason that larger factor of safety is adopted for direct stress than bending stress.

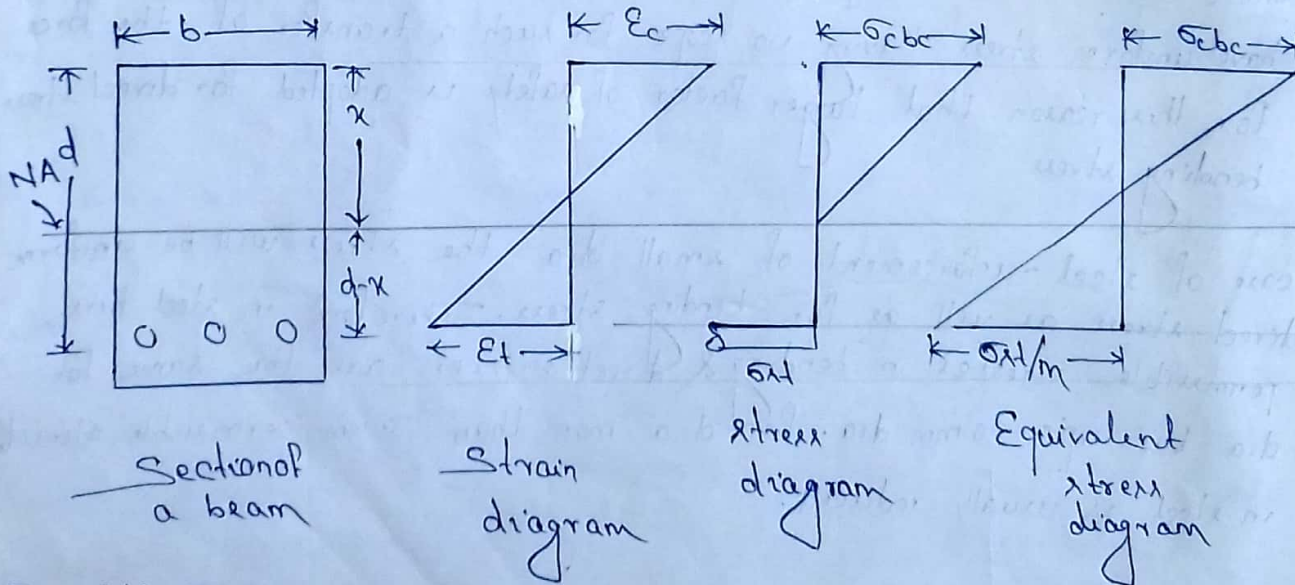
* In case of steel reinforcement of small dia, the stress will be uniform for direct stress as well as for bending stress. Therefore in steel bars, the permissible stresses in bending & direct stresses are the same for lower dia bars upto 20 mm dia. For dia more than 20 mm, permissible tensile stress in steel is usually reduced.

Assumptions in WSM (B-1.3) (Page-80)

The analysis & design of a reinforced concrete member are based on the following assumptions.

- (1) Concrete is assumed to be homogeneous.
- (2) At any c/s plane rect^s before bending remain plane after bending.
- (3) The stress-strain relationship for steel & concrete is a straight line under working loads.
- (4) Concrete area on the tension side is assumed to be ineffective.
- (5) All tensile stresses are taken up by reinforcement & none by concrete except when specially permitted.
- (6) The steel area is assumed to be concentrated at the centroid of the steel.
- (7) The modular ratio has the value $\frac{280}{3\sigma_{cbc}}$ where σ_{cbc} is permissible compressive stress due to bending in concrete in N/mm^2 .

* From the above assumptions, the equivalent stress diagram is developed as shown in fig.



$$\sigma_{cbc} = \sigma_{st}/m \text{ prove}$$

$$\frac{\text{Stress in steel}}{\text{strain in steel}} = E_t \quad \dots (1)$$

⇒

$$\frac{\text{stress in surrounding concrete}}{\text{strain " " "}} = E_c \quad \dots (2)$$

$$\therefore \frac{E_t \epsilon(1)}{E_t \epsilon(2)} > \frac{\sigma_{st}}{\sigma_{cbc}} = \frac{E_t}{E_c}$$

$$\Rightarrow \frac{\sigma_{st}}{\sigma_{cbc}} = m$$

$$\Rightarrow \sigma_{cbc} = \sigma_{st}/m$$

modular ratio

(page 106 of HJ saha)

~~This is the ratio betⁿ elastic mo~~

It is defined as the ratio of modulus of elasticity of steel (E_s) to that of concrete (E_c) & is denoted by m .

$$m = \frac{E_s}{E_c}$$

It is stated that the modulus of elasticity of all grades of steel ~~is~~ $2 \times 10^5 \text{ N/mm}^2$. The short term modulus of elasticity of concrete is given by $E_c = 5000 \sqrt{f_{ck}}$. Also the long term modulus of elasticity of concrete was shown to be $E_{ce} = \frac{E_c}{1 + \phi}$ where ϕ is the creep coefficient given in 6.2.5 (page 16 of IS 456:2000).
 (Instantaneous)
 (Creep & Shrinkage) clause

* ~~It should~~ ^{the} For design work, long term modulus of elasticity of concrete shall be taken ^{into} account for calculation of modular ratio.

→ IS:456 stipulates that for the design with working stress method, the value of modular ratio m shall be taken as $m = \frac{280}{3\sigma_{cbc}}$

where σ_{cbc} = permissible ^{compressive} stress due to bending in concrete in N/mm^2

* The modular ratio varies with the grade of concrete. No rounding off of the values of 'm' is permitted by Indian standard.

Ex Modular ratio values for different grades of concrete are as under

Grade of concrete

M15

M20

Modular ratio

$$m = \frac{280}{3 \times 5} = 18.66$$

$$m = \frac{280}{3 \times 7} = 13.33$$

etc

Analysis of singly reinforced sections

Let b = breadth/width of the rectangular beam

d = effective depth of the beam
(which is defined as the distance from the extreme compression fibre to the centroid of tensile reinforcement)

x = depth of neutral axis which is defined as the distance from the extreme compression fibre.

A_{st} = c/a area of steel in tension

σ_{cbc} = permissible compressive stress in concrete due to bending

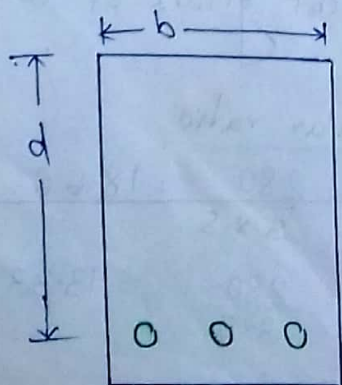
σ_{st} = permissible tensile stress in steel

m = modular ratio

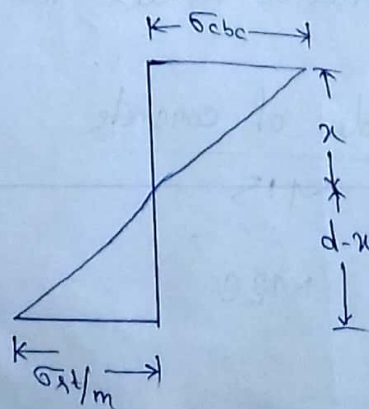
Neutral Axis: (N.A.)

Neutral axis is the axis at which the stresses are zero in the sectⁿ. The areas above & below it are subjected to compressive & tensile stresses respectively in the case of simply supported beams with vertical loads acting downwards while reverse in the case when the beam is a cantilever. Locating the position of neutral axis is of great importance to the structural Engineers. This helps in knowing the amount of strains both in concrete & steel.

* There are 2 methods for determining the neutral axis of a beam depending upon the data given.



Sectⁿ of a beam



Equivalent stress diagram

Method-I (

Given data

- (i) Dimensions of the section (b & d)
- (ii) Permissible stresses in concrete & steel (σ_{cbc} & σ_{st})
- (iii) modular ratio (m)

Referring to fig, from similar triangles in the equivalent concrete stress diagram,

$$\frac{\sigma_{cbc}}{\sigma_{cbc}/m} = \frac{x}{d-x}$$

$$\Rightarrow \sigma_{cbc} (d-x) = x \times \sigma_{st}/m$$

$$\Rightarrow \sigma_{cbc} \times d = x \cdot \frac{\sigma_{st}}{m} + x \sigma_{cbc}$$

$$\Rightarrow \sigma_{cbc} \times d = x (\sigma_{cbc} + \sigma_{st}/m)$$

$$\Rightarrow x = \frac{\sigma_{cbc} \times d}{(\sigma_{cbc} + \sigma_{st}/m)}$$

$$\Rightarrow x = \left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) \times d \dots \dots (1)$$

$$\Rightarrow x = k \times d$$

$$\text{where } k = \text{neutral axis const} = \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}}$$

To find the area of steel A_{st} , equate total compressive force (C) to total tensile force (T).

$$\begin{aligned} \text{Total compressive force (C)} &= \text{Area} \times \text{average compressive stress} \\ &= b \cdot x \times \left(\frac{\sigma_{cbc} + 0}{2} \right) = \frac{b x \sigma_{cbc}}{2} \end{aligned}$$

Total tensile force (T) = area $\times$ tensile stress

$$= A_{st} \sigma_{st}$$

$$\therefore \boxed{b \times \frac{\sigma_{cbc}}{2} = A_{st} \sigma_{st}} \dots \dots \dots (2)$$

After calculating the neutral axis x from eqn (1), the area of steel can be calculated from eqn 2.

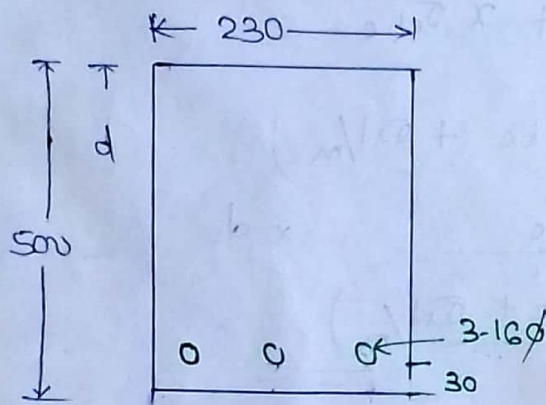
The area of tensile steel (A_{st}) is expressed as a % (P_t) of the effective section.

$$P_t = \frac{A_{st} \times 100}{bd}$$

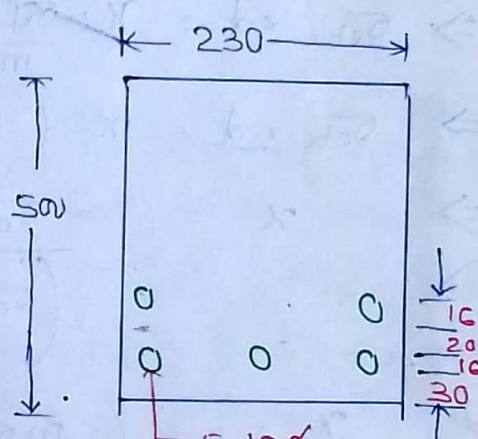
Problem-1

Determine the effective depth for the beam sect'n of

Fig



Fig(a)



Fig(b)

sol'n

For fig(a) cover = 30 mm

Overall depth (D) = 500 mm

$$\text{effective depth (d)} = 500 - 30 (\text{cover}) - 8 (\text{upto centroid of reinforcement})$$

$$= 462 \text{ mm}$$

For Fig(b)

In the fig if all the reinforcement bars are provided in a single layer, the concreting will be difficult. Therefore two layer shall be 20 mm or more. The centroid of reinforcement is assumed at centre of 2 layers.

$$d = 500 - 30 (\text{cover}) - 16 (\text{first layer}) - 10 (\text{centre of 2 layers})$$

$$= 500 - 56 = 444 \text{ mm}$$

Problem: 2

An RC beam 200mm wide has an effective depth of 350mm. The permissible stresses in concrete & steel are 5 N/mm^2 & 140 N/mm^2 respectively. Find the depth of neutral axis, area of steel & % of steel (m) = 18.66.

Given

width of beam (b) = 200mm

effective depth of beam (d) = 350mm

permissible compressive stress in concrete due to bending (σ_{cbc}) = 5 N/mm^2

permissible tensile stress in steel (σ_{st}) = 140 N/mm^2

modular ratio (m) = 18.66

method-I

From similar triangles in the equivalent ^{concrete} stress diagram

depth of neutral axis

$$x = \left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) d$$

$$= \left(\frac{18.66 \times 5}{18.66 \times 5 + 140} \right) \times 350$$

$$= 139.97 \text{ mm}$$

To find the area of steel

Total compressive force = Total tensile force

$$\Rightarrow b \times \frac{\sigma_{cbc}}{2} = A_{st} \sigma_{st}$$

$$\Rightarrow 200 \times 139.97 \times \frac{5}{2} = A_{st} \times 140$$

$$\Rightarrow A_{st} = 499.89 \text{ mm}^2$$

Percentage of steel

$$P_t = \frac{A_{st}}{bd} \times 100$$
$$= \frac{499.89}{200 \times 350} \times 100$$
$$= 0.714$$

Method-II

Given data

- (i) Dimensions of section (b & d)
- (ii) Area of tensile steel (A_{st})
- (iii) modular ratio (m)

Referring to fig, we see that the neutral axis is situated at the centre of gravity of a rect. The moments of area on either side of it are equal.

$\therefore$ Moment of area on compression side = Moment of area on tension side

Moment of area on ^{compression} tension side

$$= \text{Area of compression side} \times \text{distance of c.g. of compression area from N.A.}$$

$$= b \times x \times \frac{x}{2}$$

Moment of area on tension side

$$= \text{equivalent area of concrete} \times \text{distance of c.g. of tensile steel from N.A.}$$

$$= m A_{st} (d - x)$$

$$\text{Therefore } b \cdot x \cdot \frac{x}{2} = m A_{st} (d - x)$$

Note that $m A_{st}$ = Equivalent of concrete.

Problem - 3

Find the position of the neutral axis of a reinforced concrete beam 150mm wide & 400mm deep (effective). Area of tensile steel is 804 mm^2 , $m = 18.66$

Given data

width of beam (b) = 150mm

effective depth of beam = 400mm

Area of tensile steel (A_{st}) = 804 mm^2

modular ratio (m) = 18.66

Taking moments of area of compression & tension sides about N.A.

$$b \cdot x \cdot \frac{x}{2} = m A_{st} (d - x)$$

$$\Rightarrow 150 x^2 \frac{x}{2} = 18.66 \times 804 (400 - x)$$

$$\Rightarrow 75 x^2 = 6001056 - 15002.64 x$$

$$\Rightarrow 75 x^2 + 15002.64 x - 6001056 = 0$$

$$\Rightarrow x = 200 \text{ mm}$$

Concept of balanced, Under-reinforced, Over-reinforced section

Depending on mode of failure of a beam, three types of design are possible, viz, balanced, under-reinforced & over-reinforced design. These are now briefly discussed.

(1) Balanced design sectⁿ:

Reinforced concrete sectⁿ are designed on the assumption that the max^m stresses in concrete & steel reach their permissible stresses at the same time. Both the materials are stressed to their fullest allowable stress at one & the same time. As such the section is called 'balanced' or 'critical' or 'economic'.

* The neutral axis of this sectⁿ is called critical neutral axis & may be denoted by x_c . Critical n.A is calculated as follows

$$\frac{\sigma_{cbc}}{\sigma_{st}/m} = \frac{x_c}{d - x_c}$$

$$x_c = \left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) d$$

Under-reinforced Sectⁿ

If less steel than that required for balanced section is used, the sectⁿ is under-reinforced. In such a condition, the steel is stressed to its max^m permissible stress. Under such conditions, the beam will fail initially due to overstress in the steel.

* The neutral axis is calculated by taking moments of area of compression & tension sides about neutral axis. Thus

$$b \cdot x \cdot \frac{x}{2} = m A_{st} (d - x)$$

* This actual N.A is above the critical N.A ($x < x_c$) as shown in fig. The neutral axis moves upwards because the concrete area is greater than the area of steel, the latter being less than what it should be for perfect balance.

Over-reinforced sectⁿ

If more steel than the steel required for balanced sectⁿ is used, the sectⁿ is over-reinforced. The steel is not fully stressed to its max^m permissible tensile stress, while the compressive stress in concrete at extreme fibre reaches its max^m permissible value. In this case, the beam will fail initially due to overstress in the concrete.

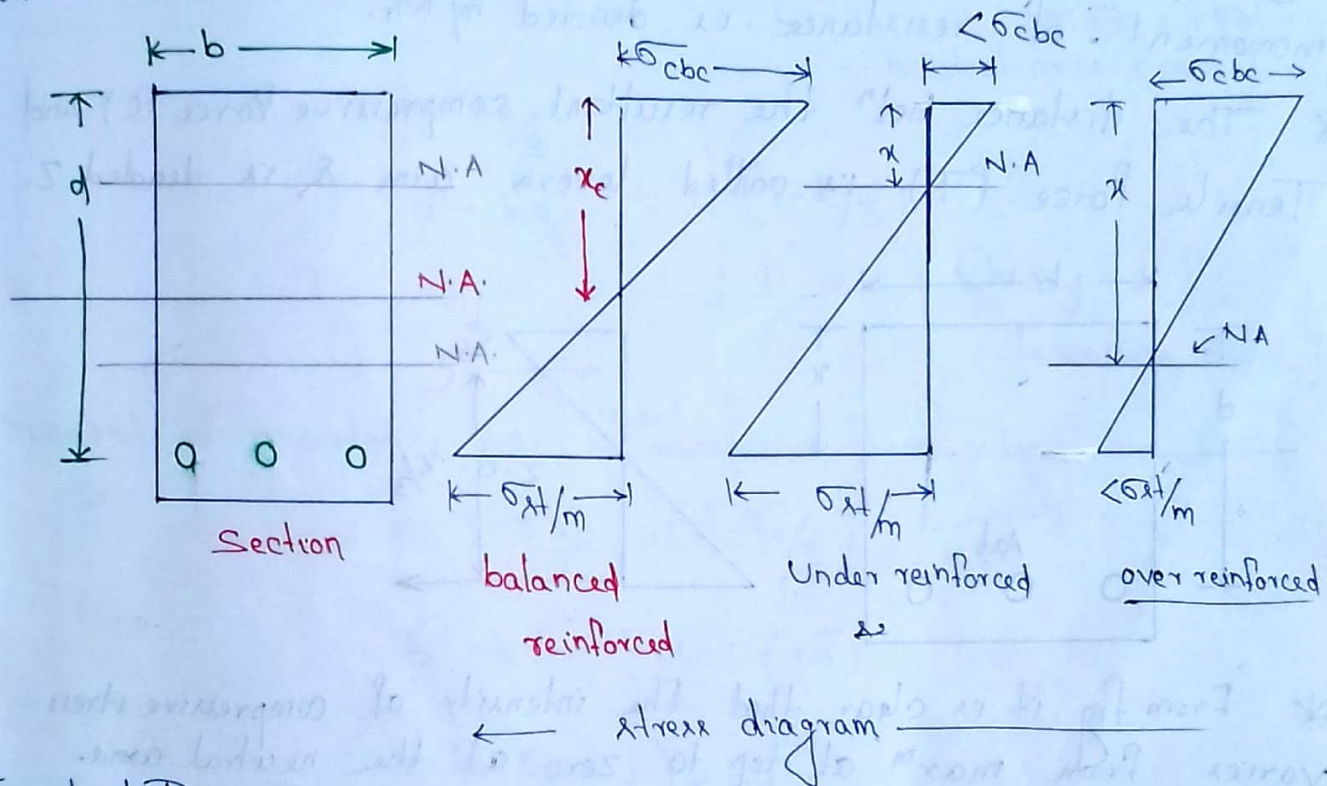
* N.A is calculated by taking moments of area of compression & tension sides about the N.A.

$$b \cdot x \cdot \frac{x}{2} = m A_{st} (d - x)$$

The failure in this case is called tension failure & the design is referred to as under-reinforced. Note that steel is a ductile material & yields before failure. Thus the under-reinforced sectⁿ suffers large deflections and concrete in tension zone cracks extensively before failure. The failure in this case is a ductile failure.

* This actual N.A. is below the critical N.A. The N.A. moves downwards because the area of steel is more than that of balanced sectⁿ. ($\chi > \chi_c$)

* The failure in this case is called compression failure and the design is referred to as overreinforced. Note that concrete is a brittle material & therefore the failure will be brittle & sudden.



Important Points

A beam, if under-reinforced, gives notice before failure as the steel yields & concrete in tension zone shows cracks. If it is over-reinforced concrete fails first.

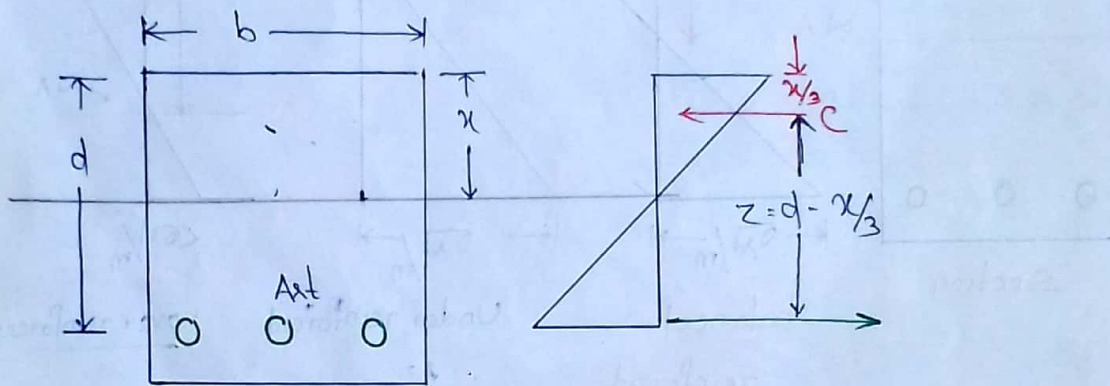
* Prior to failure, concrete does not give notice as it does not yield. Thus, over-reinforced beam may collapse without giving a notice when overloaded. Therefore normally balanced or under-reinforced design is preferred. The mode of failure in this case will be ductile.

* In fact, we should design the structure as ductile as possible so that in case of failure due to overloads, it gives notice before failure and prevention can be taken. IS 456 stipulates all provision to see that the designed structure is ductile as per the importance of the structure.

Moment of resistance.

The moment of resistance of the concrete beam section is the moment of couple formed by the total tensile force (T) in the steel acting at the c.g. of reinforcement & the total compressive force (C) in the concrete acting at the c.g. of the compressive stress diagram as shown in fig. The moment of resistance is denoted by M_r .

* The distance betⁿ the resultant compressive force (C) and Tensile force (T) is called lever arm & is denoted Z .



* From fig, it is clear that the intensity of compressive stress varies from max^m at top to zero at the neutral axis.

Therefore c.g. of the compressive force is at a distance $x/3$ from the top edge sectⁿ

$$\therefore Z = d - \frac{x}{3}$$

Moment of resistance

$$\begin{aligned} M_r &= \text{resultant / Total compressive force (C)} \times \text{lever arm (Z)} \\ &= b \cdot x \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x}{3} \right) \end{aligned}$$

or

$$\begin{aligned} M_r &= \text{resultant / Total tensile force (T)} \times \text{lever arm (Z)} \\ &= A_{st} \cdot \sigma_{st} \cdot \left(d - \frac{x}{3} \right) \end{aligned}$$

moment

For balanced sectⁿ

Critical / balanced / economic depth of neutral axis

$$x_c = \left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) d$$
$$= k d \quad \left[\text{where } k = \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right]$$

= neutral axis const

lever arm $z = d - x_c / 3$

$$= d - \frac{k d}{3} = d (1 - k/3)$$

$$= j \cdot d$$

$$\left[\text{where } j = 1 - \frac{k}{3} \right]$$

= lever arm const

moment of resistance (M_r) = Total compressive force^(c) $\times$ lever arm (z)

$$= b \cdot x_c \cdot \frac{\sigma_{cbc}}{2} \cdot (d - x_c / 3)$$

$$= b \cdot k d \cdot \frac{\sigma_{cbc}}{2} \cdot (j d)$$

$$= \left(\frac{k \cdot \sigma_{cbc} \cdot j}{2} \right) \times b d^2$$

$$= Q b d^2$$

$$\left[\text{where } Q = \frac{k \sigma_{cbc} \cdot j}{2} \right]$$

= moment of resistance

or moment of resistance (M_r) const

= Total tensile force $\times$ lever arm

$$= A_{st} \sigma_{st} \cdot \frac{(d - x_c / 3)}{(j d)}$$

For under reinforced sectⁿ

$$M_r = \text{Total tensile force (T)} \times \text{lever arm}$$

$$= A_s \cdot \sigma_{st} \left(d - \frac{x}{3} \right)$$

For over-reinforced sectⁿ

$$M_r = \text{Total compressive force (C)} \times \text{lever arm}$$

$$= b \cdot x \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x}{3} \right)$$

Problem: 4

For a rectangular beam of size 250mm wide x 520mm effective depth, find out the balanced depth of neutral axis, balanced lever arm, balanced moment of resistance & balanced steel area. The materials are M20 grade concrete & HYSD reinforcement of grade Fe415. Also find out the design const.

Solⁿ

given data

width of the beam (b) = 250mm

effective depth " (d) = 520mm

For, M20 grade concrete,

$$\sigma_{cbc} = 7 \text{ N/mm}^2 \quad (\text{Table 21, IS:456-2000})$$

for HYSD reinforcement of grade Fe415

$$\sigma_{st} = 230 \text{ N/mm}^2$$

(Assuming bar dia $\leq 20\text{mm}$)

(i) balanced depth of neutral axis (x_c) = k d

$$\text{where } k = \frac{\text{neutral axis const}}{\left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right)} = \frac{\frac{280}{3 \sigma_{cbc}} \times \sigma_{cbc}}{\frac{280}{3 \sigma_{cbc}} \times \sigma_{cbc} + \sigma_{st}}$$

$$= \left(\frac{\frac{280}{3 \times 7} \times 7}{\frac{280}{3 \times 7} \times 7 + 230} \right)$$

$$= 0.29$$

$$\therefore x_c = 0.29 \times 520 = 150.8 \text{ mm}$$

$$\begin{aligned} \text{(ii) balanced lever arm } (Z) &= d - \frac{x_c}{3} \\ &= d - \frac{k d}{3} \\ &= d(1 - k/3) \\ &= j d \end{aligned}$$

$$\text{lever arm const } (j) = 1 - \frac{k}{3} = 1 - \frac{0.29}{3} = 0.90$$

$$\therefore Z = 0.90 \times 520 = 468 \text{ mm}$$

$$\text{(iii) balanced moment of resistance } M_r = Q b d^2$$

where Q = moment of resistance const

$$\begin{aligned} &= \frac{k \sigma_{cbc} \times j}{2} \\ &= \frac{0.29 \times 7 \times 0.90}{2} \\ &= 0.91 \end{aligned}$$

$$M_r = 0.91 \times 250 \times 520^2 = 61.52 \text{ kNm}$$

(iv) balanced steel area

moment of resistance for balanced rectⁿ (M_r)

$$M_r = A_{st} \sigma_{st} (d - x_c/3)$$

$$\Rightarrow 61.52 \times 10^6 = A_{st} \times 230 \times 468$$

$$\begin{aligned} \Rightarrow A_{st} &= \frac{61.52 \times 10^6}{230 \times 468} \\ &= 571.53 \text{ mm}^2 \end{aligned}$$

$$\text{Percentage of steel } (P_t) = \frac{100 A_{st}}{b d} = \frac{100 \times 571.53}{250 \times 520}$$

$$\left[\frac{A_{st}}{b d} \times 100 \right]$$

$$= 0.44$$

Types of Problem

The following types of problems can be encountered in the design of reinforced concrete beam

- (1) To find the moment of resistance of a given sectⁿ & load carrying capacity of a beam
- (2) To find the actual stress in steel & concrete
- (3) To find the area of tensile reinforcement.
- (4) To Design of singly reinforce the section

To find the moment of resistance of a given sectⁿ

Generally the following data are given

b = breadth of a beam

d = depth of a beam

A_{st} = area of tensile steel

σ_{cbc} = permissible stress in concrete

σ_{st} = " " in steel

m = modular ratio

To solve the problem, the following procedure may be used

- (a) Find x_c by the formula

$$x_c = \left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) \times d$$

- (b) Find x by the formula

$$b \cdot x \cdot \frac{x}{2} = m A_{st} (d - x)$$

- (c) Find whether the section is under reinforced or over reinforced
If $x < x_c$, then the sectⁿ is under reinforced.

If $x > x_c$ " " " over "

If $x = x_c$ " " " balanced sectⁿ

- (d) Find M_r

a) If the sectⁿ is under-reinforced

$$M_r = T \times Z \\ = A_{st} \sigma_{st} \left(d - \frac{x}{3}\right)$$

If the sectⁿ is over-reinforced

$$M_r = C \times Z \\ = b \times x \times \frac{\sigma_{cbc}}{2} \left(d - \frac{x}{3}\right)$$

Problem-5

A singly reinforced rectangular beam is 300 mm wide & 500 mm deep to the centre of tensile steel. It is reinforced with 4 bars of 20 mm dia on tension side. If the allowable stresses in concrete & steel are 5 N/mm^2 & 140 N/mm^2 respectively. Find the moment of resistance of the section. $m = 18.66$

Given data

$$b = 300 \text{ mm}$$

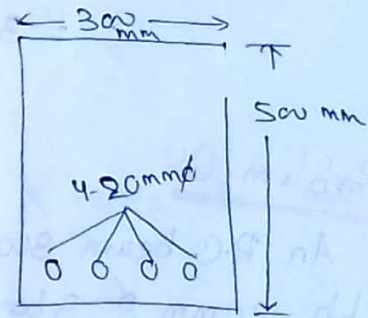
$$d = 500 \text{ mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.63 \text{ mm}^2$$

$$\sigma_{cbc} = 5 \text{ N/mm}^2$$

$$\sigma_{st} = 140 \text{ N/mm}^2$$

$$m = 18.66$$



To find the critical depth of N.A

$$x_c = \left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) d \\ = \left(\frac{18.66 \times 5}{18.66 \times 5 + 140} \right) \times 500 \\ = 199.95 \text{ mm}$$

To find the actual depth of neutral axis

$$\frac{b x^2}{2} = m A_{st} (d - x)$$

$$\Rightarrow \frac{300x^2}{2} = 18.66 \times 1256.63 (500 - x)$$

$$\Rightarrow 150x^2 = 23448.71 (500 - x)$$

$$\Rightarrow 150x^2 = 11724357.9 - 23448.71x$$

$$\Rightarrow 150x^2 + 23448.71x - 11724357.9 = 0$$

$$\Rightarrow x = 212.13 \text{ mm}$$

So $x > x_c$, the beam is overreinforced & hence concrete will reach its max^m permissible value first.

$$\begin{aligned} M_r &= C \times Z \\ &= b \times \frac{\sigma_{cbc}}{2} \left(d - \frac{x}{3} \right) \\ &= 300 \times 212.13 \times \frac{5}{2} \left(500 - \frac{212.13}{3} \right) \\ &= 68.29 \text{ kN.m} \end{aligned}$$

Problem-6

An R.C. beam 300mm wide & 700mm deep overall, is reinforced with 20mm ϕ 3 bars. The centre of bars are 50mm from the underside of the beam. The max^m stresses are not to exceed 7N/mm² for concrete and 190N/mm² for steel. Find the moment of resistance & the safety distributed load, the beam can carry. The span is 6m & weight of concrete is 25 kN/m³.

Given data

$$b = 300 \text{ mm}$$

$$D = 700 \text{ mm}$$

$$d = 700 - 50 = 650 \text{ mm}$$

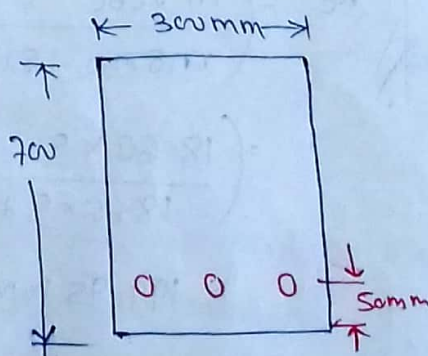
$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$\sigma_{st} = 190 \text{ N/mm}^2$$

$$m = 13.33$$

$$x_{\text{span}} (L) = 6 \text{ m}$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 20^2 = 942.47 \text{ mm}^2$$



To find the critical depth of neutral axis

$$x_c = \left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) d$$
$$= \left(\frac{13.33 \times 7}{13.33 \times 7 + 190} \right) \times 650$$
$$= 214.08 \text{ mm}$$

To find the actual depth of neutral axis

$$\frac{bx^2}{2} = m A_{st} (d - x)$$
$$\Rightarrow \frac{300x^2}{2} = 13.33 \times 942.47 (650 - x)$$
$$\Rightarrow 150x^2 = 12563.12 (650 - x)$$
$$\Rightarrow 150x^2 + 12563.12x - 8166031.31 = 0$$
$$\Rightarrow x = 195.17 \text{ mm}$$

Here $x < x_c$. So the beam is under-reinforced & hence ^{the} steel will reach its max^m permissible value first.

$$M_r = T \times Z$$
$$= A_{st} \sigma_{st} (d - x/3)$$
$$= 942.47 \times 190 \times \left(650 - \frac{195.17}{3} \right)$$
$$= 104.74 \text{ kN}\cdot\text{m}$$

Let w kN be the load on the beam per metre

$$B.M_r = \frac{wl^2}{8} = \frac{w \times 6^2}{8} = 4.5w \text{ kN}\cdot\text{m}$$

Equating M_r to B.M. we get

$$4.5w = 104.74$$
$$\Rightarrow w = 23.22 \text{ kN/m}$$

Net weight / live load the beam can carry will be

$$23.22 - 5.25 = 18.02 \text{ kN/m}$$

Problem-7

A reinforced concrete cantilever beam 2 m long & 240x400 mm is designed to support a concentrated load of 3000 N at the free end in addition to a UDL. The beam is reinforced with 4 bars 12 mm dia on the tension side. Determine the permissible UDL the beam can be support. ^{Take} $\sigma_{cbc} = 5 \text{ N/mm}^2$ & $\sigma_{st} = 140 \text{ N/mm}^2$.

Solⁿ

Given data

$$b = 240 \text{ mm}$$

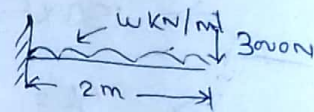
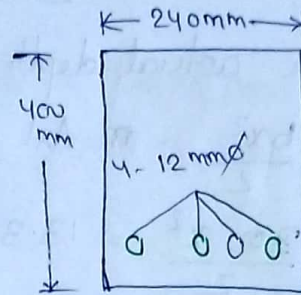
$$D = 400 \text{ mm}$$

$$\sigma_{cbc} = 5 \text{ N/mm}^2$$

$$\sigma_{st} = 140 \text{ N/mm}^2$$

$$d = 400 - 25 - 12/2 = 369 \text{ mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 12^2 = 452.38 \text{ mm}^2$$



$$\text{modular ratio } (m) = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 5} = 18.66$$

To find the critical depth of neutral axis (x_c)

$$\begin{aligned} x_c &= \left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) d \\ &= \left(\frac{18.66 \times 5}{18.66 \times 5 + 140} \right) \times 369 \\ &= 147.56 \text{ mm} \end{aligned}$$

To find the actual depth of neutral axis

$$\begin{aligned} \frac{bx^2}{2} &= m A_{st} (d - x) \\ \Rightarrow \frac{240x^2}{2} &= 18.66 \times 452.38 (369 - x) \\ \Rightarrow 120x^2 &= 8441.41 (369 - x) \\ \Rightarrow 120x^2 + 8441.41x - 3114880.58 &= 0 \Rightarrow x = 129.73 \text{ mm} \end{aligned}$$

Here $x_c > x$, so the beam is under-reinforced sectⁿ & steel reaches its max^m permissible value first.

$$M_r = T \times Z$$

$$= A_s \sigma_s \left(d - \frac{x_c}{3}\right)$$

$$= 452.38 \times 140 \times \left(369 - \frac{129.73}{3}\right)$$

$$= 20.63 \text{ kN}\cdot\text{m}$$

Bending moment of the sectⁿ

$$B.M. = \frac{wL^2}{2} \times wt$$

$$= \frac{w \times 2^2}{2} + 3 \times 2$$

Equating B.M to M_r we get

$$\frac{w \times 2^2}{2} + 3 \times 2 = 20.63$$

$$\Rightarrow 2w = 20.63 - 6$$

$$\Rightarrow w = 7.315 \text{ kN}\cdot\text{m}$$

T

To find the actual stresses in concrete & steel

Generally the following data are given

b = breadth of a sectⁿ

d = effective depth of a sectⁿ

A_{st} = area of steel

m = modular ratio

M = bending moment

To solve a problem, the following procedure may be used.

(a) Find x by the formula

$$\frac{bx^2}{2} = m A_{st} (d-x)$$

(b) Find Z from

$$Z = d - \frac{x}{3}$$

(c) Equate M_r to M

$$M_r = T \times Z$$

$$\Rightarrow BM = A_{st} \sigma_{st} \left(d - \frac{x}{3} \right)$$

The stress σ_{st} can be worked since M, A_{st} & Z are known

(d) Find the stress in concrete from

$$\frac{\sigma_{cbc}}{\sigma_{st}/m} = \frac{x}{d-x}$$

Problem 8

A sectⁿ of RC beam 300×700 mm is reinforced with 4 no's of $25\text{mm } \phi$ bars placed 30mm from the bottom of the beam. The beam is subjected to a bending moment of 130 kNm . Find the stresses set up concrete & steel, if $m = 18.66$.

Solⁿ

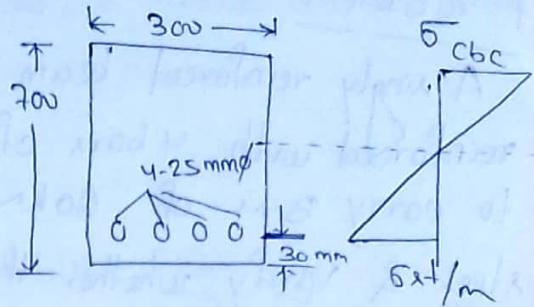
$$b = 300 \text{ mm}$$

$$d = 700 - 30 = 670 \text{ mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.49 \text{ mm}^2$$

$$B.M = 130 \text{ kN}\cdot\text{m} = 130 \times 10^6 \text{ N}\cdot\text{mm}$$

$$m = 18.66$$



To find the actual depth of neutral axis

$$\frac{bx^2}{2} = m A_{st} (d - x)$$

$$\Rightarrow \frac{300x^2}{2} = 18.66 \times 1963.49 (670 - x)$$

$$\Rightarrow 150x^2 = 36638.72 (670 - x)$$

$$\Rightarrow 150x^2 + 36638.72x - 24547944.68 = 0$$

$$\Rightarrow x = 300.44 \text{ mm}$$

$$\text{To find } z = d - \frac{x}{3} = 670 - \frac{300.44}{3} = 569.85 \text{ mm}$$

To find M_r ,

$$M_r = T \times z$$

$$= A_{st} \sigma_{st} (d - \frac{x}{3})$$

$$= 1963.49 \times \sigma_{st} \times 569.85$$

$$= 1118901.32 \sigma_{st}$$

Equating M_r to B.M we get

$$130 \times 10^6 = 1118901.32 \sigma_{st}$$

$$\Rightarrow \sigma_{st} = 116.18 \text{ N/mm}^2$$

To find σ_{cbc} from the following eqⁿ

$$\frac{\sigma_{cbc}}{\sigma_{st}/m} = \frac{x}{d - x} \Rightarrow \frac{\sigma_{cbc}}{116.18/18.66} = \frac{300.44}{670 - 300.44}$$

$$\Rightarrow \sigma_{cbc} = 5.06 \text{ N/mm}^2$$

Problem-9

A simply reinforced beam is 300mm wide & 550mm deep is reinforced with 4 bars of 20mm dia in tension side. It is designed to carry B.M. of 60kNm. Calculate the actual stress in concrete & verify whether the sectⁿ safe or not. Use concrete M_{20} & mild steel grade.

Solⁿ

$$b = 300\text{mm}$$

$$D = 550\text{mm}$$

$$d = 550 - 25 - \frac{20}{2} = 515\text{mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.63\text{mm}^2$$

$$B.M. = 60\text{kNm}$$

$$\sigma_{cbc} = 5\text{N/mm}^2$$

$$\sigma_{st} = 140\text{N/mm}^2$$

$$\text{modular ratio} = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 5} = 18.66$$

To find the critical depth of neutral axis

$$\begin{aligned} A x_c &= \left(\frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) d \\ &= \left(\frac{18.66 \times 5}{18.66 \times 5 + 140} \right) \times 515 \\ &= 205.95\text{mm} \end{aligned}$$

To find the actual depth of neutral axis

$$\begin{aligned} \frac{bx^2}{2} &= m A_{st} (d - x) \\ \Rightarrow \frac{300x^2}{2} &= 18.66 \times 1256.63 (515 - x) \end{aligned}$$

$$\Rightarrow 150x^2 = 23448.71 (515 - x)$$

$$\Rightarrow 150x^2 + 23448.71x - 12076088.64 = 0 \Rightarrow x = 216.14\text{mm}$$

Here $x > x_c$, so the beam is over reinforced & hence concrete reach its max^m permissible value first.

$$M_r = b \times \frac{\sigma_{cbc}'}{2} \left(d - \frac{x}{3} \right)$$

$$\Rightarrow 60 \times 10^6 = 300 \times 216.14 \times \frac{\sigma_{cbc}'}{2} \left(515 - \frac{216.3}{3} \right)$$

$$\Rightarrow \sigma_{cbc}' = 4.17 \text{ N/mm}^2$$

so the sectⁿ is safe.

To find σ_{xt}

$$\frac{\sigma_{cbc}'}{\sigma_{xt}'/m} = \frac{x}{d-x}$$

$$\Rightarrow \frac{4.17}{\sigma_{xt}'/18.66} = \frac{216.14}{515 - 216.14}$$

$$\Rightarrow \frac{\sigma_{xt}'}{18.66} = 5.76$$

$$\Rightarrow \sigma_{xt}' = 107.59 \text{ N/mm}^2$$

Hence the sectⁿ is safe.

Problem: 10

A beam $350 \times 650 \text{ mm}$ overall is provided with tensile reinforcement consisting of 4 nos of $22 \text{ mm } \phi$ bars. Calculate the stresses developed in steel & concrete, if the beam carries a uniformly distributed load of 20 kN/m (inclusive of self weight). Take $l_f = 7 \text{ m}$ & $m = 13.33$ & clear cover = 25 mm .

Solⁿ Given data

$$b = 350 \text{ mm}$$

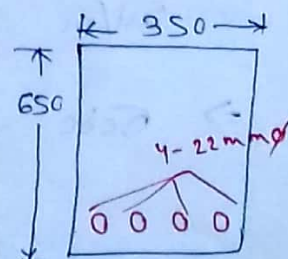
$$D = 650 \text{ mm}$$

$$d = 650 - 25 - 11 = 614 \text{ mm}$$

$$m = 13.33$$

$$l_f = 7 \text{ m}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 22^2 = 1520.53 \text{ mm}^2$$



To find the actual depth of neutral axis

$$\frac{bx^2}{2} = m A_{st} (d - x)$$

$$\Rightarrow \frac{350 x^2}{2} = 13.33 \times 1520.53 (614 - x)$$

$$\Rightarrow 175 x^2 = 20268.66 (614 - x)$$

$$\Rightarrow 175 x^2 + 20268.66 x - 12444957.24 = 0$$

$$\Rightarrow x = 214.98 \text{ mm}$$

To find lever arm

$$Z = d - \frac{x}{3} = 614 - \frac{214.98}{3} = 542.34 \text{ mm}$$

To find moment of resistance

$$M_r = A_{st} \sigma_{st} (d - \frac{x}{3})$$

$$= 1520.53 \times \sigma_{st} \times 542.34$$

$$\text{bending moment (M)} = \frac{wl^2}{8} = \frac{20 \times 7^2}{8} = 122.5 \text{ kN}\cdot\text{m}$$

Equating M_r to B.M. we get

$$122.5 \times 10^6 = 1520.53 \times \sigma_{st} \times 542.34$$

$$\Rightarrow \sigma_{st} = 148.8595 \text{ N/mm}^2$$

To find σ_{cbc}

$$\frac{\sigma_{cbc}}{\sigma_{st}/m} = \frac{x}{d - x}$$

$$\Rightarrow \sigma_{cbc} = \frac{x}{d - x} \times \sigma_{st}/m$$

$$= \frac{214.98}{614 - 214.98} \times \frac{148.85}{13.33}$$

$$= 8.00 \text{ N/mm}^2$$

To find the area of tensile reinforcement

Generally, the following data are given

b = breadth of a beam

d = depth of a beam

σ_{cbc} = permissible stress in concrete

σ_{st} = " " in steel

m = modular ratio

B.M. = bending moment

To solve the problem, the following procedure may be followed

1> Determine k, j, α for the given stress

2> Find the critical moment of resistance $M_r = \alpha b d^2$

from the dimensions of the beam

3> Compare the bending moment to the critical moment of resistance

a) If $B.M. < M_r$ the section is under reinforced

$$M_r = A_{st} \sigma_{st} \left(d - \frac{x}{3}\right)$$

To find A_{st} in term of x take moment of areas about NA

$$\frac{bx^2}{2} = m A_{st} (d - x)$$

$$\Rightarrow A_{st} = \frac{bx^2}{2m(d-x)}$$

$$\text{so } M = \frac{bx^2 \sigma_{st} \left(d - \frac{x}{3}\right)}{2m(d-x)} = \text{B.M. to be resisted}$$

solve for x & then A_{st} can be calculated.

(b) If $\frac{x}{d} > \frac{x_c}{d}$ the sectⁿ is over reinforced sectⁿ

$$M_f = \frac{b x \sigma_{cbc}}{2} (d - \frac{x}{3}) = \text{B.M. to be resisted}$$

Determine x . Then A_{st} can be obtained by taking moments of areas about N.A using.

$$A_{st} = \frac{b x^2}{2 m (d - x)}$$

Problem: 11

A singly reinforced rectangular beam 300mm wide & 600mm deep has an effective span of 6m. Find necessary tension reinforcement to enable the beam to carry a live load of 8 kN/m in addition to its own weight. Use M15 grade of concrete & steel of grade Fe 250. modular ratio = 18.66 & density of concrete = 25 kN/m³

Solⁿ

Given data $b = 300 \text{ mm}$

$$D = 600 \text{ mm}$$

$$d = 600 - 35 = 565 \text{ mm}$$

$$\sigma_{cbc} = 5 \text{ N/mm}^2$$

$$\sigma_{st} = 140 \text{ N/mm}^2$$

live load of the beam = 8 kN/m

$$\text{dead load/self weight of the beam} = 0.3 \times 0.6 \times 25 = 4.5 \text{ kN/m}$$

$$\text{So the total load on the beam (w)} \\ = 8 + 4.5 = 12.5 \text{ kN/m}$$

The max^m bending moment is at centre of the beam

$$\text{B.M.} = \frac{w l^2}{8} = \frac{12.5 \times 6^2}{8} = 56.25 \text{ kN-m}$$

$$\text{neutral axis const (rk)} = \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}}$$

$$= \frac{18.66 \times 5}{18.66 \times 5 + 140} = 0.39$$

lever arm const (j)

$$= 1 - \frac{k}{3}$$

$$= 1 - \frac{0.39}{3} = 0.87$$

moment of resistance const (Q)

$$= \frac{k \sigma_{cbc} j}{2}$$

$$= \frac{0.39 \times 5 \times 0.87}{2}$$

$$= 0.85 \text{ N/mm}^2$$

Critical moment of resistance

$$M_r = Q b d^2$$

$$= 0.85 \times 300 \times 565^2$$

$$= 81.4 \text{ kN-m}$$

Here $M_r > \text{B.M.}$ Hence the sectⁿ is under reinforced & steel will fail first.

$$M_r = A_{st} \sigma_{st} (d - \frac{x}{3})$$

To find A_{st} in terms of x take moment of areas about N.A.

$$A_{st} = \frac{b x^2}{2m(d-x)}$$

$$\therefore M_r = \frac{b x^2}{2m(d-x)} \sigma_{st} (d - \frac{x}{3})$$

$$\Rightarrow 56.25 \times 10^6 = \frac{300 x^2}{2 \times 18.66 (565 - x)} \times 140 \times (565 - \frac{x}{3})$$

$$\Rightarrow 49982.14 = \frac{x^2 (565 - \frac{x}{3})}{565 - x}$$

by solving trial & error method we get

$$x = 192.75 \text{ mm}$$

$$\begin{aligned} \text{So } A_{st} &= \frac{bx^2}{2m(d-x)} \\ &= \frac{300 \times 192.7^2}{2 \times 18.66 \times (565 - 192.7)} \\ &= 801.77 \text{ mm}^2 \end{aligned}$$

Problem-12

A singly reinforced beam $250 \times 400 \text{ mm}$ deep to the centre of steel at a span of 5 m . It carries a total UDL of 12.5 kN/m including of its weight. The stress in concrete & steel shall not exceed 5 N/mm^2 & 230 N/mm^2 .

Solⁿ

$$\text{modular ratio } (m) = \frac{280}{35 \text{ cbc}} = \frac{280}{3 \times 5} = 18.66$$

$$\text{The total load the beam can take} = 12.5 \text{ kN/m} = 12.5 \text{ kN/m}$$

The max^m bending moment is at centre of beam

$$BM = \frac{wl^2}{8} = \frac{12.5 \times 5^2}{8} = 39.06 \text{ kN-m}$$

neutral axis const (k)

$$= \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}}$$

$$\begin{aligned} &= \frac{18.66 \times 5}{18.66 \times 5 + 230} \\ &= 0.29 \end{aligned}$$

lever arm const (j) = $1 - \frac{k}{3}$

$$\begin{aligned} &= 1 - \frac{0.29}{3} \\ &= 0.90 \end{aligned}$$

moment of resistance const (R) = $\frac{k \sigma_{cbc} j}{2}$

$$= \frac{0.29 \times 5 \times 0.90}{2} = 0.65 \text{ N/mm}^2$$

Critical moment of resistance

$$M_r = Qbd^2$$

$$= 0.65 \times 250 \times 400^2$$

$$= 26 \text{ kNm}$$

Here $BM > M_r$. Hence the section is over reinforced & concrete reaches its max^m permissible value first

$$M = \frac{bn\sigma_{cbc}}{2} \left(d - \frac{x}{3}\right)$$

$$\Rightarrow 39.06 \times 10^6 = 250 \times x \times \frac{5}{2} \left(400 - \frac{x}{3}\right)$$

$$\Rightarrow 62496 = 400x - \frac{x^2}{3}$$

$$\Rightarrow x^2 - 1200x + 187488 = 0$$

$$\Rightarrow x = 184.65 \text{ mm}$$

Then A_{st} can be calculated by taking moments about NA

$$A_{st} = \frac{bx^2}{2m(d-x)}$$

$$= \frac{250 \times 184.65^2}{2 \times 18.66 \times (400 - 184.65)}$$

$$= 1060.60 \text{ mm}^2$$

$$= 1060.60 \text{ mm}^2$$

Design of singly reinforced beam

Generally the following data are given

σ_{cbc} = permissible stress in concrete

σ_{st} = " " " steel

m = modular ratio

B.M = bending moment

To solve the problem the following procedure may be followed

1> Determine the k, j, Q for the given stress

2> Find the breadth of the beam by the following relationship

$$\text{breadth of beam } (b) = \frac{\text{effective span}}{30} + 80 \text{ mm}$$

3> Find the depth of beam

$$d = 2b \text{ or } 3b$$

4> Find the critical moment of resistance

$$M_{cr} = Qbd^2$$

5> Find the sectⁿ is under reinforced or over reinforced

6> Then the area of steel can be calculated by the before procedure.

Problem: 13

A singly reinforced RCC beam has a ^{effective} span of 5m. It carries a total UDL of 12500 N/m including its self weight. The stresses in concrete & steel shall not exceed 5 N/mm² & 230 N/mm². design the beam.

Solⁿ:
Given data

$$\text{effective span (l)} = 5\text{m}$$

$$w = 12500 \text{ N/m} = 12.5 \text{ kN/m}$$

$$\sigma_{cbc} = 5 \text{ N/mm}^2$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$\text{modular ratio (m)} = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 5} = 18.66$$

for the beam max^m B.M. at centre

$$B.M. = \frac{wl^2}{8} = \frac{12.5 \times 5^2}{8} = 39.06 \text{ kN.m}$$

$$\begin{aligned} \text{neutral axis const (k)} &= \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} \\ &= \frac{18.66 \times 5}{18.66 \times 5 + 230} = 0.29 \end{aligned}$$

$$\text{lever arm const (j)} = 1 - \frac{k}{3} = 1 - \frac{0.29}{3} = 0.90$$

$$\begin{aligned} \text{moment of resistance const (Q)} &= \frac{k \cdot \sigma_{cbc} j}{2} \\ &= \frac{0.29 \times 5 \times 0.90}{2} \\ &= 0.65 \end{aligned}$$

$$\begin{aligned} \text{breadth of the beam } b &= \left(\frac{\text{effective span}}{30} + 80 \text{ mm} \right) \\ &= \left(\frac{5000}{30} + 80 \right) = 246.66 \text{ mm} \approx 250 \text{ mm} \end{aligned}$$

So effective depth of the beam (d)

$$d = 2b \\ = 2 \times 250 = 500 \text{ mm}$$

The critical moment of resistance

$$M_r = Qbd^2 \\ = 0.65 \times 250 \times 500^2 \\ = 40.62 \text{ kNm}$$

Here $M_r > BM$. So the section is underreinforced & hence steel reaches its max^m permissible value f_{st}

$$M = A_{st} \sigma_{st} (d - x/3)$$

To find A_{st} in terms of ' x ', take moment of areas about N.A.

$$\frac{bx^2}{2} = m A_{st} (d - x) \\ \Rightarrow A_{st} = \frac{bx^2}{2m(d-x)}$$

$$\text{so } M = \frac{bn^2 \sigma_{st}}{2m(d-x)} (d - x/3)$$

$$\Rightarrow 39.06 \times 10^6 = \frac{250 \times x^2 \times 230 (500 - x/3)}{2 \times 1866 \times (500 - x)}$$

$$\Rightarrow 25351.63 = \frac{x^2}{500 - x} (500 - x/3)$$

by solving trial & error method we get

$$x = 141.7 \text{ mm}$$

$$A_{st} = \frac{bx^2}{2m(d-x)} = \frac{250 \times 141.7^2}{2 \times 1866 \times (500 - 141.7)} \\ = 375.39 \text{ mm}^2$$

Advantages & Disadvantages of WSM (Working Stress Method)

This is the traditional method of design, used not only for reinforced concrete but also for structural steel & timber. This method of design was evolved around the year 1900. This method was accepted by many traditional national codes. India's code IS:456-1953 was based on this method. It was revised in 1957, 1964, 1978, 2000 & reaffirmed in 2005 & tenth reprint ^{was} in 2007. This method is based on linear elastic theory.

* This method ensures adequate safety by suitably restricting the stresses in the materials induced by expected working loads on the structure.

* In this method it is assumed that the concrete & steel are elastic. At the worst combinations of working loads, the stresses in the materials are not exceeded beyond permissible values. The permissible stresses are found out by using a suitable factor of safety to the material strength, e.g. for concrete in compression due to bending, a factor of safety equals to 3.0 is considered on 28 days characteristic strength & factor of safety equals to 1.8 is considered on the yield strength for mild steel reinforcement in tension due to bending.

* Working stress method does not consider the mode of failure of the structure that whether it is ductile or brittle. Also the reserve strength of the material beyond yield point is not considered in this method of design.

* Reinforced concrete is a composite material. The WSM assumes the strain compatibility, whereby the strain reinforcing steel is assumed to be equal to that in the adjoining concrete to which it is bonded. Consequently the stress in concrete by a const. factor, called the modular ratio, defined as the ratio of modulus of elasticity of steel to that of concrete. The WSM is therefore also known as modular ratio method.

$$\text{modular ratio } m = E_s / E_c = 280 / 35000$$

Advantages of WSM

- * It is simple, both in concept as well as in application.
- * It is reasonably reliable.
- * It is essential to have a knowledge of WSM, since it forms a part of LSM. IS: 456-2000 has incorporated LSM & WSM. But the concept of WSM is retained for checking serviceability states of deflection & cracking.
- * The design usually results in large sections of structural members as compared to LSM. Due to this, structural design by WSM gives better serviceability performance, i.e. less deflection, less crack width etc.

Dis-advantages of WSM

- It gives larger sectⁿ of structural members, thus it is uneconomical.
- It assumes that, stress & strain relationship is const for concrete, which is not true.
- WSM does not consider the mode of failure of the structure (brittle or ductile).
- Because of creep & non linear stress-strain relationship, concrete does not have definite modulus of elasticity.
- The modular ratio design results in larger percentage of compression steel than that given by the limit state design, thus it leads to uneconomical design.
- WSM fails to discriminate betⁿ different types of loads that act simultaneously but have different uncertainties.

Philosophy of Limit State Method (LSM)

The object of reinforced concrete design is to achieve a structure that will result in a safe & economical solution.

* There are four philosophies for the design of reinforced concrete, prestressed concrete as well as steel structure.

(1) Working stress method Design

(2) Ultimate load design

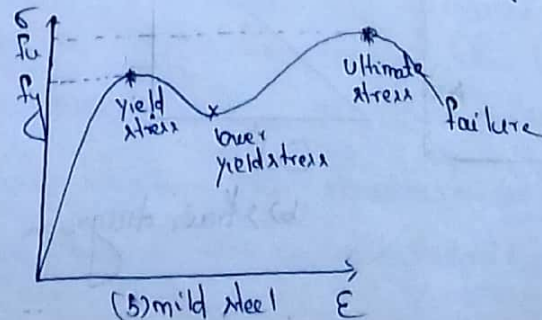
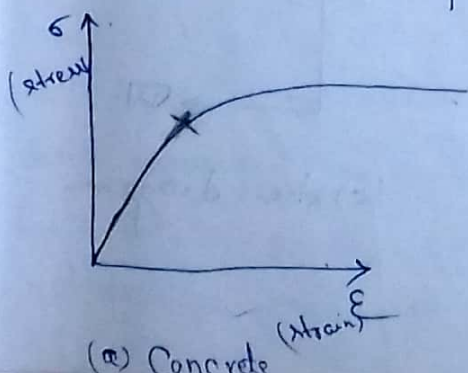
(3) Limit state design

(4) ^{for} Performance based design

Working Stress method/modular ratio method/Elastic stress method

* The working stress method was the prevalent in use from the early 1900's until the early 1960's.

* The working-stress method of design was developed first which is based on the behaviour of the structure at working load. The stress distribution in concrete & steel at working load is assumed to be linear. Hence the design is made by assuming linear stress-strain relationship ensuring that the stresses in concrete & steel do not exceed their permissible values at service load, which is taken as the fixed proportion of the ~~ultimate~~ ^{yield} strength of materials. It is obtained by dividing the yield strength of materials by the factor of safety. The factor of safety for ~~steel~~ ^{concrete} in compression in bending & in direct compression are 3.0 & 4.0 respectively. The factor of safety for steel in tension & compression is 1.8.



$1.78 - 1.8$

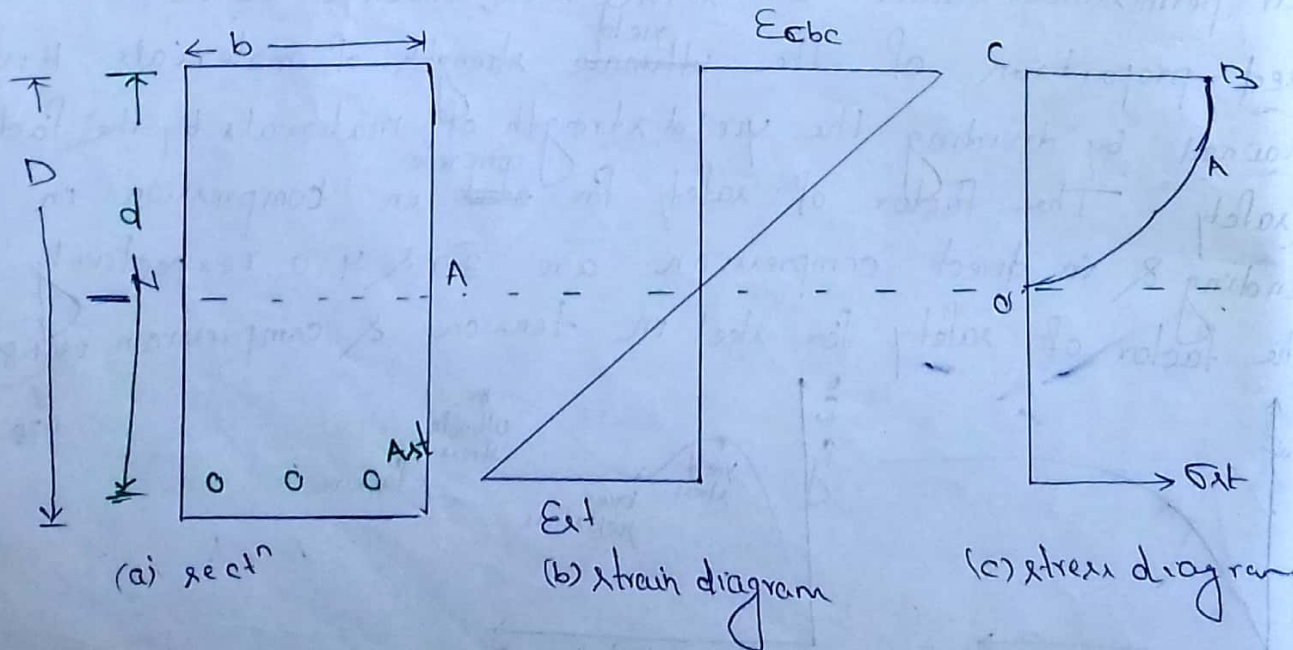
Ultimate load design / Load factor method

In the ultimate load method, the working loads are increased by suitable factors to obtain ultimate loads. These factors are called load factors. The structure is then designed to resist the desired ultimate loads. This method takes into account the nonlinear stress-strain behaviour of concrete.

* The term safety factor has been used in the working stress method to denote the ratio betⁿ the yield stress and the permissible stress. It had little meaning as far as the ratio betⁿ collapse load & working load was concerned. The term load factor has been traditionally used to denote the ratio betⁿ the collapse or ultimate load to the working load.

* The knowledge of load factor is more important than the knowledge of factor of safety. In the former case, the designer is able to predict the excess load which a given structure can carry beyond the working load without collapse.

* When, stress-strain diagram in this method is idealized into individual well defined graphs, then the stress diagram becomes parabolic, which is straight line in elastic theory or the working stress method.



Limit State method

* It takes into account the inelastic behaviour of the materials, which resulted into economical design. At the same time, it resulted in slender section, large deformations, & larger crack widths in concrete.

This created the necessity of refining the method so that in addition to strength, the structure ~~shall~~ would be serviceable also.

Limit State Method

Limit state method of design is a further improvement of ultimate load design. We know that the Ultimate load theory is based on the assumption that a structure reaches a collapse condition forming a mechanism ^{ptⁿ} when a certain load is applied. The ratio of the Ultimate load to the ~~factored~~ ^{working} load is called the load factor. This method appeared to be ~~more~~ satisfactory and acceptable & was recommended by the codes of practice of many countries.

* After more research & study it has now been felt that this method of design has to be modified, so that a structure can be designed to function satisfactorily at service condition.

* The ultimate load method appears to emphasize on the adequate strength of a member. Now it is realized that the adequate strength could not be considered as the only criterion for the satisfactory performance of the member at service conditions. In most of the cases, a member designed by ultimate load theory is too slender to be adopted & if adopted may exhibit new problems like excessive deflection, excessive crack width etc at service conditions.

* In the limit state method, a structure is designed to withstand all loads likely to act on it in the duration of its life span & also to satisfy the serviceability requirements before failure can occur. The design provides a condition that the structure will not become unfit for use for which it is meant or in other words the structure will not reach a limit state.

* All limit states which are relevant are considered in the design to ensure an adequate degree of safety & serviceability. The structure in general shall be designed on the basis of the most critical limit state & shall also be checked for other limit states.

* The limit states to be considered are
(a) Formation of a mechanism leading to collapse &
(b) excessive deflection & development of excessive cracks.

* The limit state corresponding to the formation of a mechanism depends on the ultimate strength. But the limit state corresponding to excessive deflection & excessive crack width is determined by elastic theory. Thus the limit state method combines the principles of ultimate load theory & elastic theory.

Limit State - The acceptable limit for the safety & serviceability requirements before failure occurs.

Types of limit states

The following limit states refer to the strength of the structure are considered in the design of structures.

(a) Limit state of collapse / failure → ^{Based on imaginary behaviour of concrete} [Flexure (bending), Compression, shear, Torsion]

This limit state refers to the strength of the structure. A structure or its part should be strong enough to resist the applied stress design loads. This limit state is called collapse.

* The limit state of collapse includes the design for axial forces, flexure, shear, torsion, buckling etc. The strength of each section must be more than the applied stresses on that section due to all expected combination of loads.

(b) Limit state of serviceability → ^{Based on actual behaviour of concrete} (deflection, cracking, vibration, fire resistance etc)

The structure or its part thereof shall be serviceable during its expected life span. The serviceability corresponds to the deflection & cracking of the structure.

Deflection:

Excessive deflections that can reduce the efficiency of the structure must be avoided.

Cracking:

Concrete structures have innumerable cracks, however if the crack widths are larger, the appearance of the structure will be affected. Also ingress of water & gas from the atmosphere can rusting of reinforcements. For normal structures, the surface crack width of 0.3 mm is acceptable.

* It may happen that the structure is strong & safe to resist applied design loads but is not serviceable. Increasing the section dimensions usually solves the problem.

(c) Limit state of durability: Rain, floods, fire, earthquakes, severe weather etc are forces of nature which may affect the durability of a structure. Limits in the form of specifications are provided to cover the durability of the structures.

Partial safety factors:

These are the factors which when applied to loads & materials give the design values. The partial safety factors take into account the possible overloads, the kind of limit state & inaccurate assessment of the effects of loading. There are set out in table 18 of BS 5456 accordingly the partial safety factors for various loads shall be as follows.

(A) Partial safety factors (γ_f) for loads

Sr No.	Description	Partial safety factors	
		Collapse	Serviceability (short term)
1.	Gravity loads		
	DL	1.5	1.0
	IL (IL)	1.5	1.0
2.	DL+WL (or EL) combination		
	(i) for normal case		
	DL	1.5	1.0
	WL (EL)	1.5	1.0
	(ii) for checking stability against overturning / stress reversal		
	DL	0.9	1.0
	WL (or EL)	1.5	1.0
3.	DL+LL+WL (or EL) combination		
	DL	1.2	1.0
	LL	1.2	0.8
	WL (or EL)	1.2	0.8

* For long term effect only permanent loads like dead load & a few live loads likely to be permanent should be considered

(B) Partial safety factors (γ_m) for materials

Material	Partial safety Factor
For concrete	1.5
" steel	1.15

Characteristic strength of materials

The characteristic strength of a material is that value of the strength of a material below which not more than 5 percent of the test results are expected to fall.

* The characteristic strength (f_{ck}) for various grades of concrete are given below

Grade of concrete	M15	M20	M25	M30	M35	M40
Characteristic strength (28 days compressive strength f_{ck} in N/mm^2)	15	20	25	30	35	40

* The characteristic strength for steel (f_y) shall be assumed as min^m yield stress or 0.2% proof stress.

Grade of steel	Fe 250	Fe 415	Fe 500
Characteristic strength (f_y) N/mm^2	250	415	500

Characteristic Load (F)

The term 'characteristic load' means that value of load which has a 95% probability of not ^{being} exceeded during the life of structure & is denoted by (F)

Design values

The design values are obtained when partial safety factors are applied to characteristic loads F_k and strengths. These are obtained as below.

(i) Materials:

The design strength of the materials f_d is given by

$$f_d = \frac{f}{\gamma_m}$$

where f = characteristic strength of the material

γ_m = partial safety factor appropriate to the material & limit state being considered

* While assessing the strength of a structural member for the limit state of collapse, the value of partial safety factor γ_m should be taken as 1.5 for concrete & 1.15 for steel.

(ii) Loads: The design load F_d is given by

$$F_d = F \gamma_f$$

F = characteristic load

γ_f = partial safety factor appropriate to the nature of loading & the limit state being considered.

(Table 18 of IS 456:2000)

Loading on the structure as per IS 875

(i) Characteristic dead loads are due to weight of the structure itself. These shall be found out using the unit weights of materials as given in IS:875-1987 (part I)

(ii) Characteristic live loads and wind loads in the absence of statistical data, shall be taken from IS:875-1987 (part II & III)

(iii) same

windloads (IS 875 1987 part IV)

Q) characteristic seismic loads in the absence of statistical data shall be taken from IS: 1893-2002 (part-1)

Spec 1.5 specification regarding spacing of reinforcement in slab.

The bars shall be placed in such a way that they allow the concrete to enter when placed or a vibrator can be immersed.

max^m spacing of main bars - cl - 26.3.3b)

The spacing of the main bar shall not exceed the following

i) 3 times the effective depth of the slab.

ii) 300mm, which ever is smaller

max^m spacing of distribution bars. The spacing of distribution bars shall not exceed the following

i) Five times the effective depth of the slab

ii) 300 450 mm, which ever is smaller

min^m spacing of bars: The spacing of the bars shall not be less than 75 mm for satisfactory concreting.

Cover to reinforcement

A concrete cover shall have to provided to the reinforcement for the following reasons.

(1) to protect the reinforcement from weather & fire, so that it does not corrode or melt &

(2) to ensure the grip of concrete over reinforcement so that they act as one & resist the loads.

cover (concrete cover) / ^{clear cover / nominal cover} is defined as the min^m distance betⁿ the surface of concrete to the outside of the reinforcement.

Thus Overall depth (D) = effective depth (d) + bar radius + cover

Cover to reinforcement:

slab For mild exposure, specified clear cover is 20mm; This can be reduced by 5mm when the reinforcement of 12 mm dia or less is used. In most slabs, the bar dia is not more than 12 mm. We may take clear cover of 15mm for $\phi \leq 12$ mm & clear cover of 20mm when $\phi > 12$ mm for mild exposure.

beam (Table 16) of IS 456

The cover to all reinforcement including stirrups is 20mm for mild exposure. Assuming max^m diameter of stirrups for normal beams be 10mm, the clear cover to main reinforcement for beams may be taken as $20 + 10 = 30$ mm.

column

The longitudinal reinforcing bar in a column nominal cover shall in no case not be less than 40mm or less than the diameter of such bar.

* In the case of columns of min^m dimension, of 200mm or under, whose reinforcing bar do not exceed 12mm, a nominal cover of 25mm may be used.

footing

For footing min^m cover shall be 50mm.

Min^m reinforcement

Beams
(a) Tension reinforcement: (26.5.1)

min^m reinforcement: The min^m area of tension reinforcement shall be not less than that given by the following

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

where A_s = min^m area of tension reinforcement

b = breadth of beam

d = effective depth

f_y = characteristic strength of reinforcement in N/mm²

7
b) max^m reinforcement:

The max^m area of tension reinforcement shall not exceed $0.04 bD$.

Compression reinforcement:

The max^m area of compression reinforcement shall not exceed $0.04 bD$.

min^m reinforcement in slab: (26.5.2.1)

The reinforcement in either direction in slab shall not be less than the following

$0.15\% bD$ for mild steel bar

$0.12\% bD$ for HYSD bars.

Column

The c/s area of longitudinal reinforcement, shall be not less than 0.8% nor more than 6% of the gross sectional area of column.

Effective span (22.2)

Simply supported beam or slab:

The effective span of a member that is not built integrally with its supports shall be taken as clear span plus the effective depth of slab or beam or c/c of supports whichever ever is less.

b) Cantilever^{beam/slab} - The effective length of a cantilever shall be taken as its length to the face of support plus half the effective depth except where it forms the end of a continuous beam where the length to the centre of support shall be taken.

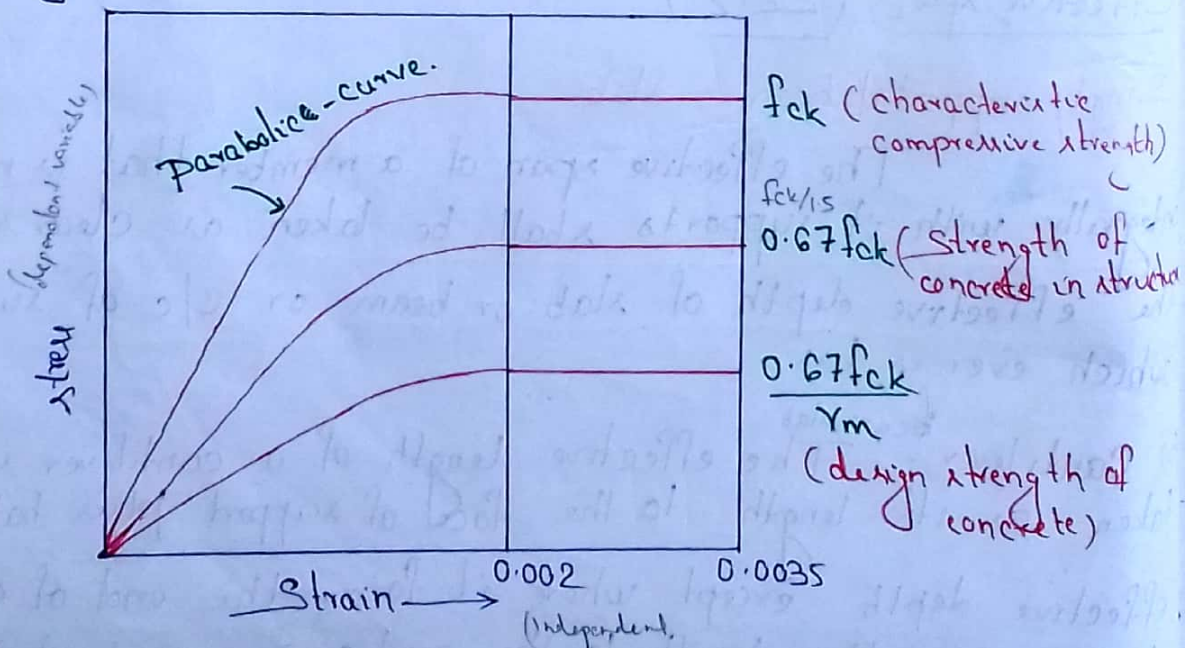
Analysis & Design of Singly & Doubly Reinforced Sections (LSM)

Limit state of collapse: Flexure:

Assumptions ^{clause} (38.1)

Design for the limit state of collapse in flexure shall be based on the assumptions given below

- a) plane sections normal to the axis remain plane after bending
- (b) The max^m strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.
- (3) The ~~stress~~ relationship betⁿ the compressive stress distribution in concrete & the strain in concrete may be assumed to be rectangular, trapezoid, parabola or any other shape which results in predictⁿ of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given in fig. For design purposes the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor $\gamma_m = 1.5$ shall be applied in addition to this.



Fig(1) Stress-strain curve for concrete

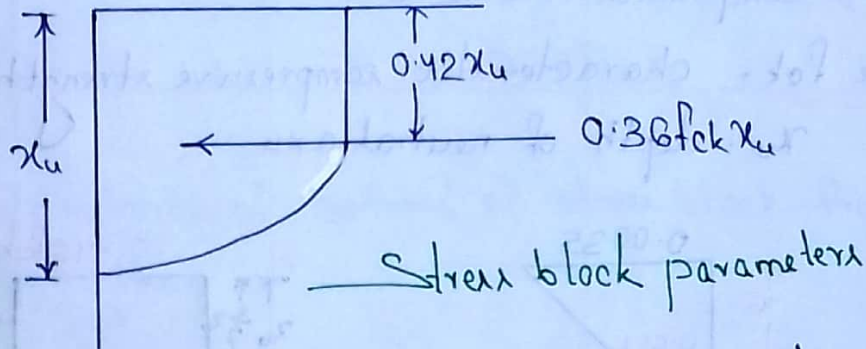
Note: For the stress-strain curve in fig (a) the design stress block parameters are as follows

$$\text{Area of stress block} = 0.36 f_{ck} x_u$$

$$\text{Depth of centre of compressive force} = 0.42 x_u$$

from the extreme fibre in compression

where f_{ck} = characteristic compressive strength of concrete &
 x_u = depth of neutral axis.



(d) The tensile strength of the concrete is ignored.

(e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in (fig 23 of IS 456). For design purposes the partial safety factor γ_m , equal to 1.15 shall be applied.

(f) The max^m strain in the tension reinforcement in the sectⁿ at failure shall not be less than.

$$\frac{f_y}{1.15 E_s} + 0.002$$

where f_y = characteristic strength of steel

E_s = modulus of elasticity of steel

Stress-strain relationship for concrete & steel

fig 21 & 23 of IS 456 2000

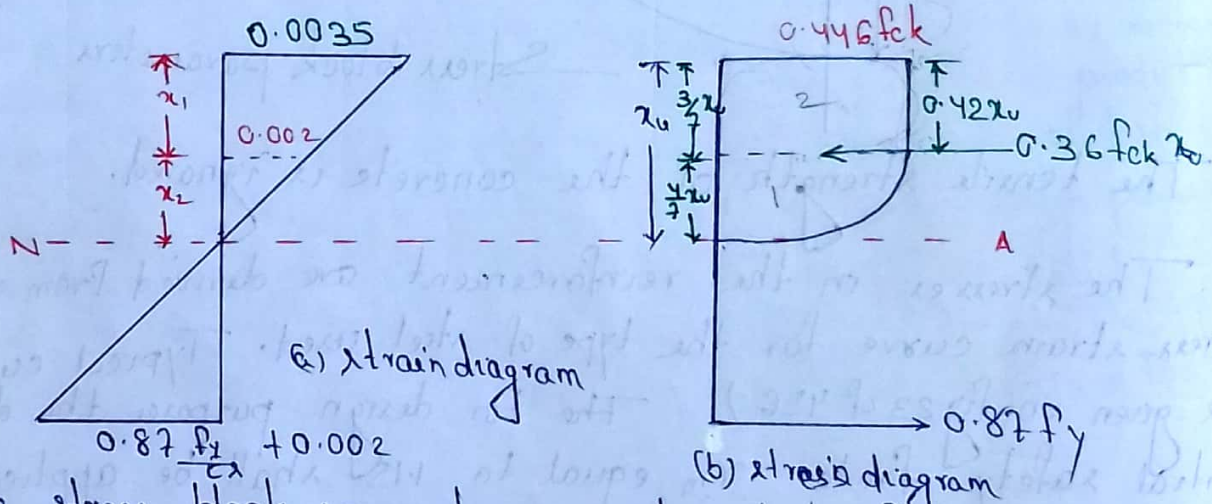
Stress block diagram & strain diagram for singly reinforced section

The strain diagram & design stress block are separately shown in Fig. The design stress block parameters are as follows

$$\text{Area of stress block} = 0.36 f_{ck} x_u$$

$$\text{depth of centroid of compressive force from the extreme fibre in compression} = 0.42 x_u$$

where f_{ck} = characteristic compressive strength of concrete
 x_u = depth of neutral axis



The stress block parameters are derived as follows

See stress block diagram & strain diagram for concrete

Let x_1 = height of rectangular part of the stress block
 x_2 = " of parabolic " " "

As the strain in concrete is proportional to the distance from the neutral axis, the

$$\text{Now } \frac{x_2}{x_u} = \frac{0.002}{0.0035} = \frac{4}{7}$$

$$\therefore x_2 = \frac{4}{7} x_u$$

$$\& x_1 = x_u - \frac{4}{7} x_u = \frac{3}{7} x_u$$

Area of the parabolic part of the stress block

$$= \frac{2}{3} \times 0.446 f_{ck} \times \frac{4}{7} x_u$$
$$= 0.17 f_{ck} x_u$$

Area of the rectangular part of the stress block

$$= 0.446 f_{ck} \times \frac{3}{7} x_u = 0.19 f_{ck} x_u$$

$$\therefore \text{Total area of the stress block} = 0.17 f_{ck} x_u + 0.19 f_{ck} x_u$$
$$= 0.36 f_{ck} x_u$$

Let $\bar{y}$ be the distance of centroid of stress block from extreme compression fibre, then

$$\bar{y} = \frac{0.17 f_{ck} x_u \times \left(\frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u \right) + 0.19 f_{ck} x_u \times \frac{x_u}{2}}{0.36 f_{ck} x_u}$$

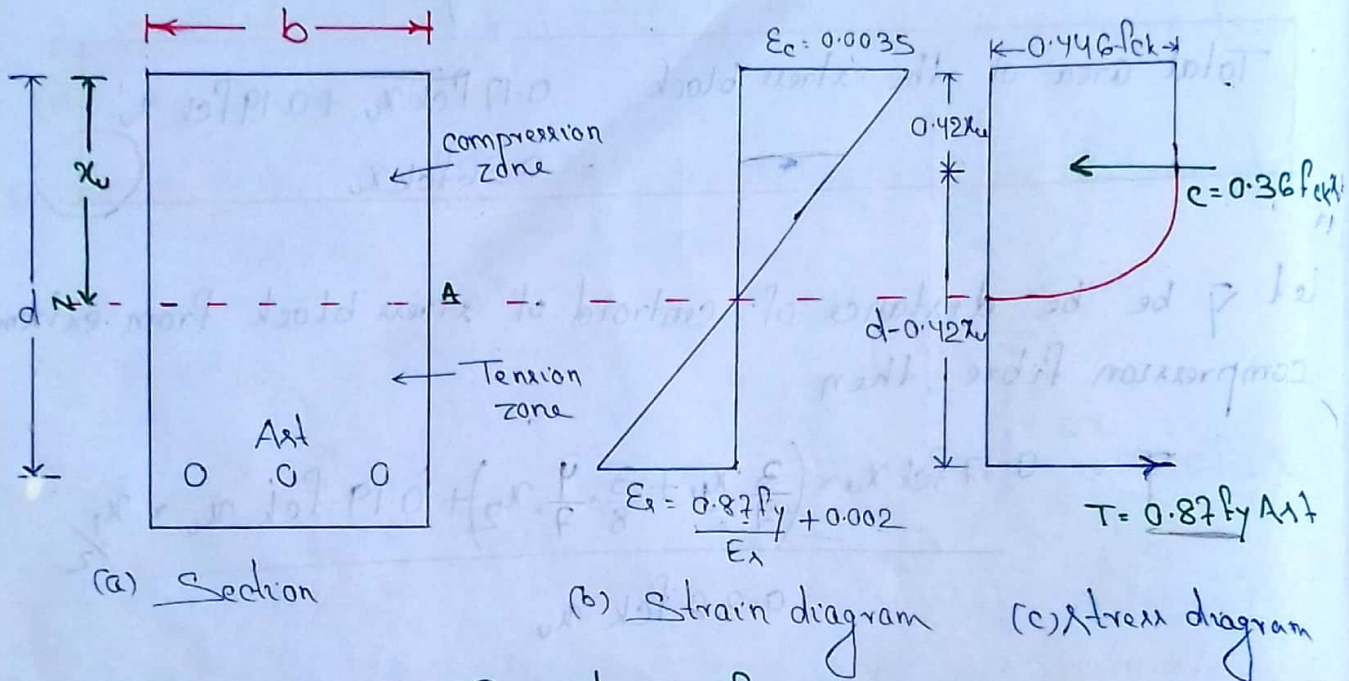
$$= \frac{0.17 f_{ck} x_u \times \left(\frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u \right) + 0.19 f_{ck} x_u \times \left(\frac{1}{2} \times \frac{3}{7} x_u \right)}{0.36 f_{ck} x_u}$$

$$\bar{y} = 0.42 x_u$$

Singly Reinforced Rectangular Beam

A singly reinforced rectangular beam sections, strain diagram & stress diagram are shown in fig.

* To find neutral



Singly reinforced beam

* To find neutral axis

Total compression = total tension

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

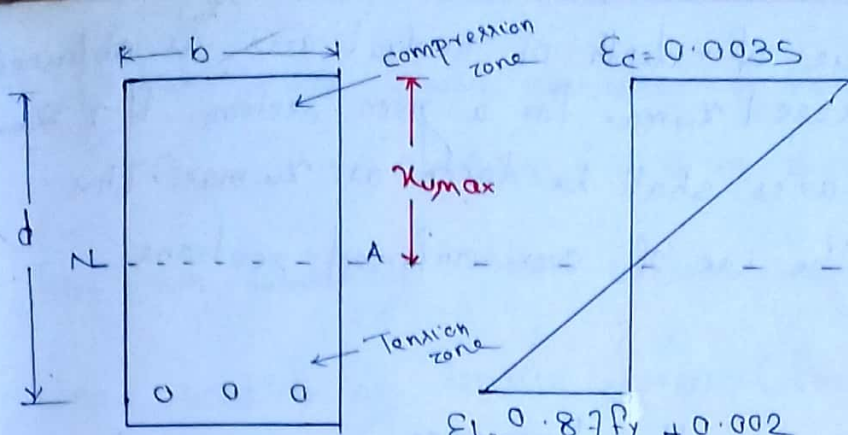
$$\text{Total compression} \\ c = \text{Area of stress block} \\ \times b$$

$$\text{Total tension / tensile force} \\ = \text{Area of steel} \times \text{stress}$$

Max^m depth of neutral axis

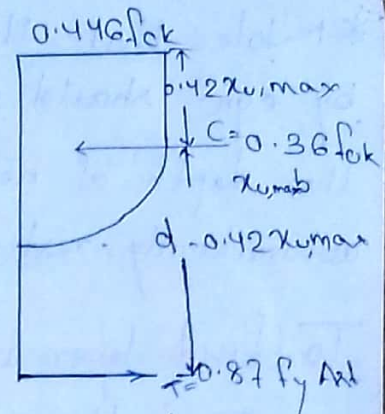
The max^m depth of neutral axis is governed by the assumption (a) & (b) in members in flexure

Fig shows the strain & the stress diagram at limiting conditions. Let $x_{u, \text{max}}$ be the max^m depth of neutral axis



(a) secⁿ

(b) strain diagram



(c) stress diagram

by geometry of the strain diagram

$$\frac{x_{u,max}}{d - x_{u,max}} = \frac{0.0035}{\frac{0.87f_y}{E_s} + 0.002}$$

$$\Rightarrow \frac{x_{u,max}}{d} = \frac{0.0035}{0.0035 + \frac{0.87f_y}{E_s} + 0.002}$$

$$\Rightarrow \frac{x_{u,max}}{d} = \frac{0.0035}{0.0055 + \frac{0.87f_y}{E_s}}$$

$$\Rightarrow \frac{x_{u,max}}{d} = \frac{0.0035 E_s}{0.0055 E_s + 0.87f_y}$$

Taking $E_s = 2 \times 10^5 \text{ N/mm}^2$, we get

$$\frac{x_{u,max}}{d} = \frac{700}{1100 + 0.87f_y}$$

The following table shows the values of $\frac{x_{u,max}}{d}$ for various grades of steel

value of $\frac{x_{u,max}}{d}$ for various grades of steel

$f_y \text{ (N/mm}^2\text{)}$	$\frac{x_{u,max}}{d}$
250	0.53
415	0.48
500	0.46

* Note that the value of depth of neutral axis x_u obtained by eqn should not exceed $x_{u,max}$ for a given section. If $x_u > x_{u,max}$, the depth of neutral axis shall be taken as $x_{u,max}$. This automatically restricts the use of over-reinforced sections.

To find lever arm:

From the stress diagram, the lever arm

$$Z = d - 0.42 x_u \quad \dots (2)$$

* To find moment of resistance:

(1) For a balanced section

$$\begin{aligned} MR &= \text{total compression} \times \text{lever arm} \\ &= \text{total tension} \times \text{lever arm} \end{aligned}$$

Considering the compressive forces (over reinforced & balanced section)

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 f_{ck} \frac{x_u}{d} (1 - 0.42 \frac{x_u}{d}) f_{ck} b d^2 \quad \dots (3)$$

For a limiting value, substitute $x_{u,max}$ for x_u & $M_{u,lim}$ for M_u

$$\text{Then } M_{u,lim} = 0.36 \frac{x_{u,max}}{d} (1 - 0.42 \frac{x_{u,max}}{d}) f_{ck} b d^2 \quad \dots (4)$$

$$Q_{lim} = \frac{M_{u,lim}}{b d^2}$$

$$\text{where } Q_{lim} = \frac{M_{u,lim}}{b d^2}$$

$$= 0.36 \frac{x_{u,max}}{d} (1 - 0.42 \frac{x_{u,max}}{d}) f_{ck}$$

Q_{lim} = limiting moment of resistance factor for balanced rectangular sectⁿ

For eg, for M20 grade concrete & Fe415 grade steel.

$$Q_{lim} = 0.38 \times 0.48 (1 - 0.42 \times 0.48) \times 20 = 2.76$$

(2) For an under-reinforced section

Now considering tensile forces (For under-reinforced sections)

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u) \dots (5)$$

$$\text{Substituting } x_u = \frac{M_u}{0.36 f_{ck} b} \left[\begin{aligned} M_u &= 0.87 f_y A_{st} \left(d - 0.42 \times \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \right) \\ M_u &= 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{b d f_{ck}} \right) \dots (6) \end{aligned} \right]$$

Dividing both the sides by $0.87 f_y b d^2$

$$\frac{M_u}{0.87 f_y b d^2} = \frac{A_{st}}{b d} \left(1 - \frac{f_y A_{st}}{f_{ck} b d} \right)$$

$$\text{Substituting } A_{st} = \frac{P_t b d}{100}$$

$$\frac{M_u}{0.87 f_y b d^2} = \frac{P_t}{100} - \frac{f_y}{f_{ck}} \left(\frac{P_t}{100} \right)^2$$

$$\Rightarrow \frac{f_y}{f_{ck}} \left(\frac{P_t}{100} \right)^2 - \left(\frac{P_t}{100} \right) + \frac{M_u}{0.87 f_y b d^2} = 0$$

which on solving gives

$$\frac{P_t}{100} = \frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_u}{b d^2}}}{2 f_y / f_{ck}}$$

$$\therefore P_t = 100 \left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_u}{b d^2}}}{2 f_y / f_{ck}} \right]$$

The positive value of the root of eqn will give larger area of steel than $P_{t,lim}$ & hence it is not considered.

Limiting value of moment of resistance.

Properties of the beam section at limiting section.

At limiting condition $\frac{x_u}{d} = \frac{x_{u,max}}{d}$

The moment of resistance at limiting condition

$$M_{u,lim} = 0.36 \frac{x_{u,max}}{d} \left(1 - 0.42 \frac{x_{u,max}}{d}\right) b d^2 f_{ck}$$

$$= 0.36 \frac{x_{u,max}}{d} \left(1 - 0.42 \frac{x_{u,max}}{d}\right) f_{ck} b d^2$$

Corresponding to the various grades of steel the moments of resistance at limiting condⁿ are given in the below

Moment of resistance at limiting condⁿ

Grade of steel	$\frac{x_{u,max}}{d}$	Limiting moment of resistance
Fe 250	0.53	$M_{u,lim} = 0.149 f_{ck} b d^2$
Fe 415	0.48	$M_{u,lim} = 0.138 f_{ck} b d^2$
Fe 500	0.46	$M_{u,lim} = 0.133 f_{ck} b d^2$

Here the const $\frac{M_{u,lim}}{f_{ck} b d^2} = 0.36 \frac{x_{u,max}}{d} \left(1 - 0.42 \frac{x_{u,max}}{d}\right)$ is known as limiting moment of resistance index

Limiting percentage of steel required for limiting reinforced section

steel requirement at limiting condⁿ

Equating the total compression to total tension at limiting condⁿ

$$0.36 f_{ck} b \frac{x_{u,max}}{d} = 0.87 f_y A_{st,lim}$$

$$\Rightarrow A_{st,lim} = \frac{0.36 f_{ck} b x_{u,max}}{0.87 f_y}$$

$$= \frac{0.36 f_{ck} x_{u,max}}{0.87 f_y} \cdot bd$$

$$= 0.414 \frac{f_{ck}}{f_y} \cdot \frac{x_{u,max}}{d} bd$$

$$\text{Limiting percentage of steel} = P_{t,lim} = \frac{A_{st,lim}}{bd} \times 100$$

$$= 41.4 \frac{f_{ck}}{f_y} \cdot \frac{x_{u,max}}{d}$$

The limiting percentages of steel for various combinations of grades of concrete & steel are given in the table below.

$$\text{Limiting percentage of steel } P_{t,lim} = 41.4 \frac{f_{ck}}{f_y} \cdot \frac{x_{u,max}}{d}$$

f_{ck} N/mm ²	f_y (N/mm ²)		
	250	415	500
15	1.32	0.72	0.57
20	1.75	0.96	0.76
25	2.19	1.20	0.95
30	2.63	1.44	1.14

Types of Problems

The different types of problems are considered for singly reinforced rectangular beams are as follows

Type 1

To find out the depth of neutral axis & specifying the type of beam

For a given section, equate total tension & total compression & thus find out the depth of neutral axis using eqn

i.e. $x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

Also find out the limiting value of depth of neutral axis $x_{u,max}$ using limiting value of $\frac{x_{u,max}}{d}$

Then If $x_u < x_{u,max}$ the sectⁿ is under-reinforced

If $x_u = x_{u,max}$ the sectⁿ is balanced.

If $x_u > x_{u,max}$ the sectⁿ is overreinforced & adopt

$$x_u = x_{u,max}$$

Alternatively the reinforcement percentage P_t shall be compared with $P_{t,lim}$

Then If $P_t < P_{t,lim}$ the section is under-reinforced.

If $P_t = P_{t,lim}$ " " is balanced.

If $P_t > P_{t,lim}$ " " is over-reinforced

Type 2

To find out moment of resistance for a given section

1) Find out the depth of neutral axis (x_u) & the type of the beam as discussed in type 1.

2) For over-reinforced & balanced sections, obtain moment of resistance using eqn (3)

i.e.

$$M_{u,lim} = \frac{0.36 x_{u,max}}{d} \left(1 - 0.42 \frac{x_{u,max}}{d} \right) f_{ck} b d^2$$

3) For under reinforced sections, obtain moment of resistance by using eqn (3) (5) or (6)

i.e.

$$M_u = \frac{0.36 x_u}{d} \left(1 - 0.42 \frac{x_u}{d} \right) f_{ck} b d^2$$

or

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} b d} \right)$$

Type 3:

To design a singly reinforced rectangular section for a given width & applied factored moment.

The width is usually decided by the functional or architectural requirements. To decide the depth, first determine the balanced depth.

$$d_{bal} = \sqrt{\frac{M_u}{Q_{lim} \times b}} = \sqrt{\frac{M_u}{0.36 \frac{x_{u,max}}{d} (1 - 0.42 \frac{x_{u,max}}{d}) f_{ck}}$$

$$\left[\because M_{u,lim} = 0.36 \frac{x_{u,max}}{d} (1 - 0.42 \frac{x_{u,max}}{d}) f_{ck} b d^2 \right]$$

* If the balanced area is found with this depth, difficulty will arise while providing the reinforcements. Selecting exact no of reinforcement with available dia. is hardly possible. We cannot provide lesser area.

* If we provide larger area, the section becomes over-reinforced. Therefore, the under-reinforced design should be followed by using larger depth than balanced one.

* The following procedure may be adopted as one of the alternatives for depths upto 1000 mm to arrive at a practical soln.

(1) Assume 5% larger effective depth for $d \leq 500$ mm (case 1) & 10% larger depth for $1000 \text{ mm} \geq d \geq 500$ mm (case 2) |

(2) Assume one layer of 20mm dia bars for (case 1) & 2 layers of 20mm dia. bars for (case 2)

(3) If the clear cover to main reinforcement is 30mm

$$\begin{aligned} \text{effective cover} &= 30 + 10 = 40 \text{ mm for case 1 (Assume one layer of } 20 \phi) \\ &= 40 + 20 = 60 \text{ mm for case 2 (Assume 2 layers of } 20 \phi) \end{aligned}$$

$$(30 + 20 + 10)$$

(4) The overall depth can be obtained as

$$D_{\text{overall}} = d + 40 \text{ mm for case 1}$$

$$= d + 60 \text{ mm " 2}$$

(5) The values thus obtained shall be rounded upto nearest 25 mm.

(6) Now $d = D_{\text{overall}} - 40 \text{ mm}$ for case 1

$$= D_{\text{overall}} - 60 \text{ mm " 2}$$

(7) Determine $\frac{M_u}{bd^2}$, P_t & A_{st} , using eqn as per the case of under-reinforced sectⁿ

$$P_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_u}{bd^2}}}{f_y/f_{ck}} \right] \quad \text{or} \quad M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{bd f_{ck}} \right)$$

then $P_t = \frac{A_{st}}{bd} \times 100$. find out A_{st}

$\Rightarrow$ find out A_{st}

Also determine $A_{st, \text{lim}}$ from the following eqn.

$$0.36 f_{ck} b x_{\text{max}} = 0.87 f_y A_{st, \text{lim}}$$

8) Select the bar size & number such that $A_{st} > A_{st, \text{required}}$ & also $A_{st} < A_{st, \text{lim}}$

Type: 4

To find the steel area for a given factored moment.

(1) For a given ultimate/factored moment & assumed width b of section, find out d from eqn

$$d = \sqrt{\frac{M_u}{Q_{\text{lim}} b}}$$

This is a balanced section & balanced steel area may be found out using eqn

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

2) For a given factored moment, width & depth of section M_u, b, d
obtain $M_{u,lim} = Q_{lim} \cdot b d^2$

If $M_u < M_{u,lim}$: design as under-reinforced

If $M_u = M_{u,lim}$: design as ~~over~~ reinforced balanced sectⁿ

If $M_u > M_{u,lim}$: redesign the section either increasing the dimension of section or design as doubly reinforced beam.

For under-reinforced sectⁿ, the steel area can be obtained by using eqn

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

or

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{b d f_{ck}} \right)$$

Problem: 1

A rectangular beam 230mm wide & 520mm effective depth is reinforced with 4 no's 16mm dia bars. Find out the depth of neutral axis & specify the type of beam. The materials are are M20 grade concrete & HYSD reinforcement of grade Fe415. Also find out the depth of neutral axis if the reinforcement is increased to 4 no 20mm dia. bars.

Solⁿ

breadth of the beam (b) = 230mm

effective depth of the beam (d) = 520mm.

characteristic comp. strength of concrete (f_{ck}) = 20 N/mm²

" strength of steel (f_y) = 415 N/mm²

Case-1

$$A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804.25 \text{ mm}^2$$

$$\begin{aligned} \text{Total compressive force (C)} &= 0.36 f_{ck} b x_u \\ &= 0.36 \times 20 \times 230 x_u \\ &= 1656 x_u \end{aligned}$$

$$\begin{aligned} \text{Total tensile force (T)} &= 0.87 f_y A_{st} \\ &= 0.87 \times 415 \times 804.25 \\ &= 290374.46 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Equating } 1656 x_u &= 290374.46 \text{ N} \\ \Rightarrow x_u &= 175.35 \text{ mm} \end{aligned}$$

Limiting value of neutral axis

$$x_{u, \max} = 0.48 d = 0.48 \times 520 = 249.6 \text{ mm}$$

Here $x_u < x_{u, \max}$

$\therefore$ Section is under-reinforced.

Case-2

$$\text{Area of steel (A}_{st}) = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$$

Equating total compressive force (C) to total tensile force

$$\Rightarrow 0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 230 \times x_u = 0.87 \times 415 \times 1256.64$$

$$\Rightarrow x_u = 273.98 \text{ mm}$$

Here $x_u > x_{u, \max}$ i.e. over reinforced section

$$\therefore x_u = x_{u, \max} = 249.6 \text{ mm}$$

Problem: 2

A singly reinforced rectangular beam of width 230mm & 460mm effective depth is reinforced with 3 no. 20mm dia bars. Find out the factored moment of resistance of the section. The materials are M20 grade concrete & HYSD reinforcement of grade Fe 415. Also find out the factored moment of resistance if it is reinforced with 4 no. of 20mm dia bars.

solⁿ

breadth of the beam (b) = 230 mm

effective depth, " (d) = 460 mm

Characteristic compressive strength of concrete (f_{ck}) = 20 N/mm²

" strength of steel (f_y) = 415 N/mm²

Case-1

$$\text{Area of steel (A}_{st}\text{)} = \frac{31}{4} \times 20^2 = 942.47 \text{ mm}^2$$

To find the depth of neutral axis (x_u)

Equating total tensile & compressive forces

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 230 \times x_u = 0.87 \times 415 \times 942.47$$

$$\Rightarrow x_u = 205.48 \text{ mm}$$

$$\text{Limiting value of neutral axis (} x_{u, \max} \text{)} = 0.48 \times 460$$

$$= 220.8 \text{ mm}$$

Here $x_u < x_{u, \max}$

acceptable value of

$\therefore$ The section is under-reinforced & $x_u = 205.48 \text{ mm}$

Moment of resistance

$$= 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 942.47 \times (460 - 0.42 \times 205.48)$$

$$= 127.16 \text{ kN-m}$$

Alternatively $M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

$$= 0.36 \times 20 \times 230 \times 205.48 (460 - 0.42 \times 205.48)$$

$$= 127.16 \text{ kN-m}$$

Case 2

Area of steel (A_{st}) = $\frac{4}{7} \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$

To find the depth of neutral axis (x_u)

Equating total compressive forces to total tensile force

$$\Rightarrow 0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 230 \times x_u = 0.87 \times 415 \times 1256.64$$

$$\Rightarrow x_u = 273.98 \text{ mm}$$

Limiting value of neutral axis ($x_{u,max}$) = 0.48×460
 $= 220.8 \text{ mm}$

Here $x_u > x_{u,max}$. So the sectⁿ is overreinforced

Acceptable value of $x_u = 220.8 \text{ mm}$

Lever arm = $d - 0.42 x_u = 460 - 0.42 \times 220.8 = 367.26 \text{ mm}$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 230 \times 220.8 (460 - 0.42 \times 220.8)$$

$$= 134.2 \text{ kN-m}$$

* The actual stress in steel f_s , when the section is subjected to $M_{u,lim}$ is determined by equating total tension & total compression.

Then

$$0.36 f_{ck} b x_u = 0.87 f_s' A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 230 \times 220.8 = 0.87 \times f_s' \times 1256.64$$

$$\Rightarrow f_s' = 336.45 \text{ N/mm}^2$$

Problem: 3 Type-1

A singly reinforced rectangular beam is subjected to a bending moment of 38 kNm at working loads. The width of the beam is 200 mm. Find the depth & steel area for balanced design. The materials are M20 grade concrete & mild steel reinforcement.

Solⁿ

Bending moment acting on the beam = 38 kNm (at working loads)

$$\text{Factored moment} = 1.5 \times 38 = 54 \text{ kNm}$$

width of the beam (b) = 200 mm

characteristic ^{compressive} strength of concrete (f_{ck}) = 20 N/mm²

characteristic strength of steel (f_y) = 250 N/mm²

$$\text{Limiting moment of resistance } (M_{u, \text{lim}}) = 0.36 f_{ck} \frac{x_{u, \text{max}}}{d} \left(1 - 0.42 \frac{x_{u, \text{max}}}{d} \right) b d^2 f_{ck}$$

$$\Rightarrow 54 \times 10^6 = 0.36 \times 20 \times 0.53 \left(1 - 0.42 \times 0.53 \right) \cdot 200 \cdot d^2 \cdot 20$$

$$\Rightarrow d = 301.86$$

Steel area can be found out by using eqn

$$M_u = 0.87 f_y A_{st} d \left(1 - 0.42 \frac{x_u}{d} \right)$$

$$\text{For balanced section } \frac{x_u}{d} = \frac{x_{u, \text{max}}}{d} = 0.53$$

$$\text{substituting } 54 \times 10^6 = 0.87 \times 250 \times A_{st} \times 301.86 \times \left(1 - 0.42 \times 0.53 \right)$$
$$\Rightarrow A_{st} = 1057.99 \text{ mm}^2$$

$$\text{Provide } 3 - 20 \phi + 1 - 16 \phi = 1143.54 \text{ mm}^2$$

$$D = 302 + 10 + 30 = 342 \text{ mm}$$

Provide overall depth = 350 mm.

Problem: 4 Type: 1

A rectangular cantilever beam of size 230mm width $\times$ 500mm effective depth is subjected to a bending moment of 80 kNm at working loads. Find the steel area required. The materials are M20 grade concrete & HYSD reinforcement of grade Fe415.

Ans

width of the beam $b = 230$ mm

effective depth $d = 500$ mm

characteristic compressive strength of concrete $(f_{ck}) = 20$ N/mm²

" strength of steel $(f_y) = 415$ N/mm²

B.M due to working loads $= 80$ kNm

factored BM $(M_u) = 80 \times 1.5 = 120$ kNm

Limiting moment of resistance, for 415 grade steel

$$M_{u,lim} = 0.138 f_{ck} b d^2$$

$$\Rightarrow 120 \times 10^6 = 0.138 \times 20 \times 230 \times 500^2$$
$$= 158.7 \text{ kNm}$$

Here $M_u < M_{u,lim}$. So the beam is underreinforced

To find steel area, using eqn

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{b d f_{ck}} \right)$$

$$\Rightarrow 120 \times 10^6 = 0.87 \times 415 \times A_{st} \times 500 \left(1 - \frac{415 A_{st}}{230 \times 500 \times 20} \right)$$

$$\Rightarrow 664.72 = A_{st} \left(1 - 1.80 \times 10^{-4} A_{st} \right)$$

$$\Rightarrow 1.80 \times 10^{-4} A_{st}^2 - A_{st} + 664.72 = 0$$

$$\Rightarrow A_{st} = 779.2 \text{ mm}^2$$

Provide 4-16 ϕ bars in tension zone

$$A_{st, provided} = 804.25 \text{ mm}^2$$

Problem: 5 ^{Type-3}

Design a singly reinforced ^{rectangular} beam for an applied factored moment of 120 kNm . The materials are M20 grade concrete & HYSD reinforcement of grade Fe 415.

Solⁿ

Characteristic ^{compressive} strength of concrete (f_{ck}) = 20 N/mm^2

characteristic strength of steel (f_y) = 415 N/mm^2

Factored BM acting on the beam (M_u) = 120 kNm

The beam will be designed so that under the applied moment both materials reach their max stress. Let us assume ratio of effective depth to breadth of the beam equal to 1.5. i.e. $\frac{d}{b} = 1.5$

For a balanced design

$$M_u = 0.38 f_{ck} \frac{x_{u,max}}{d} \left(1 - 0.42 \frac{x_{u,max}}{d} \right) f_{ck} \frac{bd^2}{2}$$

$$\Rightarrow 120 \times 10^6 = 0.38 \times 0.48 \times \left(1 - 0.42 \times 0.48 \right) \frac{d}{2} \times d^2 \times 20$$

$$\Rightarrow d^3 = 86979514.58$$

$$\Rightarrow d = 443.06 \text{ mm}$$

wrong applied
change it

Increase this depth by 5%. & add 40mm effective cover

$$D = 443.06 \times 1.05 + 40 \text{ mm} \\ = 505.21 \text{ mm}$$

[assume clear cover of
of beam = 30mm &
dia bar = 20mm]

provide overall depth (D) of beam = 525 mm (Nearest multiple of 25 mm)

Assume one layer of 20mm dia bars are provided in tension zone

$$\therefore \text{effective depth (d)} = 525 - 30 - \frac{20}{2} = 485 \text{ mm}$$

$$\& \text{width of rect (b)} = \frac{d}{1.5} = \frac{485}{1.5} = 323.33 \text{ mm} \approx 325 \text{ mm}$$

To find the steel area, using eqn

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{b d f_{ck}} \right)$$

$$\Rightarrow 120 \times 10^6 = 0.87 \times 415 \times A_{st} \times 485 \left(1 - \frac{415 \times A_{st}}{325 \times 485 \times 20} \right)$$

$$\Rightarrow 685.29 = A_{st} - 1.316 \times 10^{-4} A_{st}^2$$

$$\Rightarrow 1.316 \times 10^{-4} A_{st}^2 - A_{st} + 685.29 = 0$$

$$\Rightarrow A_{st} = 781.83 \text{ mm}^2$$

Now Limiting area of steel $A_{st, \text{lim}} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b \cdot x_{u, \text{max}}}$ is obtained by eqn

$$0.36 f_{ck} \cdot b \cdot x_{u, \text{max}} = 0.87 f_y A_{st, \text{lim}}$$

$$\Rightarrow A_{st, \text{lim}} = \frac{0.36 f_{ck} b \cdot x_{u, \text{max}}}{0.87 f_y}$$

$$= \frac{0.36 \times 20 \times 325 \times 0.48 \times 485}{0.87 \times 415}$$

$$= 1508.79 \text{ mm}^2$$

Let us provide 3-20 ϕ bar in tension zone.

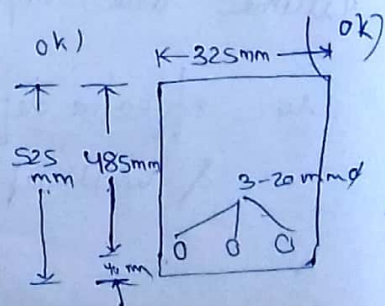
$$A_{st, \text{provided}} = 3 \times \frac{\pi}{4} \times 20^2 = 942.47 \text{ mm}^2$$

Thus $A_{st, \text{required}} < A_{st, \text{provided}}$

$$\text{Min area of steel } A_s = \frac{0.85 b d}{f_y} = \frac{0.85 \times 325 \times 485}{415}$$

$$= 322.85 \text{ mm}^2 < 942.47 \text{ mm}^2$$

Thus $A_{st, \text{required}} < A_{st, \text{provided}} < A_{st, \text{lim}}$ ok)



Doubly Reinforced beams

General Features

A doubly Reinforced concrete beams having steel reinforcement both on tensile & compression zones are known as doubly reinforced beams.

Therefore, doubly reinforced beams have moment of resistance more than singly reinforced beams of the same depth for a particular grade of steel & concrete.

Necessity of providing doubly-reinforced beam

- 1) Doubly reinforced beams are provided in order to increase the moment carrying capacity of the section.
- 2) Min^m compression reinforcement is provided to hold the shear reinforcement (stirrups) in position & for increasing the ductility of beam.
- 3) Most important reason for providing doubly reinforced beams is to ensure safety against reversal of stresses in the structure due to wind forces, seismic forces & temperature stresses.
- 4) When dimensions of the beams are restricted due to some architectural purpose, then moment of resistance of the section is increased by providing doubly reinforced sections.

Analysis of doubly reinforced beams

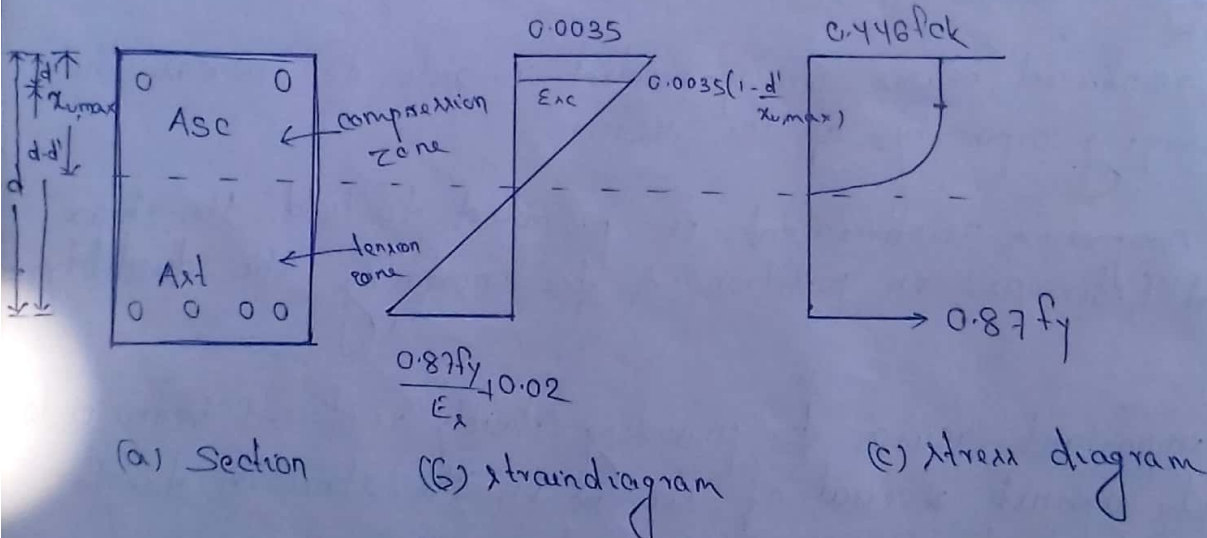
If the applied moment is greater than the M.R. of a singly reinforced section, there can be three alternatives

- (1) If possible, increase the dimensions of the section, preferably depth.
- (2) Higher grade concrete can be used to increase the M.R. of the section.

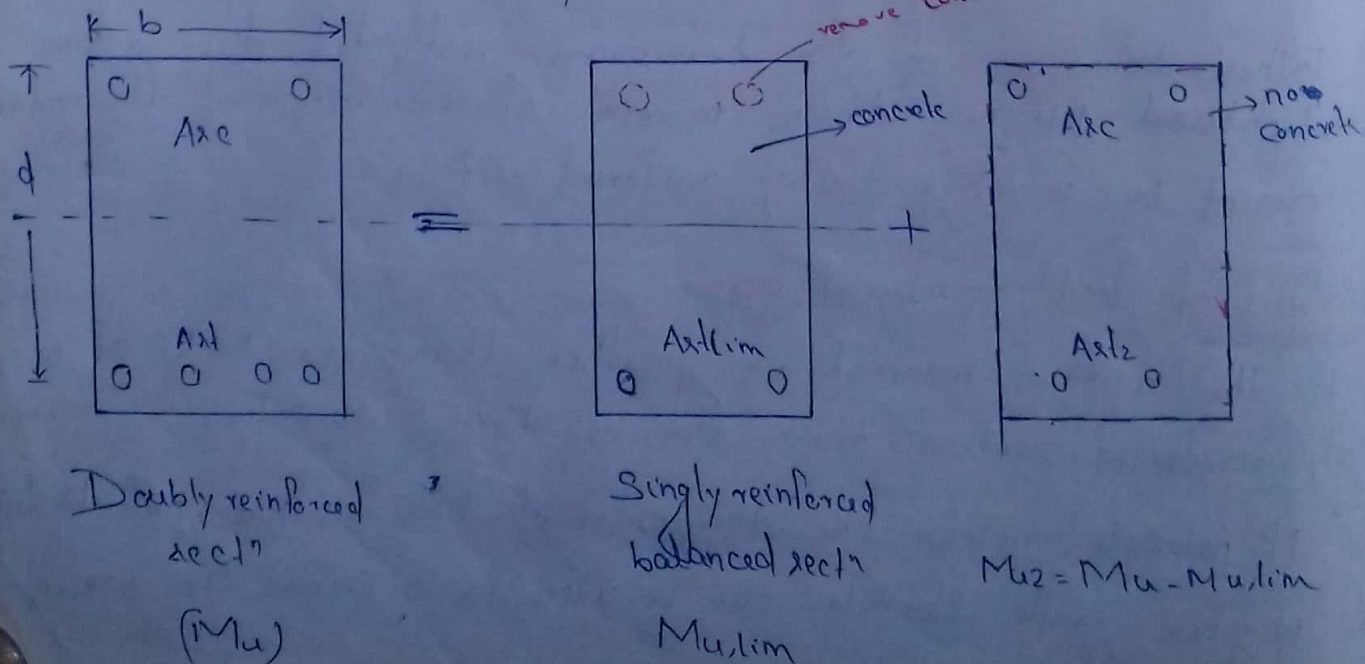
(3) Steel reinforcement may be added in compression zone to increase the M.R. of the section. This is known as doubly reinforced section.

Derivation of Formulae:

A doubly reinforced beam section, strain diagram & stress diagram are shown in fig. A doubly reinforced beam subjected to a moment M_u can be expressed as a rectangular sectⁿ with tension reinforcement $A_{st,lim}$ reinforced for balanced condⁿ giving moment of resistance $M_{u,lim}$ + an auxiliary sectⁿ reinforced with comp reinforcement A_{sc} & tensile reinforcement A_{st2} giving a moment of resistance M_{u2} such that $M_u = M_{u,lim} + M_{u2}$



Doubly reinforced beam.



Here $M_u = M_{u,lim} + M_{u2}$

$A_{st,lim}$ can be found out from $M_{u,lim}$

A_{st2} & A_{sc} $\longrightarrow$ $M_{u2} = M_u - M_{u,lim}$

Let d' = Effective cover to compression reinforcement

$d-d'$ = lever arm for additional moment.

A_{st2} = area of additional tensile reinforcement

A_{sc} = area of compression reinforcement

f_{sc} = stress in compression reinforcement

f_{cc} = compressive stress in concrete at the level of compression steel

Considering tension steel

$$M_{u2} = A_{st2} \times 0.87 f_y (d-d') \quad \dots \dots (1)$$

Considering compression steel

$$M_{u2} = A_{sc} (f_{sc} - f_{cc}) (d-d') \quad \dots \dots (2)$$

$\downarrow$
 $0.446 f_{ck}$

Now additional tension = additional compression

$$0.87 f_y A_{st2} = A_{sc} (f_{sc} - f_{cc})$$

$$\therefore A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y} \quad \dots \dots (3)$$

Any 2 of the above eqns may be used to find out the values of A_{sc} & A_{st2} : The total ^{tensile} reinforcement

$$A_{st} = A_{st,lim} + A_{st2}$$

To solve the above eqn, one needs the value of f_{sc} & p_{sc} .

To find p_{sc}

To find the strain at the level of compression reinforcement

refer to strain diagram

$$\frac{E_{sc}}{0.0035} = \frac{x_{u,max} - d'}{x_{u,max}}$$

$$\Rightarrow E_{sc} = 0.0035 \left(1 - \frac{d'}{x_{u,max}} \right)$$

Using this value of strain, the stress in steel can be found out from the representative stress-strain curves for reinforcement from Fig 23(A) & (B) of IS456-2000.

To simplify the procedure of finding out the stress in compression reinforcement let us find out the stress for different values of $\frac{d'}{d}$

For mild steel, $x_{u,max} = 0.53d$

$$\text{For } \frac{d'}{d} = 0.2$$

strain at the level of compression reinforcement

$$E_{sc} = 0.0035 \left(1 - \frac{d'}{x_{u,max}} \right)$$

$$= 0.0035 \left(1 - \frac{d'}{0.53d} \right)$$

$$= 0.0035 \left(1 - \frac{0.2}{0.53} \right) = 0.00218$$

Design stress in mild steel for strain of 0.00109 & above upto failure is 217 N/mm^2 . Therefore, for all values of $\frac{d'}{d}$ from

0 to 0.2, the stress P_{sc} will be taken as 217 N/mm^2 .

For HYSD steel of grade Fe 415

$$\kappa_{u, \max} = 0.48d$$

$$\text{for } \frac{d'}{d} = 0.05$$

strain at the level of compression steel

$$= 0.0035 \left(1 - \frac{0.05}{0.48} \right) = 0.0031135$$

Stress in compression steel for this value of strain from
fig 23(A) of IS 456 2000 is 355 N/mm^2

Stress in compression reinforcement (f_{sc}), in N/mm^2
in Doubly reinforced beam (Table F of SP 16)

$f_y (\text{N/mm}^2)$	$\frac{d'}{d}$			
	0.05	0.1	0.15	0.20
250	217	217	217	217
415	355	353	342	329
500	424	412	395	370
550	458	441	419	380

Types of problems:

Two basic types of problems in doubly reinforced beam are discussed below

Type 1: To find out the moment of resistance of a given section.

Referring to fig, the equilibrium of forces yields an equation

total compression = total tension

$$\text{i.e. } C_1 + C_2 = T$$

$$\Rightarrow 0.36 f_{ck} b x_u + A_{sc} \times f_{cc} + A_{sc} \times f_{sc} = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 f_{ck} b x_u + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

for $\frac{d'}{d}$ upto 0.2, $f_{cc} = 0.446 f_{ck}$ & f_{sc} may be found out from fig 23(A) & (B) by knowing E_{sc} .

→ find out x_u . Find $x_{u\max}$ & type of beam.

→ If it is ^{under} reinforced, then moment of resistance of the section can be found out by taking moments of compressive forces about the centroid of tensile reinforcement.

$$\text{Then } M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} (f_{sc} - f_{cc}) (d - d')$$

The value of f_{cc} is very small as compared to the value of f_{sc} & can be neglected.

$$\text{Then } M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} f_{sc} (d - d')$$

^R Moment of resistance can also be calculated by taking moments of tensile forces about the centroid of compressive forces

$$M_u = 0.87 f_y A_{st} x_u (d - 0.42 x_u) + 0.87 f_y A_{st} (d - d')$$

→ If it is over-reinforced/balanced, the depth of neutral axis obtained will be more than $x_{u,max}$ i.e. ($x_u > x_{u,max}$). In such cases $x_u = x_{u,max}$, shall be used for calculation

$$\text{i.e. } M_u = 0.36 f_{ck} b x_{u,max} (d - 0.42 x_{u,max}) + A_{sc} f_{sc} (d - d')$$

or

$$M_u = 0.87 f_y A_{st1} (d - 0.42 x_{u,max}) + 0.87 f_y A_{st2} (d - d')$$

Type 2

To find out reinforcement for flexure for a given section and factored moment

(1) find out $M_{u,lim}$ & reinforcement $A_{st,lim}$ for a given section by using eqn

$$M_{u,lim} = 0.36 f_{ck} b x_{u,max} (d - 0.42 x_{u,max})$$

$$\text{and } A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,max})}$$

(2) find obtain moment $M_{u2} = M_u - M_{u,lim}$

(3) Find compression steel from eqn

$$M_{u2} = A_{sc} f_{sc} (d - d')$$

$$\Rightarrow A_{sc} = \frac{M_{u2}}{f_{sc} (d - d')}$$

(4) corresponding tension steel A_{st2} may be found out from

$$A_{sc} f_{sc} = 0.87 f_y A_{st2} \Rightarrow A_{st2} = \frac{A_{sc} f_{sc}}{0.87 f_y}$$

or

$$M_{u2} = 0.87 f_y A_{st2} (d - d') \quad \text{or} \quad A_{st2} = M_{u2} / 0.87 f_y (d - d')$$

$$5) A_{st} = A_{st,lim} + A_{st2}$$

Problem: 1

Find the factored moment of resistance of a beam section 230mm wide x 460mm effective depth reinforced with 2-16mm dia bars as compression reinforcement at an effective cover of 40mm & 4-20mm dia bars as tension reinforcement as shown in fig. The materials are M20 grade concrete & mild steel reinforcement.

Solⁿ

width of the section (b) = 230mm

effective depth of the section (d) = 460mm

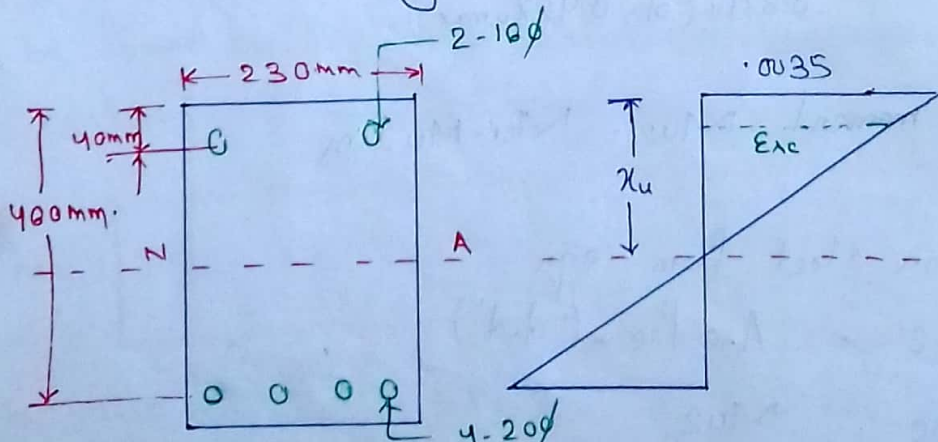
$$\text{Area of compression reinforcement } (A_{sc}) = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$$

$$\text{Area of tension " } (A_{st}) = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$$

effective cover to compression reinforcement (d') = 40mm

characteristic strength of concrete (f_{ck}) = 20 N/mm²

characteristic strength of steel (f_y) = 250 N/mm²



(a) Section

(b) ~~Strain~~ strain diagram

Equating total compressive forces to total tensile forces

$$0.36 f_{ck} b x_u + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

$$\rightarrow 0.36 \times 20 \times 230 \times x_u$$

$$\begin{aligned} \text{for } \frac{d'}{d} &= \frac{40}{460} = 0.086 < 0.2 \quad \text{so } f_{cc} = 0.446 f_{ck} \\ &= 0.446 \times 20 \\ &= 8.92 \text{ N/mm}^2 \end{aligned}$$

Strain at the level of compression reinforcement

$$\begin{aligned} (E_{sc}) &= 0.0035 \left(1 - \frac{d'}{x_{u, \max}} \right) \\ &= 0.0035 \left(1 - \frac{d'}{0.53d} \right) \quad \left[\text{For Fe250 grade steel } x_{u, \max} = 0.53d \right] \\ &= 0.0035 \left(1 - \frac{40}{0.53 \times 460} \right) \\ &= 0.00293 \end{aligned}$$

corresponding stress in compression steel can be obtained by interpolation using Fig 23(B) of IS 456:2000

$$f_{sc} = 0.87 f_y = 0.87 \times 250 = 217.5 \text{ N/mm}^2$$

$$\begin{aligned} 0.36 \times 20 \times 230 \times x_u + 402.12 \times (217.5 - 8.92) \\ = 0.87 \times 250 \times 1258.64 \end{aligned}$$

$$\Rightarrow x_u = 114.40 \text{ mm}$$

$$\begin{aligned} \text{Limiting value } (x_{u, \max}) \text{ of neutral axis} &= 0.53d = 0.53 \times 2 \times 460 = 243.8 \text{ mm} \end{aligned}$$

$\therefore x_u < x_{u, \max}$. Hence the section is under-reinforced

$$\text{so } x_u = 114.40 \text{ mm}$$

Taking moments of compressive forces about centroid of tensile reinforcement

$$\begin{aligned}
 M_u &= 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} (f_{sc} - f_{ec}) (d - d') \\
 &= 0.36 \times 20 \times 230 \times 114.40 (460 - 0.42 \times 114.40) + \\
 &\quad 402.12 \times (217.5 - 8.92) (460 - 40) \\
 &= 113.27 \text{ kN-m}
 \end{aligned}$$

Problem: 2

A rectangular beam of size 230mm wide x 500mm effective depth is subjected to a factored moment of 200 kNm. Find the reinforcement for flexure. The materials are M20 grade concrete & HYSD reinforcement of grade Fe 415.

Solⁿ

width of the section (b) = 230mm

effective depth of the section (d) = 500mm

characteristic strength of concrete (f_{ck}) = 20 N/mm²

characteristic strength of steel (f_y) = 415 N/mm²

factored bending acting on the beam (M_u) = 200 kNm

Limiting moment of resistance for balanced section ($M_{u,lim}$)

$$\begin{aligned}
 &= 0.36 f_{ck} b x_{u,max} (d - 0.42 x_{u,max}) \\
 &= 0.36 \times 20 \times 230 \times (0.48 \times 500) \times (500 - 0.42 \times 0.48 \times 500) \\
 &= 158.66 \text{ kNm}
 \end{aligned}$$

& Limiting area of steel ($A_{st,lim}$) = $\frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,max})}$

$$\begin{aligned}
 &= \frac{158.66 \times 10^6}{0.87 \times 415 \times (500 - 0.42 \times 0.48 \times 500)} \\
 &= 1100.80 \text{ mm}^2
 \end{aligned}$$

Here additional moment M_{u2} of the section $(M_{u2}) = M_u - M_{u,lim}$

$$= 200 - 158.66$$

$$= 41.34 \text{ kNm}$$

Let the compression reinforcement be provided at an effective cover of 40 mm

$$\therefore \frac{d'}{d} = \frac{40}{500} = 0.08$$

strain at the level of compression reinforcement (ϵ_{sc})

$$= 0.0035 \left(1 - \frac{d'}{x_{u,max}} \right)$$

$$= 0.0035 \left(1 - \frac{40}{0.48 \times 500} \right)$$

$$= 0.00292$$

Corresponding stress in compression steel can be obtained by

Fig 23(A) of IS 456:2000

$$f_{sc} = 0.85 f_y = 0.85 \times 415 = 352.75 \text{ N/mm}^2$$

Now, considering ~~tension~~ ^{compression} steel

$$M_{u2} = A_{sc} (f_{sc} - f_{cc}) (d - d')$$

$$\Rightarrow 41.34 \times 10^6 = A_{sc} (352.75 - 0.446 \times 20) (500 - 40)$$

$$\Rightarrow A_{sc} = 261.38 \text{ mm}^2$$

Provide 2-20 ϕ bar in compression zone $A_{sc, provided} = 2 \times \frac{\pi}{4} \times 20^2 = 628.32 \text{ mm}^2$

Now, considering ~~compression~~ ^{tension} steel

$$M_{u2} = A_{st2} \times 0.87 f_y (d - d')$$

$$\Rightarrow 41.34 \times 10^6 = A_{st2} \times 0.87 \times 415 (500 - 40)$$

$$\Rightarrow A_{st2} = 248.91 \text{ mm}^2$$

$$\therefore A_{st} = A_{st, lim} + A_{st2} = 1100.80 + 248.91 = 1349.71 \text{ mm}^2$$

provide 5-20 ϕ bar in tension zone $A_{st, provided} = 5 \times \frac{\pi}{4} \times 20^2 = 1570.80 \text{ mm}^2$

for the designed sectⁿ, determine x_u by equating total compression & total tension

$$\text{i.e. } 0.36 f_{ck} b x_u + A_{sc} (f_{sc} - f_{ec}) = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 230 \times x_u + 628.32 (352.75 - 0.446 \times 20)$$

$$= 0.87 \times 415 \times 1570.80$$

$$\Rightarrow x_u = 212.02 \text{ mm}$$

$$\text{Also } x_{u, \max} = 0.48 \times d = 0.48 \times 500 = 240 \text{ mm}$$

$$\text{Now } x_u < x_{u, \max}$$

$\therefore$ The beam is under-reinforced. The design is accepted

Q Design a rectangular beam for an effective span of 6m. The super-imposed load is 60 kNm & size of the beam is limited to 30 cm x 60 cm overall. Use M20 mix & Fe415 grade steel.

Shear Bond & Development Length (LSM)

Shear in structural elements:

The section of a structural member may be subjected to shear force due to flexure, punching or torsion. Accordingly the shear may be flexural shear, punching shear or torsional shear.

(1) Flexural shear:

The shear associated with change of bending moment along the span is known as flexural shear. The beams are usually subjected to flexural shear. For beams, the flexural shear is commonly referred to as 'shear'.

(2) Punching shear:

The shear associated with the possibility of ^{punching} a thin member by a concentrated load is called punching shear. A slab carrying a concentrated wall load, a beamless floor slab supported directly by columns (called flat slab) or a footing slab carrying a concentrated column load are subjected to punching shear.

→ The flexural shear is referred to as one-way shear.

→ punching shear " " " " two-way shear

(3) Torsion shear:

When a member is subjected to torsion, it is subjected to torsion shear. In RCC structures, the torsion shear is usually accompanied with flexural shear.

* The beams are the line (uni-dimensional) elements and are usually subjected to flexural shear. However, sometimes they are subjected to all the types of shear as in ^{case} of with torsion shear also. The slabs are the plate elements and usually subjected to flexural shear. However, sometimes they are subjected to all the types of shear as in case of restrained two-way slabs & flat slabs.

* The beams are classified as shallow & deep depending on the ratio of effective span l , to overall depth D , i.e. l/D ratio. According the beams are classified as deep if

$\frac{l}{D} < 2$ for simply supported beams.

$\frac{l}{D} < 2.5$, for continuous beams.

The beams are otherwise classified as shallow.

Limit State theory:

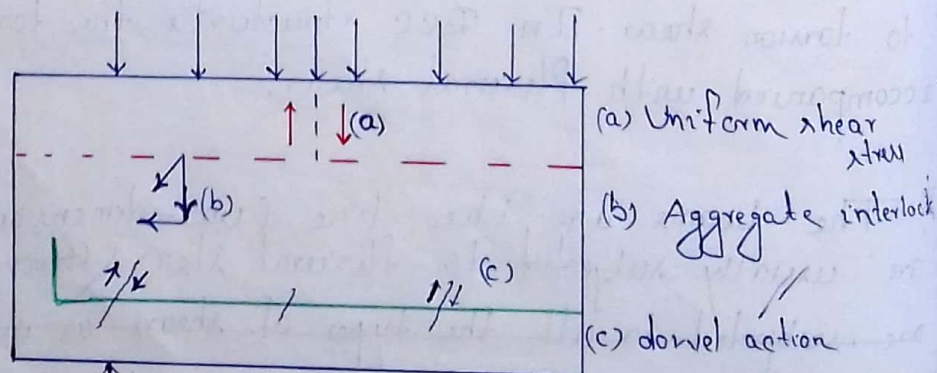
The mechanics of shear transfer in concrete beams without shear reinforcement, when concrete cracks due to shear is illustrated in fig 7.5.

* The shear is resisted by

(1) Above neutral axis shear resistance is provided by the uniform shear stress in uncracked concrete (20 to 40% of total resistance)

(2) Along the crack, shear resistance is provided by the vertical component of force due to the interlocking of aggregates (33 to 50% of total resistance)

(3) At the tensile reinforcement, shear is resisted by the dowel action of the longitudinal bars (15 to 25% of total resistance)



Shear resistance of a cracked beam

* The behaviour of concrete is thus complicated & the shear stress is found out in limit state theory by a simple formula similar to that of elastic theory where τ_v is defined as nominal shear stress.

$$\text{Then } \tau_v = \frac{V_u}{bd}$$

where τ_v = nominal shear stress

V_u = design shear

b = width of section, which for the case of a flanged beam, is width of the web, b_w

d = effective depth.

Design shear strength of concrete:

The design shear strength of concrete shall be taken as follows

(i) Without shear reinforcement:

The design shear strength of concrete in beams without shear reinforcement for some cases is given in table 19 of IS 456: 2000 (page 73).

* From this table we can observe that design shear strength τ_c in concrete depends on the percentage of tension steel. This is because.

(i) When the amount of tension steel increases, the depth of neutral axis increases & thus, the depth of uncracked concrete increases. This increases the capacity of concrete in shear.

(ii) When the amount of tension steel increases, the cracks formed are smaller, which improves the aggregate interlock. Also because of larger steel area the dowel action is improved. This further improves the capacity of section in shear.

* Again from table, it can be observed that shear strength of concrete (τ_c) is also related to the compressive strength of concrete.

* When the longitudinal bars are not required to resist moment they are sometimes curtailed or bent up. If the bars are curtailed they create complicated stresses at the point of curtailment, thereby reducing the shear capacity of the section. Therefore while using table 19 of IS 456:2000, any longitudinal bars which are terminated within a distance d of the section under consideration, shall not be considered for calculation of $\frac{100 A_s}{bd}$.

* If nominal shear stress (τ_v) does not exceed τ_c found from table 19 the section is safe for shear & reinforcements are not required theoretically. However, some minimum shear reinforcement shall be provided in accordance as clause 26.5.1.6.

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

* If the nominal shear stress (τ_v) exceeds the design shear strength (τ_c) the section shall be suitably reinforced with shear reinforcement in accordance as clause 40.4.

Note: $\tau_v < \tau_c \rightarrow$ Section is safe for shear and shear reinforcements are not required (But min^m shear reinforcement shall be provided as stirrups)

$\tau_v > \tau_c \rightarrow$ Need to design for shear reinforcement

2) With shear-reinforcement:

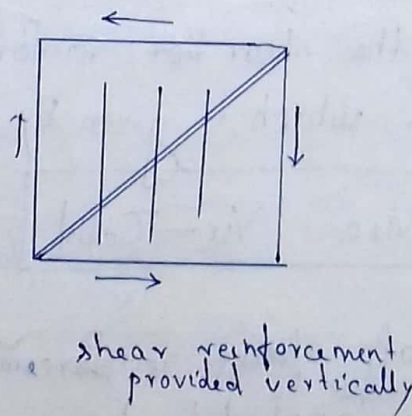
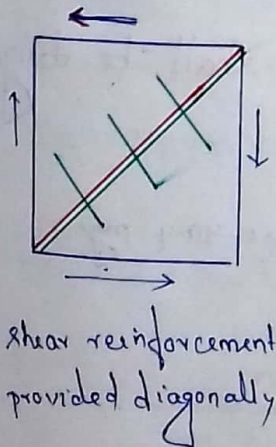
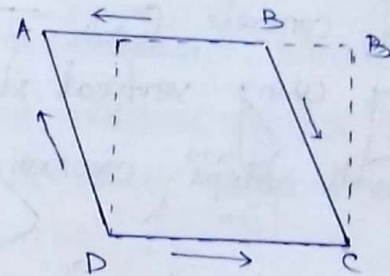
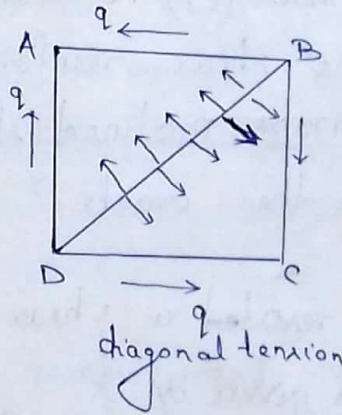
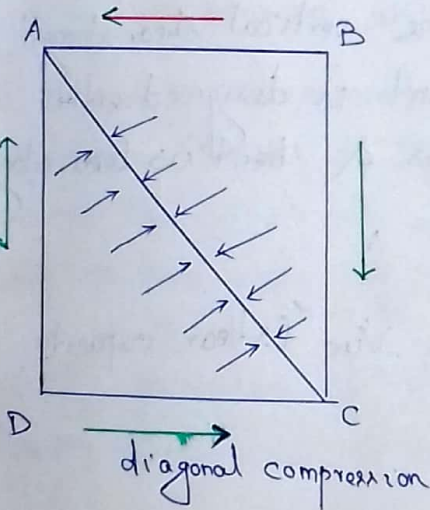
Shear reinforcements are provided against diagonal tension. As the concrete is strong in compression, generally a beam is safe for diagonal compression.

* However for any beam, suitably reinforced against diagonal tension, the nominal shear (τ_v) shall not exceed the max^m shear stress ($\tau_{c, max}$) values given in table 20 of IS 456 2000. The max^m shear stress is given by $\tau_{c, max} = 0.62 \sqrt{f_{ck}}$.

* By this provision the failure of the beam by diagonal compression is prevented. For any section if nominal shear stress (τ_{cv}) exceeds the max shear stress $(\tau_{c,max})$ value, the section shall be redesigned either by using richer concrete mix or by increasing the size of the section. For practical purpose, it is recommended that nominal shear stress may not exceed 85% of the $\tau_{c,max}$ value.

Maximum shear stress, $\tau_{c,max}$ (N/mm²)

Concrete grade	M15	M20	M25	M30	M35	M40 & above
$\tau_{c,max}$ (N/mm ²)	2.5	2.8	3.1	3.5	3.7	4.0



Design for shear:

due to shear stresses, there are cracks in concrete perpendicular to the diagonal tension. As the load increases, these cracks move upward. Most of the cracks stop when they reach the heavily stressed region of compression zone. If there is a further increase of load, a critical diagonal crack may tear through the beam.

As a result, the beam splits into 2 sections & a sudden brittle failure occurs. Thus the diagonal tension failure occurs without giving notice & may cause the beam to collapse completely. ^{Therefore} beam shall be suitably reinforced with shear reinforcements.

* The shear failure of the section occurs on a vertical plane when the nominal shear stress (τ_v) exceeds the vertical shear strength of concrete (τ_c). The shear reinforcement is designed either by using vertical stirrups, inclined stirrups & bent up bars along with stirrups crossing the cracks.

* Concrete alone can resist a shear force V_{uc} (shear capacity of concrete) which is given by

$$V_{uc} = \tau_c b d$$

According to IS: 456, the shear ~~for~~ reinforcement shall be designed for a shear force, V_{us} , which is given by

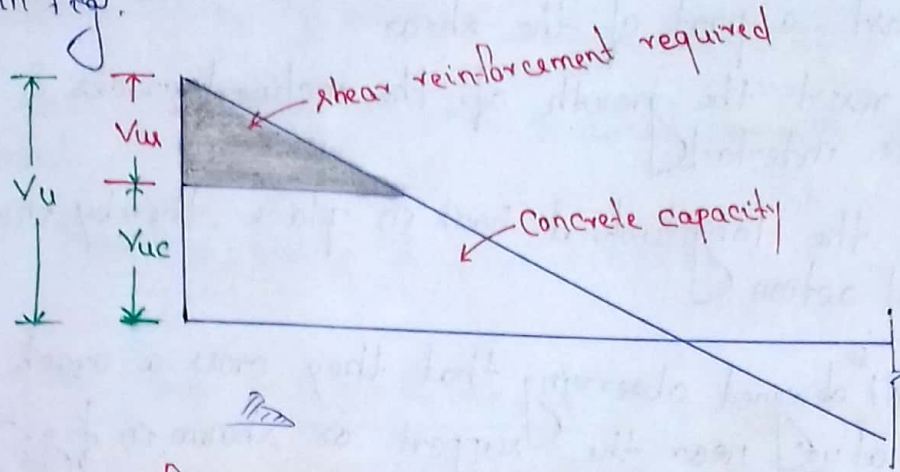
$$V_{us} = V_u - V_{uc} = V_u - \tau_c b d$$

clause 40.4 page-73

where V_{us} = strength of shear reinforcement
 V_u = shear force ^{due to} applied loads.

$V_{uc} = \tau_c b d$ = shear capacity of concrete without shear reinforcement.

The shearing of shear resistance in simply supported RCC beam is shown in fig.



Shear reinforcement in beams

Shear reinforcements shall be designed to resist a shear force V_{ux} . As concrete is weak in tension, the shear failure is caused by diagonal tension with cracks running at 45° to the beam axis. The shear reinforcement shall be provided by reinforcement which cross the cracks. These shear reinforcement shall be provided by minimize the size of diagonal tension crack & carry diagonal tension stress from one side of the crack.

* The provision of shear reinforcement is made by any of the following forms

- (1) Vertical stirrups
- (2) Inclined stirrups
- (3) Bent up bars along with stirrups

These are now briefly discussed.

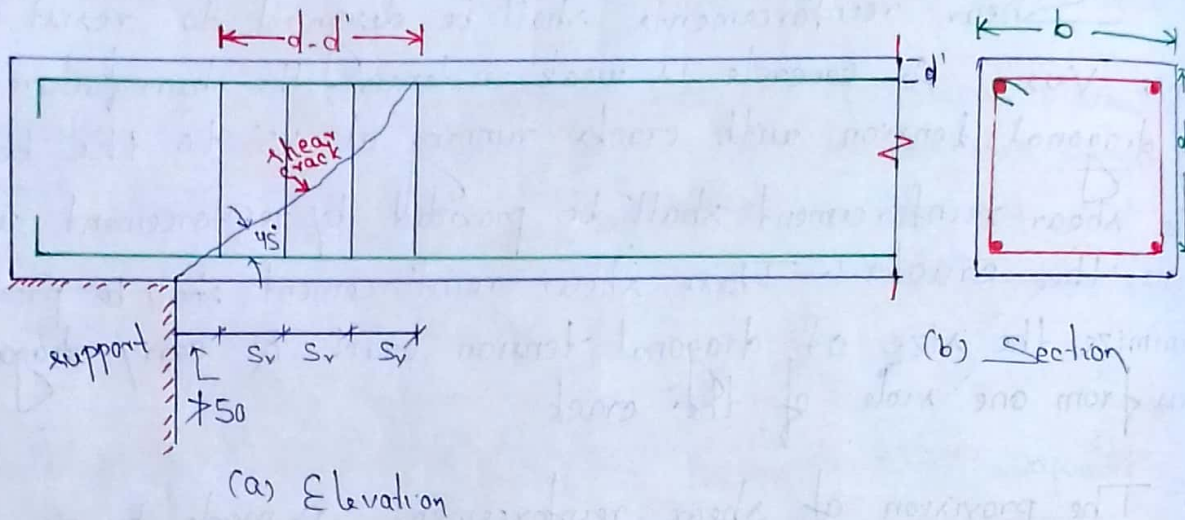
(1) Vertical stirrups

Stirrups are the commonly used shear reinforcement. These consist of a series of vertical closed bars spaced along the beam span as shown in fig. The first stirrups shall be provided within 50mm from the face of supports.

The primary functions of the stirrups are

- (i) to resist a part of the shear
- (ii) to resist the growth of the inclined cracks & improve aggregate interlock
- (iii) to tie the longitudinal bars in place, thereby increase the dowel action.

The stirrups will be designed observing that they cross a crack, say the first crack at 45° near the support as shown in fig. The stirrups are spaced at a distance S_v apart & they cross the crack.



Vertical stirrups as shear reinforcement

Let A_{sv} = area of legs of one stirrup

f_y = characteristic strength of stirrup (or bent up bar) which shall not be greater than 415 N/mm^2

$0.87 f_y$ = design tensile strength of shear reinforcement.

V_{sv} = strength of shear reinforcement

Now, The horizontal length of crack = $d - d' \approx d$ (d' neglected)

No of stirrups crossing the crack $n = d/s_v$

$\therefore$ Shear taken by stirrups $= \frac{d}{s_v} \times A_{sv} \times 0.87 f_y$

This shall be equal to the required strength V_{ux} of the shear reinforcement.

$$\text{Equating } V_{ux} = \frac{d}{s_v} \times A_{sv} \times 0.87 f_y$$

$$\Rightarrow V_{ux} = \frac{0.87 f_y A_{sv} d}{s_v}$$

cl. 40.4(a) page 73

(2) Inclined stirrups:

~~It consists of a series of inclined stirrups or~~
A series of inclined stirrups near the support crossing a 45° crack shown in fig.

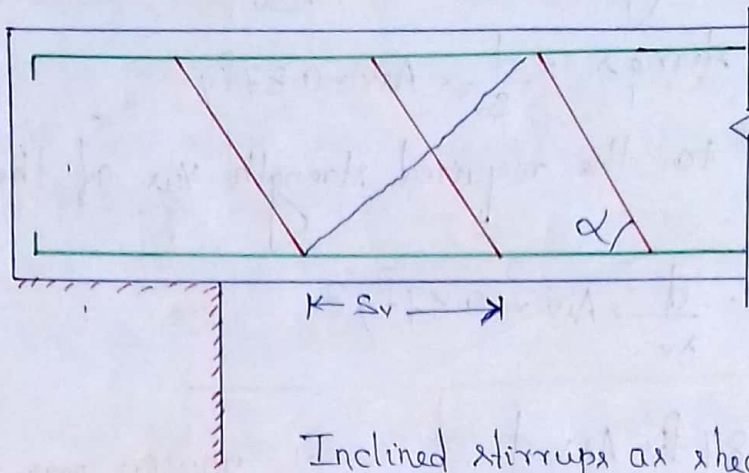
If A_{sv} is the area of legs of inclined stirrups & $0.87 f_y$ is the design tensile strength in the shear reinforcement, the tensile force in an inclined stirrup $= 0.87 f_y A_{sv}$

For inclined stirrups or a series of bars bent-up at different cross-sections, the spacing (s_v) can be found from following eqn.

$$V_{ux} = \frac{0.87 f_y A_{sv} d (\sin \alpha + \cos \alpha)}{s_v}$$

cl. 40.4(b) page 73

* The inclined stirrups are very effective in carrying flexural shear & restrain the inclined crack very effectively. However in case of stress reversal (due to earthquake loads for example), the direction of crack will be reversed & inclined stirrups become ineffective.



Inclined stirrups as shear reinforcement.

③ Bent bars.

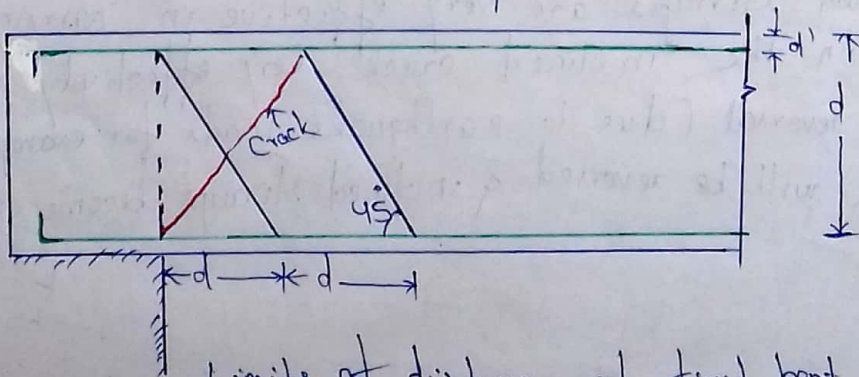
For single bar or single group of parallel bars, all bent up at the same cross-section. The strength of shear reinforcement is given by

$$V_{us} = 0.87 f_y A_{sv} \sin \alpha$$

* Usually the bars are bent at 45° or 60° to the longitudinal axis of the beam.

* Note that if a bar is bent at a distance from the support, less than d the bent portion of the bar will enter into the support which is not practicable. Therefore the first bent, if its contribution to resist the shear at support is to be counted, should start at a distance from support betⁿ d & $2d$.

* Note that if a second bar or a group of bars is to be bent, this shall have to start at a distance d or less than d from the first bent bar as restricted by the code.



Limits of distance of first bent from support

Maximum Spacing:

The shear reinforcements are provided to prevent the shear cracks in the beam. The horizontal distance b/w two successive cracks is approximately equal to the effective depth d . Stirrups shall be provided such that they cross the crack & also no crack shall remain unreinforced.

* The maximum spacing of shear reinforcement measured along the axis of the member shall not exceed

1) $0.75 d$ for vertical stirrups

2) d for inclined stirrups at 45°

where d is the effective depth of the section under consideration.

— cl 26.5.1.5

page 47

In no case shall the spacing exceed 300 mm.

Minimum Shear Reinforcement

Minimum shear reinforcement in the form of vertical stirrups shall be provided such that

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

— cl - 26.5.1.6 page (48)

$$\Rightarrow s_v \leq \frac{0.87 f_y A_{sv}}{0.4 b}$$

where

A_{sv} = total c/s area of stirrup legs effective in shear

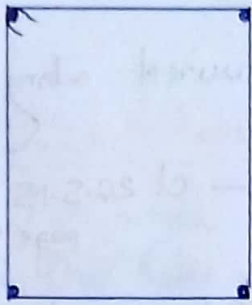
s_v = stirrup spacing along the length of the member

b = breadth of the beam or breadth of the web of flanged beam

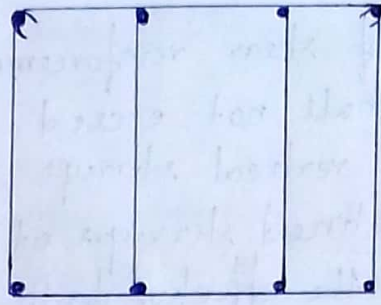
f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than $415 N/mm^2$

Forms of shear reinforcements

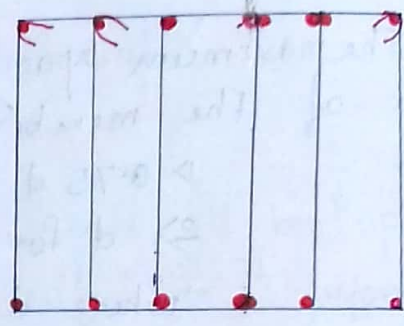
The stirrups are usually 2 legged. However stirrups of 4 legged, six-legged etc also can be formed as shown in fig. More than 2 legged stirrups are used for heavy shear. Unless otherwise specified, stirrups are always 2 legged.



(a) Two-legged stirrup



(b) Four-legged stirrup



(c) Six-legged stirrup

Problem: 1.

A simply supported reinforced concrete beam is 250 mm wide and 500 mm effective depth and is reinforced with 5 bars of 18 mm diameter as tensile steel. If the beam is subjected to a factored shear of 62.5 kN at the support, find the nominal shear stress at the support & design the shear reinforcement. Use M20 concrete & Fe 250 steel.

Sol:

Given data:

width of beam (b) = 250 mm

effective depth " (d) = 500 mm

$$\text{Area of tensile steel } (A_t) = 5 \times \frac{\pi}{4} \times 18^2 = 1272.35 \text{ mm}^2$$

$$\text{Factored shear force at the support } (V_u) = 62.5 \text{ kN}$$

$$\begin{aligned} \text{Characteristic compressive strength of concrete } (f_{ck}) &= 20 \text{ N/mm}^2 \\ \text{strength of steel } (f_y) &= 250 \text{ N/mm}^2 \end{aligned}$$

$$\text{Nominal shear stress } (\tau_v) = \frac{V_u}{bd} = \frac{62.5 \times 10^3}{250 \times 500} = 0.5 \text{ N/mm}^2$$

$$\text{Percentage of tensile steel provided} = \frac{A_{st}}{bd} \times 100 = \frac{1272.35}{250 \times 500} \times 100 = 1.02\%$$

So design strength of concrete (τ_c) from table 19 of IS 456: 2000

$$= 0.62 + \frac{0.67 - 0.62}{1.25 - 1.00} \times (1.02 - 1.00) = 0.6282 \text{ N/mm}^2$$

As $\tau_v < \tau_c$, the section is safe for shear & shear reinforcements are not required theoretically. However, some minimum shear reinforcement shall be provided in accordance as clause 26.5.1.6 given by

$$\frac{A_{sv}}{b_{sv}} \geq \frac{0.4}{0.87 f_y}$$

$$\Rightarrow \frac{A_{sv} \times 0.87 f_y}{0.4 b} \geq S_v$$

$$\Rightarrow S_v \leq \frac{A_{sv} \times 0.87 f_y}{0.4 b} \quad \left[\text{Using 6mm diameter of Fe 250 grade, two legged stirrups} \right]$$

$$\Rightarrow S_v \leq \frac{\left(2 \times \frac{\pi}{4} \times 6^2 \right) \times 0.87 \times 250}{0.4 \times 250}$$

$$\Rightarrow S_v \leq 122.99 \text{ mm} \approx 123 \text{ mm}$$

The spacing of stirrups (S_v) shall not exceed

$$i) 0.75d = 0.75 \times 500 = 375 \text{ mm}$$

$$ii) 300 \text{ mm}$$

$$iii) 123 \text{ mm as calculated above}$$

Provide 6mm diameter two legged stirrups @ 120 mm c/c.

Problem: 2

A simply supported reinforced concrete beam is 250mm wide and 500mm deep to the centre of tensile reinforcement & is reinforced with 4 bars of 20mm diameter as tensile steel. If the beam is subjected to a factored shear force of 95kN at the support, design the shear reinforcement consisting of stirrups. Use M20 concrete & Fe415 steel.

Solⁿ

Width of the section (b) = 250mm
effective depth (d) = 500mm

Area of tensile steel (A_{st}) = $4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$

Factored shear force at the support (V_u) = 95kN

characteristic compressive strength of concrete (f_{ck}) = 20 N/mm^2

" strength of steel (f_y) = 415 N/mm^2

$$\text{Nominal shear stress } (\tau_v) = \frac{V_u}{bd} = \frac{95 \times 10^3}{250 \times 500} = 0.76 \text{ N/mm}^2$$

$$\text{Percentage of steel provided } (P_t) = \frac{A_{st}}{bd} \times 100$$

$$= \frac{1256.64}{250 \times 500} \times 100$$
$$= 1.01\%$$

So design shear strength of concrete (τ_c) from table 19 of IS 456:2000 = $0.62 + \frac{0.67 - 0.62}{1.25 - 1.00} \times (1.01 - 1.00)$

$$= 0.622 \text{ N/mm}^2$$

Now $\tau_v > \tau_c$, therefore shear reinforcement shall be designed.

Shear resistance of concrete (V_{uc}) = $\tau_{cb} b d = 0.622 \times 250 \times 500$
 $= 77.75 \text{ kN}$

shear to be resisted by stirrups

$$V_{us} = V_u - \tau_{cb} b d = 95 \text{ kN} - 77.75 \text{ kN}$$

$$= 95 \times 10^3 - (0.622 \times 250 \times 500)$$

$$= 17.25 \text{ kN}$$

Using 6mm diameter two-legged ^{vertical} stirrups. (Fe415 grade steel)

spacing of stirrups

$$(S_v) = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times (2 \times \frac{\pi}{4} \times 6^2) \times 500}{17.25 \times 10^3}$$

$$= 591.79 \text{ mm}$$

The spacing of stirrups shall not exceed

a) $0.75d = 0.75 \times 500 = 375 \text{ mm}$

b) 300 mm

(c) due to minm shear reinforcement required

$$= \frac{0.87 f_y A_{sv}}{0.4 b} = \frac{0.87 \times 415 \times (2 \times \frac{\pi}{4} \times 6^2)}{0.4 \times 250} = 204.17 \text{ mm}$$

(d) required spacing = 591.79 mm

provide 6mm ϕ - two legged stirrups about 200mm c/c.

Problem: 3

A simply supported reinforced concrete beam is 250mm wide, 500mm effective depth and is reinforced with 5 bars of 18mm dia as tensile steel. If the beam is subjected to a factored shear of 140kN at the support & two of the main bars are cranked up at 45°, find the spacing of 2 legged 6mm diameter stirrups at support. Use M20 grade concrete & Fe250 steel.

Solⁿ

Width of the section (b) = 250mm
effective depth " (d) = 500mm

$$\text{Area of tensile steel } (A_{st}) = 5 \times \frac{\pi}{4} \times 18^2 = 1272.35 \text{ mm}^2$$

Factored shear force at the support (V_u) = 140kN.

characteristic strength of concrete (f_{ck}) = 20 N/mm²

" " " steel (f_y) = 250 N/mm².

$$\text{total c/s area of stirrup leg } (A_{sv}) = 2 \times \frac{\pi}{4} \times 6^2 = 56.55 \text{ mm}^2$$

$$\text{Nominal shear stress } (\tau_v) = \frac{V_u}{bd} = \frac{140 \times 10^3}{250 \times 500} = 1.12 \text{ N/mm}^2$$

$$\begin{aligned} \text{percentage of tensile steel available after cranking up two bars} \\ = \frac{A_{st}}{bd} \times 100 = \frac{(3 \times \frac{\pi}{4} \times 18^2)}{250 \times 500} \times 100 = 0.61\% \end{aligned}$$

$$\begin{aligned} \text{So design shear strength of concrete } (\tau_c) \text{ from table 19 of IS 456:2000} \\ = 0.48 + \frac{0.56 - 0.48}{0.75 - 0.50} \times (0.61 - 0.50) \end{aligned}$$

$$= 0.512 \text{ N/mm}^2$$

$$\& \tau_{cmax} = 2.8 \text{ N/mm}^2$$

Here $\tau_v > \tau_c < \tau_{cmax}$, therefore shear reinforcement shall be designed.

Shear resistance of the 2 cranked up bars

$$= 0.87 f_y A_{sv} \sin \alpha = 0.87 \times 250 \times 2 \times \frac{\pi}{4} \times 18^2 \times \sin 45^\circ$$
$$= 78.27 \text{ kN}$$

Shear resistance of concrete (V_{uc}) = $\tau_{cbd} = 0.5152 \times 250 \times 500$

$$= 64.40 \text{ kN}$$

shear resistance to be provided by shear reinforcement

$$V_{us} = V_u - V_{uc} = 140 - 64.40 = 75.6 \text{ kN}$$

Shear resistance to be provided by bent up bars

$$= 50\% V_{us} = 50\% \times 75.6$$
$$= 37.8 \text{ kN}$$

Shear resistance to be provided by stirrups = 50% V_{us}

$$= 50\% \times 75.6$$
$$= 37.8 \text{ kN}$$

Spacing of 2 legged 6-mm diameter vertical stirrups (S_v)

$$= \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 250 \times 2 \times \frac{\pi}{4} \times 6^2 \times 500}{37.8 \times 10^3}$$
$$= 162.69 \text{ mm}$$

The spacing of stirrups shall not exceed

1) $0.75d = 0.75 \times 500 = 375 \text{ mm}$

2) 300 mm

3) due to min^m shear reinforcement required

$$= \frac{0.87 f_y A_{sv}}{0.46} = \frac{0.87 \times 250 \times (2 \times \frac{\pi}{4} \times 6^2)}{0.46 \times 250} = 122.99 \text{ mm}$$

4) required spacing = 162.69 mm

Provide 6mm ϕ two legged stirrups about 120 mm c/c

Problem: 4

Determine the shear capacity of the beam section in each of the following cases

- (a) Size 230mm x 500 effective depth reinforced with 3 no 16mm diameter Fe 415 grade reinforcement & 6mm ϕ M.S. stirrups @ 130mm c/c. Concrete is of M20 grade.
- (b) Size 230mm x 720mm effective depth reinforced with 5 no 16mm diameter Fe 415 grade reinforcement (3 straight & 2 bent at 45°) with 8mm # stirrups at 300mm c/c Fe 415 grade. Consider M20 grade concrete.
- (c) section same as (b) but stirrups 10mm # Fe 415 grade at 300mm c/c.

Solⁿ

- (a) Width of section (b) = 230mm
effective depth of section (d) = 500mm

$$\text{Area of tensile steel (A}_{st}\text{)} = 3 \times \frac{\pi}{4} \times 16^2 = 603.19 \text{ mm}^2$$

$$\text{spacing of stirrup (S}_v\text{)} = 130 \text{ mm}$$

$$\text{total c/c area of stirrup legs (A}_{sv}\text{)} = 2 \times \frac{\pi}{4} \times 6^2 = 56.55 \text{ mm}^2$$

$$\text{Percentage of tensile steel (P}_t\text{)} = \frac{100 A_{st}}{bd} = \frac{603.19}{230 \times 500} \times 100 = 0.525$$

so design shear strength of concrete (τ_c) from table 19 of IS 456:2000

$$= 0.48 + \frac{0.56 - 0.48}{0.75 - 0.50} \times (0.525 - 0.50) = 0.488 \text{ N/mm}^2$$

Shear to be resisted by vertical stirrups (V_{u1})

$$= \frac{0.87 f_y A_{sv} d}{S_v} = \frac{0.87 \times 415 \times 100 \times 53}{300} = 87.11 \text{ kN}$$

shear to be resisted by

Capacity of bent up bar (V_{u2}) = $0.87 f_y A_{sv} \sin \alpha$

$$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 16^2 \times \sin 45^\circ$$
$$= 102.66 \text{ kN}$$

but $V_{u2} > V_{u1}$

$$\therefore V_{u2} = 87.11 \text{ kN}$$

Total shear capacity = $V_{uc} + V_{u1} + V_{u2}$

$$= 68.724 + 87.11 \text{ kN} + 87.11 \text{ kN}$$

$$= 242.94 \text{ kN}$$

(c) ~~design~~ Same data as (b) ^{question} except

spacing of stirrup (S_v) = 300 mm

total c/s area of stirrup legs (A_{sv}) = $2 \times \frac{\pi}{4} \times 10^2 = 157.08 \text{ mm}^2$

Shear resistance of concrete (V_{uc}) = $T_{cbl} = 56.12 \text{ kN}$ (same as b)

Shear to be resisted by vertical stirrups (V_{u1})

$$= \frac{0.87 f_y A_{sv} d}{S_v} = \frac{0.87 \times 415 \times 157.08 \times 720}{300} = 136.11 \text{ kN}$$

shear to be resisted by bent up bars (V_{u2}) = 102.66 kN same as (b)

$$\text{Here } V_{u2} < V_{u1}$$

$$\therefore V_{u2} = 102.66 \text{ kN}$$

So Total shear capacity = $V_{uc} + V_{us1} + V_{us2}$
 $= 56.12 \text{ kN} + 136.11 \text{ kN} + 102.66 \text{ kN}$
 $= 294.89 \text{ kN}$

Problem: 5

A simply supported normal tee beam of 4.5 m clear span is loaded with characteristic load of 40 kN/m . It is reinforced with 4 no. 20 mm dia bars at support. The section of the beam is 230 mm wide $\times$ 560 mm effective depth. Design the shear reinforcement at the support. The materials are M20 grade concrete & HYSD reinforcement of grade Fe 415.

Solⁿ

clear span of the beam = 4.5 m

characteristic load acting on the beam = 40 kN/m

Area of tensile steel (A_{st}) = $4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$

width of the section (b) = 230 mm

effective depth of the section (d) = 560 mm

characteristic ^{compressive} strength of the concrete (f_{ck}) = 20 N/mm^2

characteristic strength of steel (f_y) = 415 N/mm^2

(a) checking the max^m shear stress at the face of the support:

The factored load = $1.5 \times 40 = 60 \text{ kN/m}$

shear force at the support (V_u) = $\frac{60 \times 4.5}{2} = 135 \text{ kN}$

Nominal shear stress (τ_v) = $\frac{V_u}{bd} = \frac{135 \times 10^3}{230 \times 560} = 1.05 \text{ N/mm}^2$

Max^m shear stress ($\tau_{c, \text{max}}$) for M20 grade concrete = 2.8 N/mm^2
 $> 1.05 \text{ N/mm}^2$ _{OK}

(b) Shear stress at the critical section

The beam is normal tee beam, i.e., the beam ends are confined by compressive reaction. The critical section for shear at a distance d from the face of the support.

$$\text{Critical shear (V}_u\text{)} = 135 - 0.56 \times 60 \\ = 101.4 \text{ kN}$$

$$\text{Nominal shear stress } (\tau_v) = \frac{101.4 \times 10^3}{230 \times 560} = 0.787 \text{ N/mm}^2$$

(c) Design shear strength of section:

$$\text{Area of tensile steel (A}_{st}\text{)} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$$

$$\text{Percentage of tensile steel (P}_t\text{)} = \frac{A_{st}}{bd} \times 100 \\ = \frac{1256.64}{230 \times 560} \times 100 \\ = 0.976$$

$$\text{Design shear strength of concrete } (\tau_c) \text{ from table 19 of IS 456:2000} = 0.56 + \frac{0.62 - 0.56}{1.00 - 0.75} \times (0.976 - 0.75) \\ = 0.614 \text{ N/mm}^2$$

Now $\tau_v > \tau_c$. So ^{shear} design is necessary

(d) Minimum shear reinforcement:

Assuming 8 mm $\#$ 2 legged stirrups.

The spacing of min^m shear reinforcement is given by

$$S_v \leq \frac{0.87 f_y A_{sv}}{0.4 b}$$

$$\leq \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2}{0.4 \times 230}$$

$$S_v \leq 392.53 \text{ mm}$$

1) Simplified approach

At d from face, $V_u = 101.4 \text{ kN}$, $\tau_v = 0.787 \text{ N/mm}^2$, & $\tau_c = 0.614 \text{ N/mm}^2$

$$\begin{aligned} \text{Shear capacity of concrete } V_{uc} &= \tau_c b d = 0.614 \times 230 \times 560 \\ &= 79.08 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{The shear resisted by stirrups } (V_{us}) &= V_u - V_{uc} \\ &= 101.4 - 79.08 = 22.32 \text{ kN} \end{aligned}$$

Using 8mm # 2 legged stirrups, the spacing is given by

$$\begin{aligned} S_v &= \frac{0.87 f_y A_{sv} d}{V_{us}} \\ &= \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2 \times 560}{22.32 \times 10^3} \\ &= 910.67 \text{ mm} \end{aligned}$$

spacing should not exceed

$$\rightarrow 0.75d = 0.75 \times 560 = 420 \text{ mm}$$

$$\text{(ii)} \rightarrow 300 \text{ mm}$$

$$\text{(iii)} \text{ due to min shear reinforcement} = 392.53 \text{ mm}$$

$$\text{(iv)} \quad 910.67 \text{ mm as designed above}$$

So provide 8mm # @ 300mm c/c.

Bond & development length:

While analysing & designing the reinforced concrete structures, the basic assumption is that there is a perfect bond betⁿ concrete & i.e. there is absolutely no slippage betⁿ the concrete & steel. The grip of the reinforcement & concrete due to adhesion or bearing is termed as bond.

* The magnitude of the bond stress at a point is called local bond stress which varies with the bending moment. Local bond is therefore, also termed as the flexural bond.

* To transfer the design force from steel to concrete or vice versa, the reinforcement is anchored or embedded in concrete for some length. A length of reinforcement embedded in concrete so that it can develop the stress by bond is termed as development length. The anchorage length is thus equal to development length.

Development length: pull out test.

To find out development length, consider a pull out test as indicated in fig.

* A rod of diameter ϕ is embedded in concrete. An axial pull P is applied & increased uniformly.

* The pull out test indicates that the maximum stress is induced in the materials at the point where the bar enters the concrete & it diminishes to zero at the end of the bar. However for calculation of development length average bond resistance is considered.

Let τ_{bd} = average design bond stress in concrete.

σ_s = working stress in bar.

ϕ = diameter of bar.

Then equating bond resistance of concrete = strength of bar:

$$\tau_{bd} \times (\pi \phi) \times L_d = \sigma_s \times \frac{\pi \phi^2}{4}$$

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

where L_d is known as development length.

The factors influencing the stress transfer by bond are

- 1> Adhesion of concrete & steel
- 2> Shear strength of concrete
- 3> interlocking of surface ribs of HYSD (CTD or TMIT) bars with concrete

Code provision

According to IS:456, the bond stresses are assumed to be uniform over the effective surface area of the bar. Thus according to clause 26.2, the calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length or end anchorage or by a combination thereof, where the development length is defined as

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

..... clause 26.2.1. (page 42)

where

ϕ = nominal dia of the bar

σ_s = stress in the bar at the sectⁿ considered at design load

τ_{bd} : design bond stress given in clause 26.2.1.1

Design bond stress (τ_{bd}) for plain bars in tension

Grade of concrete	M20	M25	M30	M35	M40 & above
Design bond stress τ_{bd} (N/mm ²)	1.2	1.4	1.5	1.7	1.9

Note 1: τ_{bd} shall be increased by 25% for bars in compression.

Note 2: For HYSD bars, the value of τ_{bd} shall be increased by 80%.

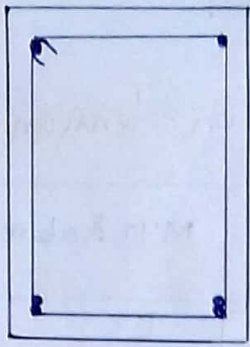
* It is also stated that development length includes the anchorage values of the hook in tension reinforcements. For bars of section other than circular, the development length should be sufficient to develop the stress in the bar by bond.

bars in bundled in contact (not in syllabus)

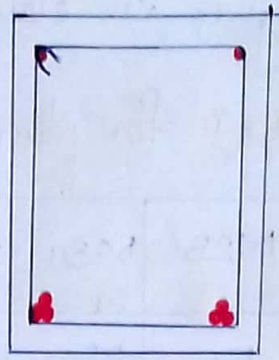
Bars larger than 36 mm diameter shall not be bundled except in columns. The bundled bars are shown in Fig(1).

* The effective perimeter of a single bar in bond is its actual perimeter $\pi \phi$. When the bars are bundled, effective perimeter having a bond with concrete is decreased. This is shown in Fig(2).

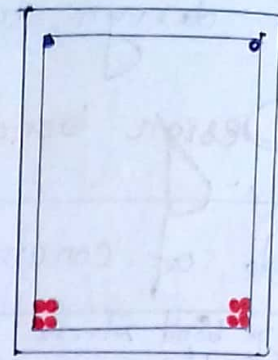
* Thus total effective perimeter for a group of bars is less than the sum of perimeters of individual bars. Therefore the development length of each bar of bundled bars shall be increased by 10% for 2 bars in contact, 20% for 3 bars in contact & 33% for 4 bars in contact, for the individual bar.



(a) Two bars in contact

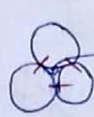


(b) Three bars in contact



Four bars in contact

Fig(1) bundled bars



Ineffective perimeter in bond.

Fig(2) bundled bars

Anchoring reinforcement

The bars are anchored in tension, compression or shear as shown below.

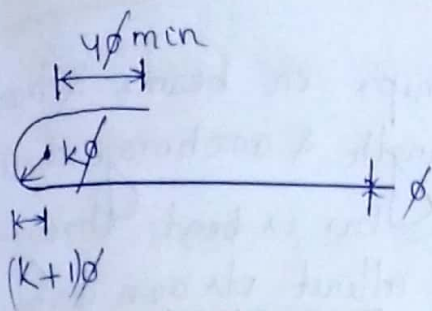
(i) Anchoring bars in tension. (cl 26.2.2. page 43)

(i) Deformed bars may be used without end anchorages provided that development length requirement is satisfied. Hooks should be normally provided for plain bars in tension.

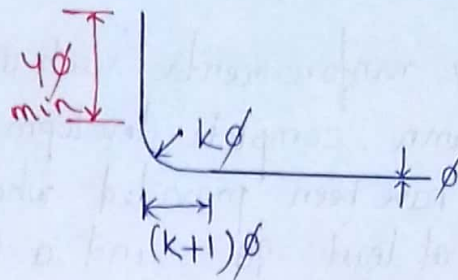
(ii) Bends: The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 45° bend subjected to a max of 16 times the diameter of the bar.

(iii) Hooks: The anchorage value of a standard U type hook shall be equal to 16 times the diameter of the bar.

Standard hook & bend are shown in fig



(a) Standard hook



(b) Standard 90° bend

Standard hook & bend

* The value of k shall be taken as follows

Type of steel	Minimum value of k
mild steel	2
Cold worked steel	4

(2) Anchoring bars in compression

The anchorage length of straight bar in compression shall be equal to the development length of bars in compression. The projected length of hooks, bends & straight lengths beyond bends if provided for bars in compression, shall be considered for development length.

(3) Anchoring bars in shear:

(i) Inclined bars:

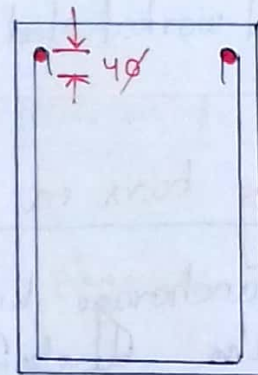
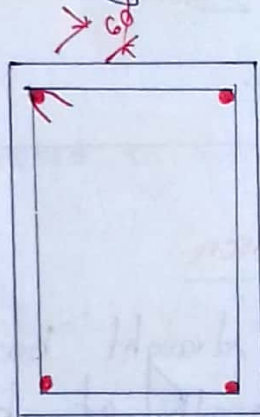
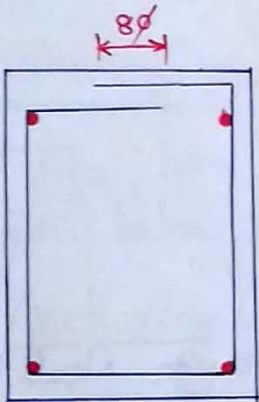
The ^{development} length shall be as for bars in tension. This length shall be measured as under:

In the tension ^{zone}, from the end of the sloping or inclined portion of the bar.

In the compression zone, from the mid depth of the beam

(ii) Stirrups:

For secondary reinforcements such as stirrups in beams & transverse ties in a column, complete development length & anchorage shall be deemed to have been provided when the bar is bent through an angle of at least 90° round a bar of at least its own diameter & is continued beyond the end of the curve for a length of at least 8ϕ or when the bar is bent through an angle of 135° & is continued beyond the end of the curve for a length of at least six diameters (6ϕ) or when the bar is bent through an angle of 180° and is continued beyond the end of the curve for a length of at least 4ϕ .



Anchoring vertical stirrups

Problem: 1

- Calculate the anchorage length in tension and compression for
- Single mild steel bar of diameter ϕ in concrete of grade M20
 - an HYSD bar of grade Fe415 of diameter ϕ in concrete of grade M20

Solⁿ

(a) Mild Steel bar

(1) Tension

$$\text{Design stress } (\sigma_s) = 0.87 f_y = 0.87 \times 250 = 217.5 \text{ N/mm}^2$$

$$\text{design bond stress } (\tau_{bd}) \text{ for M20 grade concrete} = 1.2 \text{ N/mm}^2$$

anchorage length = development length

$$= \frac{\phi \times \sigma_s}{4 \tau_{bd}} = \frac{\phi \times 0.87 f_y}{4 \tau_{bd}} = \frac{\phi \times 217.5}{4 \times 1.2}$$

$$= 45.3 \phi,$$

$$\approx 46 \phi$$

(2) Compression

(i) flexural compression (beam bars)

$$\text{design bar stress } (\sigma_s) = 0.87 f_y = 0.87 \times 250 = 217.5 \text{ N/mm}^2$$

$$\text{design bond stress } (\tau_{bd}) = 1.2 \times 1.25 \text{ (for compression)} \\ = 1.5 \text{ N/mm}^2$$

anchorage length = development length (L_d)

$$= \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{\phi \times 217.5}{4 \times 1.5}$$

$$= 36.25 \phi \approx 37 \phi$$

(ii) Direct compression (column bars)

$$\text{design stress } (\sigma_x) = 0.75 f_y = 0.75 \times 250 = 187.5 \text{ N/mm}^2$$

$$\text{design bond stress } (\tau_{bd}) = 1.2 \times 1.25 \text{ (for compression)} \\ = 1.5 \text{ N/mm}^2$$

$$L_d = \frac{\phi \sigma_x}{4 \tau_{bd}} = \frac{\phi \times 187.5}{4 \times 1.5} = 31.25 \phi \approx 32 \phi$$

(b) HYSD bar of grade Fe 415 & M20 grade concrete

(1) Tension

$$\text{Design bond stress } (\sigma_x) = 0.87 f_y = 0.87 \times 415 = 361.05 \text{ N/mm}^2$$

$$\text{design bond stress } (\tau_{bd}) = 1.2 \times 1.6 \text{ (for HYSD bars)} = 1.92 \text{ N/mm}^2$$

Anchorage length = development length

$$L_d = \frac{\phi \sigma_x}{4 \tau_{bd}} = \frac{\phi \times 361.05}{4 \times 1.92} = 47.01 \phi \approx 47 \phi$$

(2) Compression

(i) Flexural compression (beam bars)

$$\text{design stress } (\sigma_x) = 0.87 f_y = 0.87 \times 415 = 361.05 \text{ N/mm}^2$$

$$\text{design bond stress } (\tau_{bd}) = 1.2 \times 1.6 \text{ (for HYSD bars)} \times 1.25 \text{ (for compression)} \\ = 2.4 \text{ N/mm}^2$$

$$L_d = \frac{\phi \sigma_x}{4 \tau_{bd}} = \frac{\phi \times 361.05}{4 \times 2.4} = 37.61 \phi \approx 38 \phi$$

(ii) Direct compression (column bars)

$$\text{design stress } (\sigma_x) = 0.75 f_y = 0.75 \times 415 = 311.25 \text{ N/mm}^2$$

design bond stress (τ_{bd}) = 1.2×1.6 (for HYSD bar) $\times 1.25$ (for compression)

$$= 2.4 \text{ N/mm}^2$$

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{\phi \times 311.25}{4 \times 2.4} = 32.42 \phi \approx 33 \phi$$

Problem: 2

Calculate the anchorage length of a group of 3 bundled bars in contact & of equal diameter in tension. Concrete grade is M20 & mild steel reinforcement

Solⁿ

Design stress (σ_s) due to tension = $0.87 f_y = 0.87 \times 250 = 217.5 \text{ N/mm}^2$
 bond stress (τ_{bd}) for M20 grade concrete = 1.2 N/mm^2

$$\begin{aligned} \text{Anchorage length} = \text{development length} &= \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{\phi \times 0.87 f_y}{4 \tau_{bd}} \\ &= \frac{\phi \times 217.5}{4 \times 1.2} \\ &= 45.3 \phi \end{aligned}$$

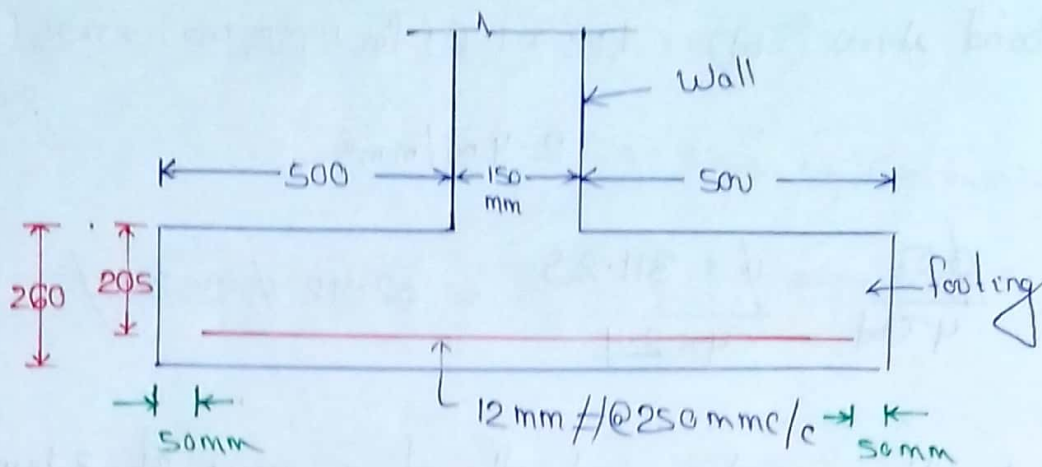
for single bar.

For 3 bars in bundle, the development length for each bar

$$L_d = 45.3 \phi \times 1.2 = 54.36 \phi \approx 55 \phi$$

Problem: 3

The main reinforcement of one-way footing is shown in fig. Verify the anchorage of main bars. The point of maximum stress is assumed to occur at the face of wall. The materials are M20 grade concrete & HYSD reinforcement of grade Fe415.



Solⁿ

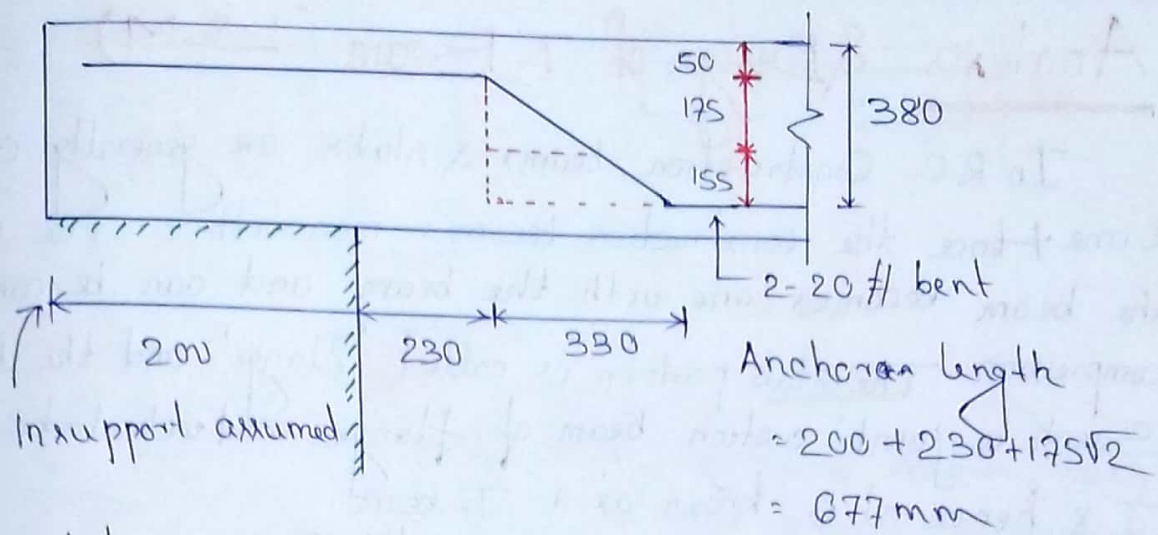
The anchorage available = $500 - 50 = 450 \text{ mm}$

$$\begin{aligned} \text{The anchorage required (Ld)} &= \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{\phi \times 0.87 f_y}{4 \tau_{bd}} \\ &= \frac{\phi \times 0.87 \times 415}{4 \times 1.2 \times 1.6} \\ &= 47.01 \phi \\ &= 47.01 \times 12 = 564.12 \text{ mm} \\ &\approx 564 \text{ mm} \end{aligned}$$

The arrangement is thus not satisfactory. To have proper anchorage in this case hook or bend may be used. The other alternative is to use the lower size diameter.

Problem: 4

Check the anchorage of the bent bar of fig(1), if shear taken by them is 50 kN. 2 bars are bent about 560 mm from support. In tension zone, sufficient length is available. In compression zone, the available anchorage is calculated in fig(1). The materials are M20 grade concrete & HYSD reinforcement of grade Fe415.



Solⁿ

shear taken by two bent bars = 50 kN

$$\text{stress in bar} = \frac{50 \times 10^3}{2 \times 314 \cos 45^\circ} = 112.60 \text{ N/mm}^2$$

$$\text{development length (Ld)} = \frac{\# \times \sigma_s}{4 \tau_{bd}} = \frac{\# \times 112.60}{4 \times 1.2 \times 1.6} = 14.66 \#$$

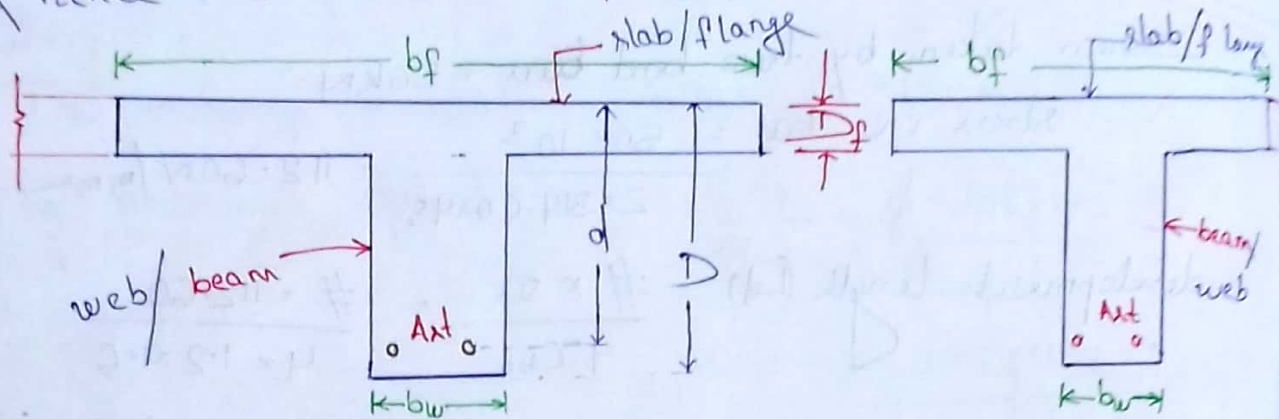
$$= 14.66 \times 20 = 293.20 \text{ mm}$$

Anchorage provided = 677 mm

The effect of 45° bend in compression zone is neglected.
The anchorage of bent bar is thus satisfactory.

Analysis & Design of T-Beam (LSM)

In R.C. Construction, beams & slabs are generally cast at one time. Hence the construction becomes monolithic. The slab above the beam becomes one with the beam and can be considered for compression. The slab portion is called 'flange' and the beam 'web'. Such a joint action beam of flange and web looks like a letter 'T' & hence it is known as a T-beam.



where b_f = effective width of flange

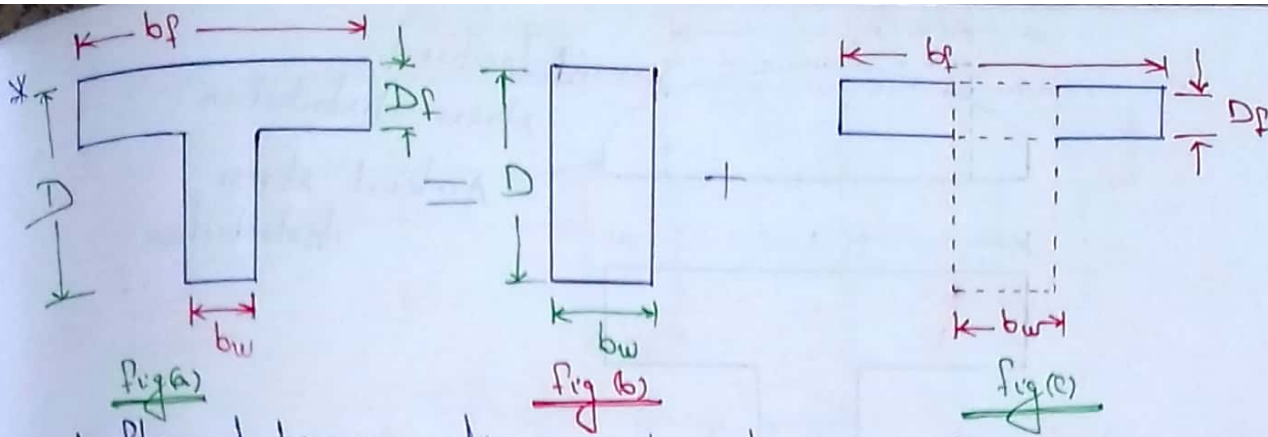
b_w = breadth of web

d = effective depth of beam

D_f = thickness of flange (slab)

* The flanged beam analysis & design are analogous to doubly reinforced rectangular beam. In doubly reinforced beams, additional compressive force is provided by ~~adding~~ adding reinforcement in compression zone, whereas in flanged beams, this is provided by the slab concrete.

* The beam of fig(a) can be thought of as a singly reinforced beam (b) plus beam (c) which provides additional compressive force.



* A Flanged beam also can be doubly reinforced. However, doubly reinforced flanged beams are rare as a large amount of compressive resistance is provided by the slab.

* The moment of resistance of a tee beam of Fig (a) is a sum of the moment of resistance of beam (b) plus the moment of resistance of beam (c).

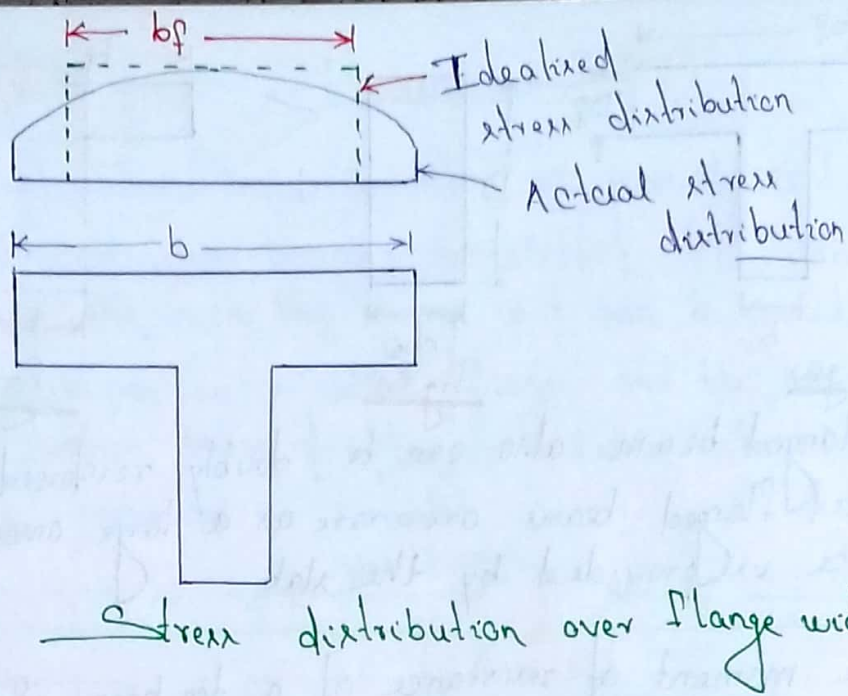
* Similarly the steel area required for beam (a) shall be equal to the sum of the steel area required for the beam (b) & the steel area required for beam (c).

Effective width of flange

Effective width instead of actual width of a flanged beam is used for calculating the moment of resistance as given in clause 23.1 of the code.

* The actual width of a flanged beam is equal to the distance betⁿ the middle points of the adjacent spans of the slab.

* The effective width is equal used to allow for the fact that the compressive stress in the flange higher above the rib than at some distance away from the rib. The effect width concept makes use of constant compressive stress over the entire effective width such that the force of compression remains same.



Clause 23.1.2 of the recommends that effective width of flange of beams may be taken as follows

(a) for T-beams, lesser of

(i) $b_f = \frac{l_0}{6} + b_w + 8D_f$ ✓

(ii) $b_f = x_1 + x_2 + b_w$

(b) for isolated T-beam, lesser of

(i) $b_f = \frac{l_0}{(l_0/b) + 4} + b_w$

(ii) b

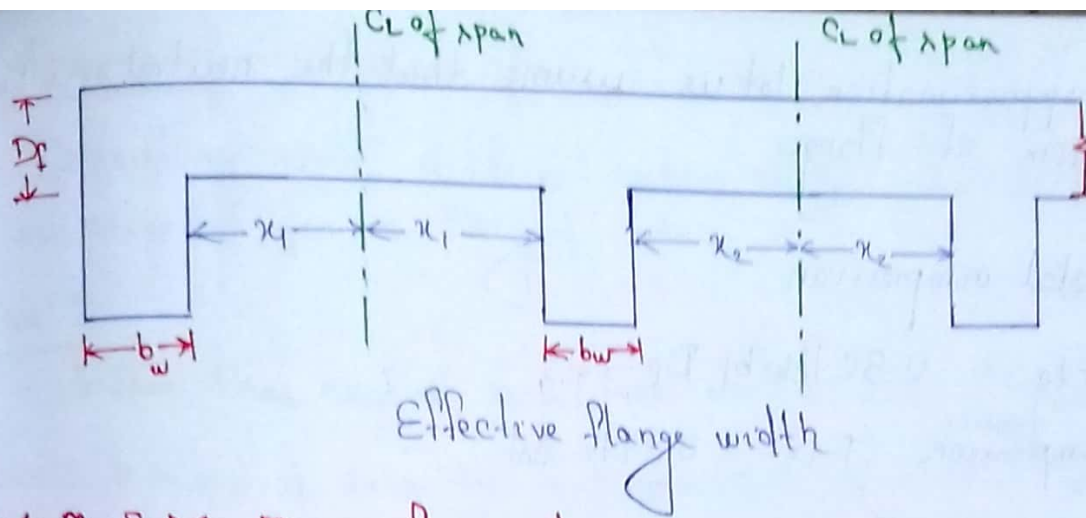
where b_f = effective width of flange

b_w = breadth of the rib or web.

b = actual width of flange

l_0 = distance betⁿ points of zero moment in a beam;
for continuous beams l_0 may be taken as 0.7 times effective span.

x_1, x_2 = half the clear distance betⁿ two adjacent beams as shown in fig.



Min^m & Max^m reinforcement

clause 26.5.1. of the code requires that min^m tension reinforcement should not be less than that given by the following

$$\frac{A_s}{b_w d} = \frac{0.85}{f_y}$$

where A_s = min^m area of tension reinforcement

b_w = breadth of the web of a flanged beam

d = effective depth

f_y = characteristic strength of reinforcement in N/mm²

* Max^m area of tension steel or compression steel should not exceed $0.04 b_w D$.

Position of neutral axis ✓

For a flanged beam, the neutral axis either (a) lies in flange or (b) lies in web. For a given section, to decide whether the neutral axis lies in flange or web, the flange force and the total tension may be compared as explained below.

As a first approximation, let us assume that the neutral axis lies at the bottom of flange.

Now total compression

$$F_{tc} = 0.36 f_{ck} b_f D_f$$

& Total ^{tenison} compression (F_{tx}) = 0.87 f_y A_{st}

Then

(i) If $F_{tc} > F_{tx}$; neutral axis lies in the flange

ii) If $F_{tc} = F_{tx}$; neutral axis lies at the bottom of the flange.

iii) If $F_{tc} < F_{tx}$; neutral axis lies in the web.

* By equating total compression with total tension, the actual depth of neutral axis can be found out

ie $0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

To find out the type of the beam, $x_{u,max}$ shall be found and compared with actual value of neutral axis x_u

Then

i) If $x_u < x_{u,max}$; the section is underreinforced

ii) If $x_u = x_{u,max}$; the section is ~~overreinforced~~ balanced.

iii) If $x_u > x_{u,max}$; the section is overreinforced & take $x_u = x_{u,max}$

Moment of resistance of a T-beam

Depending upon different values of x_u & D_f , 4 different cases are arisen for a flanged beam.

Case: 1

When the neutral axis lies within the flange. ($x_u < D_f$)
— Only in the syllabus

Case: 2 → Neutral axis lies in web ($x_u > D_f$): the section is balanced
(Limiting value of moment of resistance)

Case: 3 → Neutral axis lies in the web, section is under-reinforced

Case: 4 → Neutral axis " " " " & the section is over-reinforced

Case: 1

When the neutral axis lies within the flange ($x_u < D_f$)

When the neutral axis lies in the flange, the size of the compression zone becomes $b_f \times x_u$. As concrete does not resist any tension, the width of tension zone has no effect on the MR of the section. Therefore, this beam can be thought of as a rectangular beam of dimensions $b_f \times d$.

(i) For a singly reinforced flanged beam.

(ii) Equating total compression & total tension & find out the actual depth of N.A. (x_u)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

(iii) Then find the max^m depth of neutral axis ($x_{u, \max}$) & compare betⁿ x_u & $x_{u, \max}$. Then state the type of section.

(iv) For under-reinforced section

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$\text{or } M_u = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$$

(2) For a doubly reinforced flanged beam

for M.

(ii) For balanced or over-reinforced section

$$M_{u,lim,T} = 0.36 f_{ck} b_f x_{u,max} (d - 0.42 x_{u,max})$$

$$\text{or } M_{u,lim,T} = 0.87 f_y A_{st,lim} (d - 0.42 x_{u,max})$$

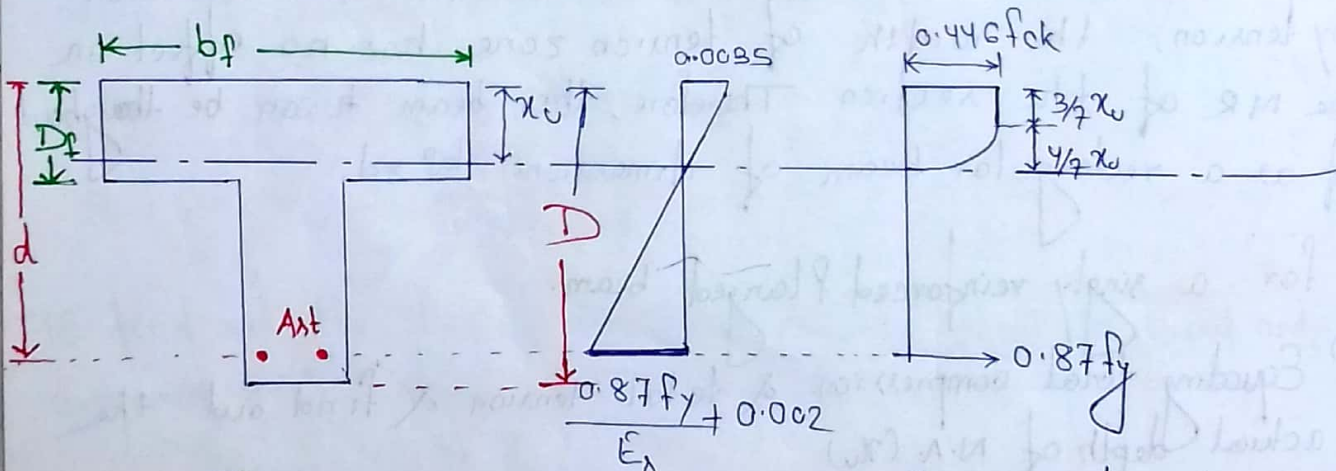
(2) For doubly reinforced flanged beam

$$(i) M_u - M_{u,lim,T} = f_{ck} A_{sc} (d - d')$$

$$(ii) A_{st1} = A_{st,lim}$$

$$(iii) A_{st2} = \frac{A_{sc} f_{ck}}{0.87 f_y}$$

$$(iv) A_{st} = A_{st1} + A_{st2}$$



(a) T-Section

(b) Strain diagram

(c) Stress diagram

Problem: 1

A T-beam of effective flange width 1200 mm, thickness of slab 100 mm, width of rib 300 mm and effective depth of 560 mm is reinforced with 4 no 25 mm diameter bars. Calculate the factored moment of resistance. The materials are M20 grade concrete & HYSD reinforcement of grade Fe415.

Solⁿ

Given data:

effective flange width (b_f) = 1200 mm

thickness of slab (D_f) = 100 mm

width of rib (b_w) = 300 mm

effective depth (d) = 560 mm

Area of steel (A_{st}) = $4 \times \frac{\pi}{4} \times 25^2 = 1963.50 \text{ mm}^2$

characteristic compressive strength of concrete (f_{ck}) = 20 N/mm^2

characteristic strength of steel (f_y) = 415 N/mm^2

To find whether the neutral axis lies in the flange or the web, flange compression & tensile force are compared.

(Total compression)
 $F_{tc} = 0.36 f_{ck} b_f D_f = 0.36 \times 20 \times 1200 \times 100 = 864000 \text{ N}$
 $= 864 \text{ kN}$

Total tension (F_{tx}) = $0.87 f_y A_{st} = 0.87 \times 415 \times 1963.50 = 708921.68 \text{ N}$
 $= 708.92 \text{ kN}$

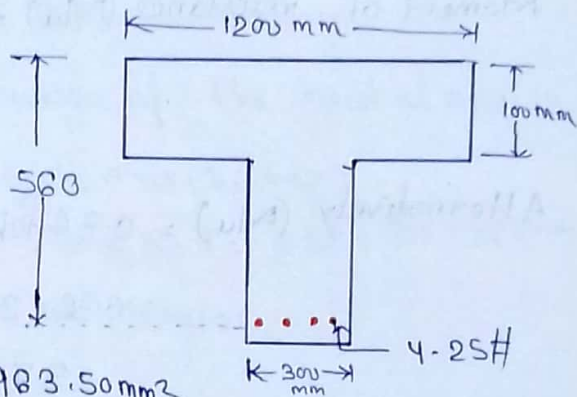
As $F_{tc} > F_{tx}$

$\therefore$ Neutral axis lies in flange.

To find the actual depth of neutral axis (x_u) Equating the total compression to total tension

i.e. $0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$

$\Rightarrow 0.36 \times 20 \times 1200 \times x_u = 0.87 \times 415 \times 1963.50 \Rightarrow x_u = 82.05 \text{ mm}$
 $< 100 \text{ mm (OK)}$



$$\text{Max}^m \text{ depth of neutral axis } (x_{u, \max}) = 0.48 d \quad [\text{For Fe 415 grade Steel}]$$

$$= 0.48 \times 560$$

$$= 268.8 \text{ mm}$$

$$A_x \quad x_u < x_{u, \max}$$

$\therefore$ Section is under-reinforced.

$$\text{Moment of resistance } (M_u) = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 1983.50 (560 - 0.42 \times 82.05)$$

$$= 372.56 \text{ kNm}$$

$$\text{Alternatively } (M_u) = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 1200 \times 82.05 (560 - 0.42 \times 82.05)$$

$$= 372.56 \text{ kNm}$$

Problem: 2

A tee-beam having an effective flange width of 2500 mm is required to resist an ultimate moment of 1200 kNm. The thickness of the flange is 150 mm, the width of the rib is 300 mm and effective depth is 900 mm. Using M20 concrete and Fe 250 steel, determine the area of reinforcement required.

Soln

Given data:

effective flange width (b_f) = 2500 mm

Thickness of flange (D_f) = 150 mm

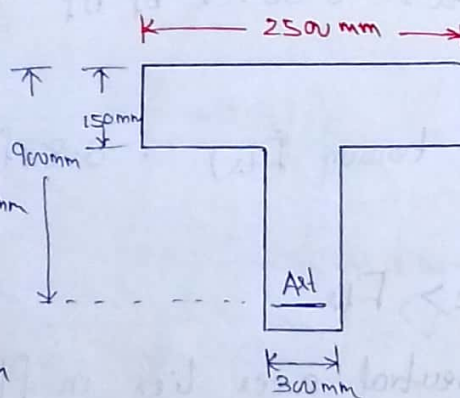
Width of rib (b_w) = 300 mm

effective depth (d) = 900 mm

Ultimate moment resisted by the beam (M_u) = 1200 kNm

Characteristic compressive strength of concrete (f_{ck}) = 20 N/mm²

" strength of steel (f_y) = 250 N/mm²



Assuming the neutral axis to lie within the flange ($x_u < D_f$)

$$\text{Moment of resistance } (M_u) = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$$

$$\Rightarrow 1200 \times 10^6 = 0.36 \times 20 \times 2500 \times x_u (900 - 0.42 x_u)$$

$$\Rightarrow 66666.67 = x_u (900 - 0.42 x_u)$$

$$\Rightarrow 0.42 x_u^2 - 900 x_u + 66666.67 = 0$$

$$\Rightarrow x_u = 76.83 \text{ mm} < D_f = 150 \text{ mm}$$

Hence our assumption about the position of the neutral axis is correct.

Also max depth of neutral axis ($x_{u, \max}$)

$$= 0.53 d = 0.53 \times 900 = 477 \text{ mm}$$

Equating total compression to total tension

$$0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$$

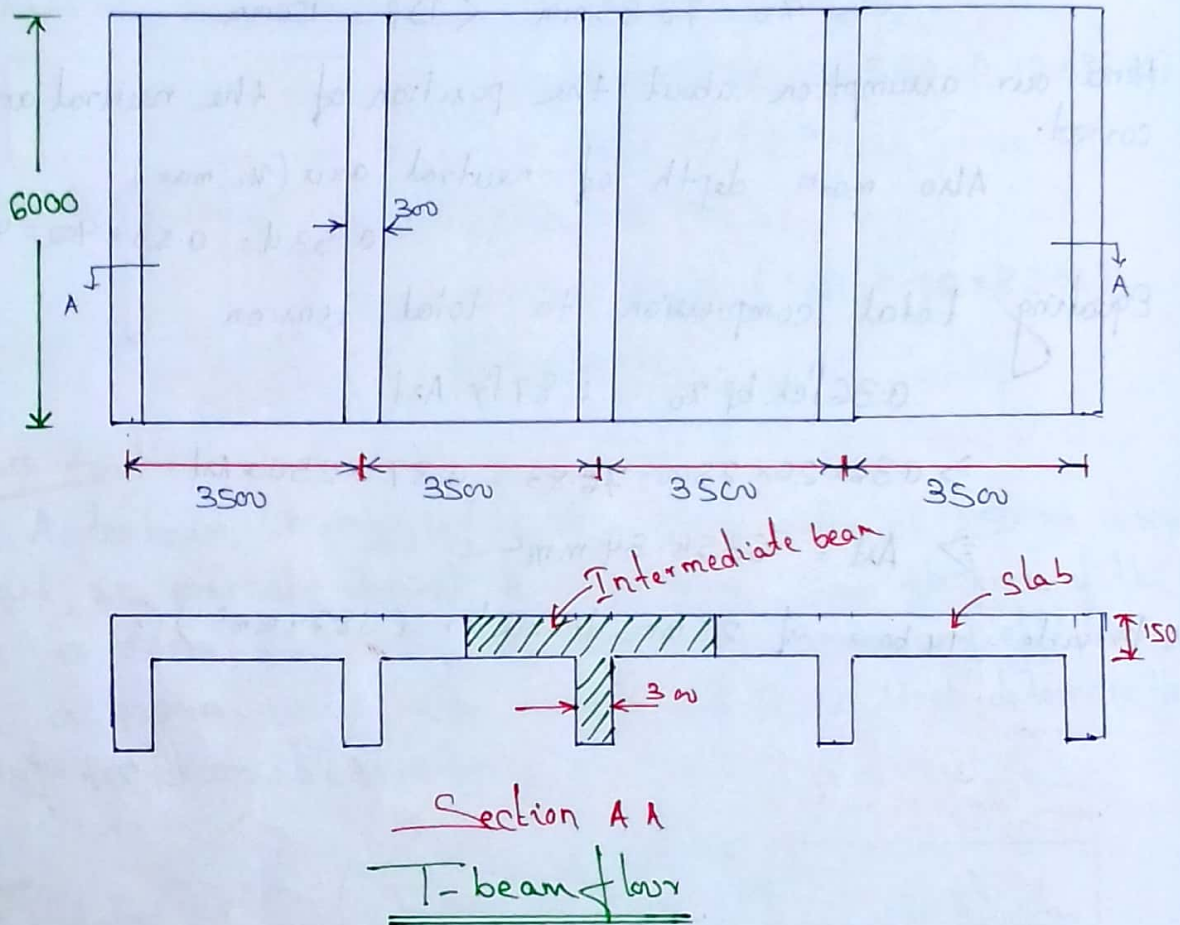
$$\Rightarrow 0.36 \times 20 \times 2500 \times 76.83 = 0.87 \times 250 \times A_{st}$$

$$\Rightarrow A_{st} = 6358.34 \text{ mm}^2$$

∴ Provide 14 bars of 25 mm diameter (6874 mm^2).

Problem: 3

A T-beam floor consists of 15 cm thick RC slab monolithic with 30 cm wide beams. The beams are spaced at 3.5 m centre to centre and their effective span is 6 m as shown in fig. If the superimposed load on the slab is 5 kN/m^2 , design an intermediate beam. Use M20 mix & Fe250 grade steel.



Soln
Given data.

Thickness of slab/plange (D_f) = 15 cm

Width of rib/web (b_w) = 30 cm

effective span of beam (l) = 6 m

c/c distance betⁿ betⁿ of beam = 3.5 m

super imposed load on the slab = 5 kN/m^2

characteristic compressive strength of concrete (f_{ck}) = 20 N/mm^2
" strength of steel (f_y) = 250 N/mm^2

$$\text{Dead load of slab} = 0.15 \times 3.5 \times 25 = 3.75 \text{ kN/m}^2$$

$$\text{superimposed load on slab} = 5 \text{ kN/m}^2$$

$$\text{Total load on slab} = 8.75 \text{ kN/m}^2$$

$$\begin{aligned} \text{Load per meter run of beam} &= \text{load on slab per unit area} \times \\ &\quad \text{c/c distance bet}^n \text{ beams} \\ &= 8.75 \times 3.5 = 30.63 \text{ kN/m} \end{aligned}$$

$$\begin{aligned} \text{effective width of flange (b}_f) &= \frac{l_0}{6} + b_w + 6D_f \\ &= \frac{6000}{6} + 300 + 6 \times 150 \\ &= 2200 \text{ mm} \end{aligned}$$

$$\text{Max}^m \text{ value of flange width} = 3500 \text{ mm} > 2200 \text{ mm}$$

$$\therefore \text{Effective width of flange (b}_f) = 2200 \text{ mm}$$

Let us adopt overall depth (D) of beam equal to 400 mm & effective cover equal to 40 mm so that effective depth is 360 mm

$$\begin{aligned} \text{Dead load of web of beam} &= \text{width of web} \times \text{depth of web} \\ &\quad \times \text{concrete density} \\ &= 0.30 \times 0.25 \times 25 = 1.875 \text{ kN/m} \end{aligned}$$

$$\text{Total load on beam per meter run} = 30.6 + 1.875 = 32.475 \text{ kN/m}$$

$$\text{Factored max}^m \text{ B.M.} = \frac{wl^2}{8} = \frac{(1.5 \times 32.475) \times 6^2}{8} = 219.2 \text{ kNm}$$

Let us assume that neutral axis lies in the flange, that is, then depth of neutral axis (x_u) is

$$x_u = \frac{0.87 f_y A_{st}}{0.36 \times f_{ck} \times b_f} = \frac{0.87 \times 250 \times A_{st}}{0.36 \times 20 \times 2200} = 0.0137 A_{st}$$

Now factored B.M. = ~~0.87~~ f_y Force of tension $\times z$

$$\Rightarrow 219.2 \times 10^6 = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$\Rightarrow 219.2 \times 10^6 = 0.87 \times 250 \times A_{st} (360 - 0.42 \times 0.0137 A_{st})$$

$$\Rightarrow 1007816.092 = 360 A_{st} - 5.754 \times 10^{-3} A_{st}^2$$

$$\Rightarrow 5.754 \times 10^{-3} A_{st}^2 - 360 A_{st} + 1007816.092 = 0$$

$$\Rightarrow A_{st} = 2937.40 \text{ mm}^2$$

$$\& x_u = 0.0137 \times 2937.40 = 40.24 \text{ mm} < 150 \text{ mm} \quad (\text{OK})$$

$\therefore$ NA lies in the flange

Use 5-28 mm bar reinforcement in tension side

$$(A_{st, \text{provided}} = 3078.76 \text{ mm}^2 > 2937.40 \text{ mm}^2) \quad \text{OK}$$

$$\text{Min}^m \text{ area of tension steel } A_{st} = \frac{0.85}{f_y} \times b_w d$$

$$= \frac{0.85}{250} \times 300 \times 360$$

$$= 367.2 \text{ mm}^2 < 3078.76 \text{ mm}^2 \quad (\text{OK})$$

$$\text{Max}^m \text{ area of tension steel } A_t = 0.04 b_w D$$

$$= 0.04 \times 300 \times 400$$

$$= 4800 \text{ mm}^2 > 3078.76 \text{ mm}^2 \quad (\text{OK})$$

Control of deflection Analysis & Design of slab (CH-6)

A structural member deflects when, it carries a load. According to clause 23.2 IS:456, the deflection of a structure

see [clause - 23.2]

* The first limit is specified to avoid psychological upsetting of the occupants.

* The second limit is intended to avoid damage to partitions and finishes.

In the clause 23.2.1 of IS:456 suggested for checking that deflection is not excessive.

These are

(i) Control of span/effective depth ratio and is discussed in clause 23.2.1 which shall be used modified for.

(ii)

Span/effective depth ratio

This method is based on the calculation of deflection and tests on practical beams/slabs. This is a semi-empirical method and is covered in clause 23.2.1 of IS:456. Accordingly the basic span/effective depth ratios for diff. support condⁿ are given and shall be modified for

- (1) Amount of tension steel
- (2) the amount of compression steel.
- (3) the type of beam.

* For beams and slabs, the vertical deflection limits as stated above may generally be assumed to be satisfied provided that the span to depth ratios are not greater than the values obtained as below.

Clause 23.2.1 (a), (b), (c), (d), (e):

Now allowable $\frac{\text{span}}{d}$ ratio is given by

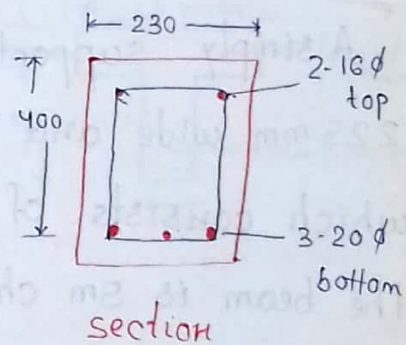
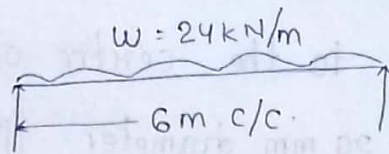
$\left(\frac{\text{span}}{d}\right)_{\text{allowable}} = \text{basic} \left(\frac{\text{span}}{d}\right)_{\text{ratio}} \times \text{modification factor for reinforcement} \times \text{modification factor for compression reinforcement} \times \text{reduction factor for flanged beam}$

The actual $\frac{\text{span}}{\text{depth}}$ ratio shall be less than or equal to the allowable $\frac{\text{span}}{d}$ ratio.

Problem-1

A simply supported R.C.C. beam is shown in fig. If the span of the beam is 6m, check the deflection of the beam using the rules. The area required in tension is 920 mm^2 and 120 mm^2 in compression. The materials are M20 grade concrete and HYSD reinforcement of grade Fe 415.

Solution



For simply supported beam basic $\frac{\text{span}}{d}$ ratio = 20

$$A_{st} = 942.47 \text{ mm}^2$$

$$f_s = 0.58 f_y \times \frac{A_{st, \text{required}}}{A_{st, \text{provided}}}$$

$$= 0.58 \times 415 \times \frac{920}{942.47}$$

$$= 234.96 \text{ N/mm}^2$$

$$P_t = \frac{100 \times 942.47}{230 \times 400} = 1.02$$

So modification factor = 1.0

$$A_{sc} = 402.12 \text{ mm}^2$$

$$P_c = \frac{100 \times 402.12}{230 \times 400} = 0.44$$

$$\text{modification factor} = 1.13$$

$$\text{Then permissible } \frac{\text{span}}{d} \text{ ratio} = 20 \times 1.0 \times 1.13 = 22.6$$

$$\text{actual } \frac{\text{span}}{d} \text{ ratio} = \frac{6000}{400} = 15 < 22.6$$

The beam is satisfactory in respect of deflection.

Problem: 2

A simply supported beam reinforced on the tension side is 225 mm wide and 400 mm deep to the centre of tensile steel which consists of 3 bars of 20 mm diameter. If the span of the beam is 5 m check if the beam is satisfactory from deflection control. Use M 20 concrete and Fe 415 steel.

Solution

$$\text{Basic } \frac{\text{span}}{d} \text{ ratio for simply supported beams} = 20$$

$$\text{percentage of tensile steel provided} = \frac{3 \times \frac{\pi}{4} \times 20^2 \times 100}{225 \times 400} = 1.05\%$$

$$\text{modification factor} = 0.97$$

$$\therefore \text{permissible } \frac{\text{span}}{\text{depth}} \text{ ratio} = 20 \times 0.97 = 19.4$$

$$\text{Actual } \frac{\text{span}}{\text{depth}} \text{ ratio} = \frac{5000}{400} = 12.5 < 19.4$$

Hence the beam is satisfactory from deflection consideration.

Problem - 3

A tee beam with $b_f = 1250 \text{ mm}$, $b_w = 250 \text{ mm}$, $D_f = 100 \text{ mm}$, $d = 450 \text{ mm}$ requires 1370 mm^2 area of tension reinforcement. It is reinforced with 5-20 ϕ in tension. Check the beam in respect of deflection. The materials are M20 grade concrete and reinforcement of grade Fe 415. The beam is simply supported over a span of 5.6m.

Solution

$$b_f = 1250 \text{ mm}$$

$$b_w = 250 \text{ mm}$$

$$D_f = 100 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 1570.79 \text{ mm}^2$$

$$P_t = \frac{100 \times 1570.79}{1250 \times 450} = 0.279$$

$$f_s = \frac{0.58 \times 415 \times 1370}{1570.79} = 209.93 \text{ N/mm}^2$$

So modification factor = 1.8

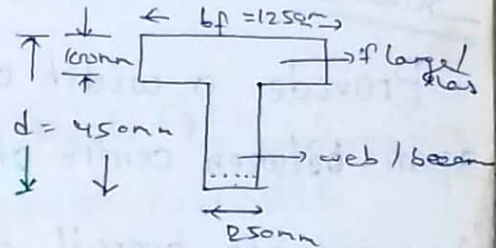
$$\text{Also } \frac{b_w}{b_f} = \frac{250}{1250} = 0.2$$

Reduction factor = 0.8

Basic $\frac{\text{span}}{d}$ ratio = 20

$$\text{permissible } \frac{\text{span}}{d} \text{ ratio} = 20 \times 1.8 \times 0.8 = 28.8$$

$$\text{Actual } \frac{\text{span}}{d} \text{ ratio} = \frac{5600}{450} = 12.44 < 28.8$$



Design of slab

* Slabs are plate elements having the depth D much smaller than its span and width. They usually carry a uniformly distributed load and form the floor or roof of the building.

* Like beams, slabs are horizontal except in the case of stair cases and ramps.

* Like beams, slabs also may be simply supported, cantilever or continuous depending upon the support conditions. They are classified according to the system of supports used as under

1) One-way spanning slabs

2) Two-way "

3) Flat slabs supported directly on columns without beams.

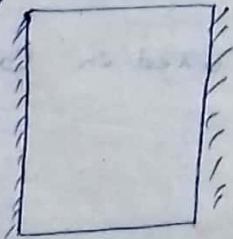
(4) Grid slabs.

(5) Circular & other shapes

(6) Ribbed & Waffle slabs.

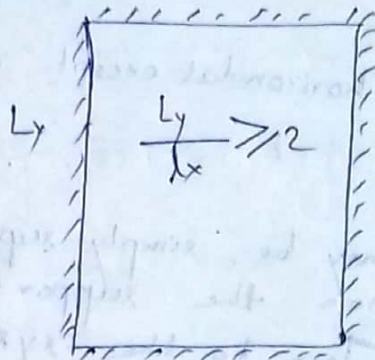
(1) One-way spanning slabs

* The slabs supported on 2 opposite supports is a one-way spanning slab. In short, a slab which transfers its load on one of the set of two opposite edge supports is called one-way slab. In fig (a) by virtue of the geometry, the slab is a one



way spanning slab, where it is supported only on 2 opposite edge supports

* The slab where $\frac{L_y}{L_x} \geq 2$, is called one way slab. provided that it is supported on all four edges. In one way slab, the load on the slab is taken as carried by the short span.



L_x = span in shorter dir
 L_y = " " longer dir

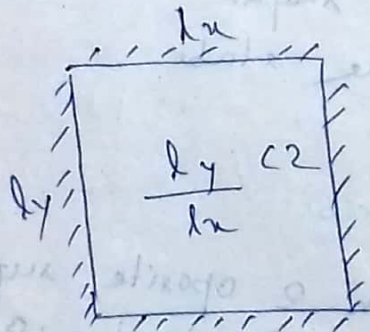
Ex Generally all the cantilever slabs are one way slabs. chajja and verandah are ex of one way slab

(2) Two way slab

if the slab is supported on all four edges and if

$\frac{L_y}{L_x} < 2$ the tendency of the slab is to bend in both

dir's. Such slabs are called two way spanning slabs



* In 2 way slabs, load will be carried in both the dir's. $\therefore$ So main reinforcement is provided in both dir's for 2 way slabs

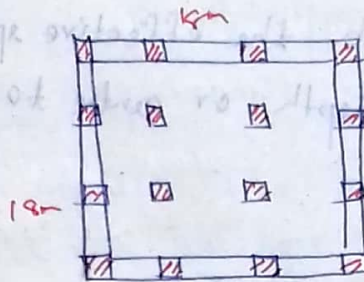
Ex: These types of slabs are used in construction floors of multi storied building

Flat slabs

Sometimes, beam free areas are required to reduce the floor height & to permit large amount of light which might be obstructed by the larger depth of the beams spanning on comparatively larger spans (5-6 metres). Flat slabs are used in such cases.

* Flat slabs are thicker slabs supported directly on columns.

* For office building in commercial areas, flat slabs are beneficial.



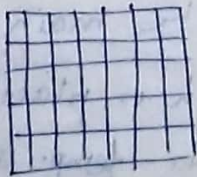
(a) Flat slab

Grid slabs

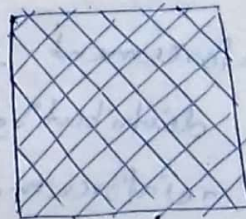
Sometimes, in large halls, public places, marriage halls, auditoriums etc. a large column free area is required. In these cases, large depth beams may be permitted only on periphery.

* In such cases grid slabs and grid beams are provided.

The grid slabs and grid beams are provided. The grid slabs may have rectangular (or square) grid or diagonal.



Rectangular grid



Circular slabs

The slab may be circular by virtue of its shape. Cover slab of a circular water tank is an excellent example of such slab.

Ribbed & waffle slabs

Thin ribs spanning in one dirⁿ & placed not more than 15 c/c constitute ribbed slab. If such slab ribs are provided in both dirⁿ, the waffle slab is obtained.

One-way spanning slab

The following are the design considerations for one-way slab.

(1) Effective span

* For simply supported slab, the effective span is equal to clear span plus effective depth or centre to centre of supports whichever is less.

(2) General

* A simply supported one-way slab. The reinforcement in the dirⁿ of span is the moment steel & is designed in the same way as beams. Slabs are usually not designed as doubly reinforced except for some special cases.

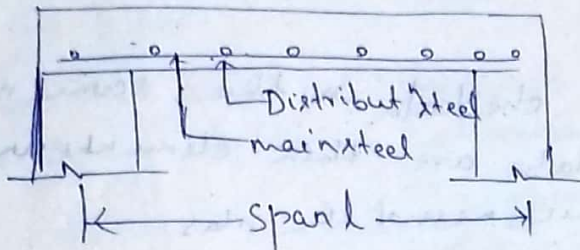
* The moment steel is known as main reinforcement and is placed in the first layer near to the extreme tension fibre, keeping clear cover as per requirement to get max^m effective depth.

* The reinforcement far to the main reinforcement is known as distribⁿ steel which is placed in second layer. This reinforcement resists temp. and shrinkage stresses, keeps the main reinforcement in posⁿ & distrib^{utes} the concentrated or non uniform loads throughout the slab.

The spacing of slab bars is given by

$$\text{spacing} = \frac{\text{area of one bar} \times 1000}{\text{required area in mm}^2 \text{ per metre}}$$

Diagram - 380 (fig 10.3.(a))



Simply supported one-way slab

3) Reinforcement requirements

a) Min^m reinforcement: The reinforcement in either dirⁿ in slab shall not be less than following

0.0015 bD for mild steel bars.

0.0012 bD " HYSD bars.

b) max^m diameter: The diameter of reinforcing bar shall not exceed $\frac{1}{8}$ th. of the total thickness of the slab.

c) According to SP: 34, for main bars the min^m dia shall be 10 mm for plain bars and 8 mm for deformed bars. For distributⁿ bar, the min^m dia shall be

6 mm in any case.

(4) Shear stress

In normal cases the shear in slab is not critical. ~~However~~

Page 41 - 40.2.1.1

→ It can be seen from the above values that higher shear stresses are permitted for thinner slabs. This is due to the observatⁿ made that thin plates can resist more shear per unit area. If shear is critical in slabs usually the shear reinforcements are not provided but

the depth of slab, or increased

* According to clause 40.2.3, the nominal shear stress should not exceed $0.5 \tau_{c, \max}$

$$\tau_v \nless 0.5 \tau_{c, \max}$$

Deflection

This shall be checked in the same manner as beams. The slabs are thin elements and deflection may govern the thickness of the slab.

Cracking

To ensure that the cracking of the slab is not excessive, spacing of the reinforcements shall be limited to the following

For main bars, spacing $\nless 3d$
 $\nless 300 \text{ mm}$

For secondary bars, spacing $\nless 5d$
 $\nless 450 \text{ mm}$

d = effective depth of slab

Cover

page - 382 HJ 1 kda

1) Development length - 382

515

The first trial of depth of slab can be arrived at by considering deflection criterion.

short span betⁿ centres of bearings = $3.5 + 0.25$
 3.75m

long span betⁿ "light" and "dark" = 3.75m
= 8 + 0.25 = 8.25m

Assuming 0.3% steel as a first trial with service stress ~~modification~~ factor =

Basic span ratio = 20

Then permissible $\frac{\gamma_{\text{pan}}}{q}$ ratio = $20 \times 145 = 29$

The depth of =

$$\text{Effective depth required} = \frac{\text{span}}{20 \times \text{modification factor}}$$

$$= \frac{3.75}{29}$$

Considering mild exposure & ~~max~~ dia of reinforcement = $129.31 \approx 130 \text{ mm}$
Overall depth (D) = $130 + 15 + 5 = 150 \text{ mm}$

Consider a 1m wide strip of the slab parallel to its shorter span.
 DL of the slab = $0.15 \times 1.0 \times 25 = 3.75 \text{ kN/m}$

super imposed load = $5 \times 1 = 5 \text{ kN/m}$

floor finish = $1 \times 1 = 1 \text{ kN/m}$

Assume floor finish = 1 kN/m

Total load = 9.75 kN/m

factored load = $1.5 \times 9.75 = 14.625 \text{ kN/m}$

Effective span is taken as the lesser of the following

(i) Distance betⁿ centre of bearing = $3.5 + 0.25 = 3.75 \text{ m}$

(ii) clear span + effective depth = $3.5 + 0.13 = 3.63 \text{ m}$

Effective span in shorter dirⁿ = 3.6 m

Moment & shear

Max^m BM at centre of shorter span = $\frac{wul^2}{8}$

$$= \frac{14.625 \times 3.6^2}{8}$$

$$= 24.08 \text{ kNm}$$

Max^m shear force = $\frac{wul}{2}$ (Based on clear span in short dirⁿ)

$$= \frac{14.625 \times 3.6}{2} = 26.325 \text{ kN}$$

$$= 26.325 \text{ kN}$$

Depth of the slab as given by B.M. = 0.138 fck

or $d = \sqrt{\frac{BM}{0.138 \text{ fck} b}}$

$$= \sqrt{\frac{24.08 \times 10^6}{0.138 \times 25 \times 1000}} = 83.59 \text{ mm}$$

(d provided = 130 mm (ok))

Effective depth required for flexure

Design for Flexure

Area of tension steel is given by

$$M = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$\Rightarrow 24.08 \times 10^6 = 0.87 \times 415 \times A_{st} \times 130 \times \left(1 - \frac{A_{st} \times 415}{100 \times 130 \times 15} \right)$$

$$\Rightarrow 513.03 = A_{st} - 4 \times 10^{-5} A_{st}^2$$

$$\Rightarrow A_{st} = 524.01 \text{ mm}^2$$

$$\text{spacing of } 10 \text{ mm } \phi \text{ bar} = \frac{\pi/4 \times 10^2 \times 1000}{524.01}$$
$$= 149.88 \text{ mm}$$

provide 10 mm ϕ bar @ 150 mm c/c

$$A_{st, \text{provided}} = 560.99 \text{ mm}^2$$

Bend alternate bars at $0.1L$ from the face (ok) of support where moment reduces to less than half of its maxⁿ value.

check for spacing & dia of main reinforcement

Temperature / distributⁿ reinforcement is equal to 0.12% of the gross concrete area provided in the longitudinal dirⁿ

$$A_{tr} = 0.12\% \times b \times D = 0.12\% \times 1000 \times 150$$
$$= 180 \text{ mm}^2$$

$$\text{spacing of } 6 \text{ mm HYSD steel } \phi = \frac{\pi/4 \times 6^2 \times 1000}{180}$$

$$= 154.07 \text{ mm}$$

provide 6 mm ϕ bar @ 150 mm c/c [$A_{tr, \text{provided}} = 168 \text{ mm}^2$ (ok)]

Check for shear

shear force at the edge of the support (V_u) = $w_u \times l_d$
Nominal shear stress (τ_v)

$$= \frac{V_u}{bd}$$

$$= \frac{25.59 \times 10^3}{1000 \times 130} = 0.20 \text{ N/mm}^2$$

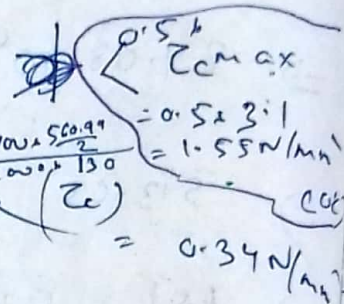
$$= \frac{14.625 \times 3.5}{2} = 25.59 \text{ KN}$$

$$0.1908 \approx 0.20$$

$$\text{N/mm}^2$$

Percentage of steel available near support = $\frac{100 \times A_{st}}{bd} = \frac{100 \times 560.99}{1000 \times 130} = 0.4315$

Design shear strength of concrete (τ_c) = 0.34 N/mm^2



For 150mm thick slab $k = 1.3$

for solid slabs, the design shear strength for concrete (τ_c') = $k \times \tau_c = 1.3 \times 0.34$

$$= 0.442 \text{ N/mm}^2$$

$$\therefore \tau_v < \tau_c' \quad (\text{OK})$$

The design is safe against shear.

Check for development length

Moment of resistance of the sectⁿ after 50% bent up of steel

$$M_r = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$= 0.87 \times 415 \times \left(\frac{560.99}{2} \right) \times 130 \left(1 - \frac{(560.99)}{1000 \times 130} \right) \times 415$$

$$M_1 = 12.697 \text{ KN-m}$$

$$= 12.$$

$$V = 25.59 \text{ kN}$$

let us assume $l_d = 8\phi$ [l_d = anchorage length beyond the centre of support & not exceeding 40×12 times the bar dia]

$$\text{dev } l_d = \frac{0.87 f_y \phi}{4 \times 26d} = \frac{0.87 \times 415 \phi}{4 \times 1.4 \times 1.6} = 40.29 \phi$$

As per code

$$l_d \leq 1.3 \frac{M_1}{V_u} + l_e$$

Assume $\rightarrow 8\phi$

$$40.29 \phi \leq \frac{1.3 \times 12.694 \times 10^3}{25.59 \times 10^3} + 8\phi$$

$$\Rightarrow 32.29 \phi \leq 644.86$$

$$\Rightarrow \phi \leq 19.97 \text{ mm}$$

$$\phi_{\text{provided}} = 10 \text{ mm} \quad (\text{safe})$$

check for deflection

$$\text{Basic } \frac{\text{span}}{d} \text{ ratio} = 20$$

-1. of tensile reinforcement

$$= \frac{100 A_{st}}{b d} = \frac{100 \times 560.91}{1000 \times 130} = 0.43 \%$$

$$f_x = 0.58 \times f_y \times \frac{A_{st, \text{required}}}{A_{st, \text{provided}}} = 0.58 \times 415 \times \frac{524.01}{560.91}$$

modification factor

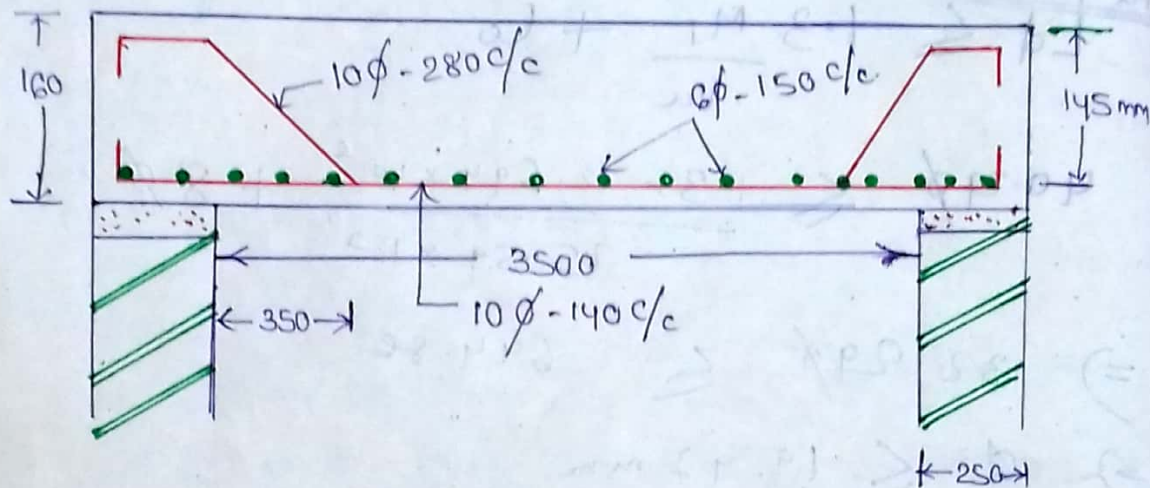
$$(\gamma) = 1.3 \text{ at a service stress of } 240 \text{ N/mm}^2 \text{ in Fe 415 grade steel}$$

$$\frac{415}{0.25} \text{) } \text{ permissible } \frac{\text{span}}{d} \text{ ratio} = 20 > 1.3 = 26$$

$$\text{actual } \frac{\text{span}}{d} \text{ ratio} = \frac{3630}{130} = 27.92 < 26 \text{ OK}$$

∴ There is need to increase the depth of slab.
 Let us increase the effective depth to 145 mm
 & overall depth to 160 mm.

The details of reinforcement are shown in fig.



Reinforcement in one-way slab.

Two-way slabs:

The two-way spanning slab occurs when the slab is supported on all four edges. The ^{two} way actⁿ can be well studied by considering a thin flexible membrane supported on all four edges.

* If l_y is much greater than l_x , the membrane deflects in dirⁿ of l_x . Now keeping l_x const, l_y is reduced & the deflectⁿ of the membrane is observed.

* When $l_y = 2l_x$; from this point it can be observed the dirⁿ of l_y also. If l_y is further reduced, the deflectⁿ decreases in the dirⁿ of l_x & increases in the dirⁿ of l_y .

* When $l_y = l_x$, the membrane deflects as a saucer.
When $l_y \gg 2l_x$, the case is a one way spanning &
when $l_y < 2l_x$ or $\frac{l_y}{l_x} < 2$ the case is a two way spanning slab.

* The two-way slabs can be simply supported or restrained slabs.

* In simply supported slabs, the corners can lift away from the support while for restrained slabs, the corners are held down by edge beams.

* The corners are held down by beams of the stiffness of the beam and therefore at corners, torsion is induced. The slab should be capable of resisting torsion at corners. Also the edge beams shall be designed for torsion.

$$= 4 + 0.2 \text{ or } 4 + 0.015 \text{ which ever is lesser}$$

$$\therefore l_x = 4.15 \text{ m}$$

similarly $l_y = 6 + 0.2 \text{ or } 6 + 0.141 \text{ which ever is lesser}$

$$\therefore l_y = 6.141 \text{ m}$$

$$\text{span ratio} = \frac{l_y}{l_x} = \frac{6.141}{4.15} = 1.48 < 2$$

so it is a two way slab.

$$\text{so for } \frac{l_y}{l_x} = 1.48, \alpha_x = 0.103 \text{ and } \alpha_y = 0.047$$

Load considers 1m wide strip

$$\text{DL of slab} = 0.17 \times 1 \times 25 = 4.25 \text{ kN/m}^2$$

$$\text{floor finish} = 1 \times 1 \times 2 = 1 \text{ kN/m}^2$$

$$\text{live load} = 3 \times 1 = 3 \text{ kN/m}^2$$

$$\text{Total load} = 8.25 \text{ kN/m}^2$$

$$\text{factored load} = 1.5 \times 8.25 = 12.38 \text{ kN/m}^2$$

~~Max~~ Moments along short span & along long span is given by

$$M_x = \alpha_x w l_x^2 = 0.103 \times 12.38 \times 4.15^2$$

$$= 21.96 \text{ kN-m}$$

$$M_y = \alpha_y w l_y^2 = 0.047 \times 12.38 \times 6.141^2$$

$$= 10.02 \text{ kN-m}$$

required
The effective depth of the slab is given by
equating M_u, lim to M_x

$$0.138 f_{ck} b d^2 = 21.96 \times 10^6$$

$$\Rightarrow 0.138 \times 20 \times 1000 \times d^2 = 21.96 \times 10^6$$

$$\Rightarrow d = 89.19 \text{ mm} < d_{\text{short, provided}} = 150 \text{ mm}$$

$d_{\text{long, provided}} = 141 \text{ mm}$

Area of steel along short span

$$M_x = 0.87 f_y A_{st} d \left(\frac{1 - A_{st} f_y}{6 d f_{ck}} \right) \quad (OK)$$

$$\Rightarrow 21.96 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150 \times \left(1 - \frac{A_{st} \times 415}{1000 \times 150 \times 20} \right)$$

$$\Rightarrow 405.48 = A_{st} - 1.38 \times 10^{-4} A_{st}^2$$

$$A_{st} = 431 \text{ mm}^2$$

$$\text{spacing of } 10 \text{ mm } \phi \text{ bars} = \frac{\pi/4 \times 10^2 \times 1000}{431.13} = 182.17 \text{ mm}$$

provide 10 mm ϕ bars @ 180 mm c/c

$$\text{Actual area of steel provided} = \frac{\pi/4 \times 10^2 \times 1000}{180} = 436.33$$

$$\text{Actual \% of steel provided} = \frac{100 \times 436.33}{1000 \times 150} = 0.29\%$$

$$\text{percentage of steel originally assumed} = 0.3\%$$

The design is safe from serviceability cond.

Area of
Steel ~~for~~ along ~~span~~ long span

$$M_y = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_c k} \right)$$

$$\Rightarrow 10.02 \times 10^6 = 0.87 \times 415 \times A_{st} \times 141 \times \left(1 - \frac{A_{st} \times 415}{1000 \times 141 \times 20} \right)$$

$$\Rightarrow 196.83 = A_{st} - 1.47 \times 10^{-4} A_{st}^2$$

$$\Rightarrow 1.47 \times 10^{-4} A_{st}^2 - A_{st} + 196.83 = 0$$

$$\Rightarrow A_{st} = 202.88 \text{ mm}^2$$

$$\text{spacing of } 8 \text{ mm } \phi \text{ bars} = \frac{\frac{\pi}{4} \times 8^2 \times 1000}{202.88} = 247.75 \text{ mm}$$

provide 8mm ϕ bars @ 240mm c/c

Actual area of steel provided

$$= \frac{\frac{\pi}{4} \times 8^2 \times 1000}{245} = 208.33 \text{ mm}^2$$

The code requires that the min area of steel should be $0.12\% b \times D = 0.0012 \times 1000 \times 170 = 204 \text{ mm}^2$

< A_{st} , in long span

< A_{st} in short span

curtail alternate bars at $\frac{1}{10}$ th of effective span in each dirⁿ as with clause D-2.11 of the code. Provide 50% of the ~~the~~ ~~moment~~ reinforcement maximum ~~the~~ steel at top near the supports to resist bending moment due to partial fixity. This steel is provided in c/c from the face of the supports

The bars cannot be bent or curtailed because if 50% of long span bars are curtailed the remaining bars will be less than the min.

* At top at support, provide 50% of bars of respective span to take into account any possible ~~due to~~ ~~negative~~ ~~moment~~ ~~created~~ ~~in~~ ~~the~~ ~~middle~~ ~~of~~ ~~the~~ ~~span~~

Check for shear force at short edges

Max shear force intensity in either dirⁿ can be taken as $\frac{1}{2} W L_x$ where L_x is clear short span

$$\text{Max}^m \text{ SF} = \frac{1}{2} W L_x = \frac{1}{2} \times 12.38 \times 4$$
$$= 24.76 \text{ kN/m}$$

$$\text{Nominal shear stress } \tau_v = \frac{24.76 \times 1000}{1000 \times 150}$$
$$= 0.17 \text{ N/mm}^2$$

r. of tensile steel near support

$$= \frac{100 A_{st}}{b d}$$

Check for development length = 100 x

$$\text{long span } V_u = 12.38 \times \frac{4.15}{2} = 25.69 \text{ kN}$$

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$
$$= 0.87 \times 415 \times 208.33 \times 141 \left(1 - \frac{208.33 \times 415}{1000 \times 141 \times 20} \right)$$
$$= 10.28 \text{ kN-m}$$

Assuming $\phi = 8$

$$l_d = \text{development length} = \frac{0.87 f_y \phi}{4 \times 25}$$
$$= \frac{0.87 \times 415 \times \phi}{4 \times 1.2 \times 1.6}$$
$$= 47.01 \phi$$

As per code

$$l_d \leq \frac{1.3 M_1}{V} + l_e$$

$$\Rightarrow 47.01 \phi \leq \frac{1.3 \times 10.28 \times 10^6}{25.69 \times 10^3} + 8\phi$$

$$\Rightarrow 39.01 \phi \leq 520.20$$

$$\Rightarrow \phi \leq 13.34 \text{ mm} \quad (\phi_{\text{provided}} = 8 \text{ mm}) \quad (\text{ok})$$

Short span $V_u = \frac{12.38 \times 4.15}{2} = 25.69 \text{ kN}$

$$\begin{aligned} M_u &= 0.87 f_y A_s d \left(1 - \frac{A_s f_y}{b d f_{ck}} \right) \\ &= 0.87 \times 415 \times 436.33 \times 150 \left(1 - \frac{436.33 \times 415}{1000 \times 150 \times 20} \right) \\ &= 23.62 \text{ kN-m} \end{aligned}$$

Assuming $l_e = 8\phi$

As per code

$$l_d \leq \frac{1.3 M_1}{V} + l_e$$

$$\Rightarrow 47.01 \phi \leq \frac{1.3 \times 23.62 \times 10^6}{25.69 \times 10^3} + 8\phi$$

$$\Rightarrow 39.01 \phi \leq 1195.25$$

$$\Rightarrow \phi \leq 30.64 \text{ mm} \quad (\phi_{\text{provided}} = 10 \text{ mm}) \quad (\text{ok})$$

check for shear

This is critical along long span.

$$\begin{aligned} \text{Shear stress } (T_v) &= \frac{V_u}{b d} = \frac{25.69 \times 10^3}{1000 \times 141} \\ &= 0.182 \text{ N/mm}^2 \end{aligned}$$

$$\% \text{ of steel near support} = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 208.33}{1000 \times 141}$$

$$= 0.148\%$$

$$\tau_c = 0.28 \text{ N/mm}^2$$

for a slab thickness of 170mm $k = 1.25$

$$\tau'_c = 0.28 \times 1.25 = 0.35 \text{ N/mm}^2 > \tau_v \quad \dots (\text{ok})$$

$\therefore$ The slab is safe in shear

check for deflection

The deflection shall be checked along short span

$$\text{Basic span ratio} = 20$$

$$\% \text{ of tensile reinforcement} = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 436.33}{1000 \times 150}$$

$$= 0.291\%$$

service stress $= 240 \text{ N/mm}^2$ in Fe 415 grade steel

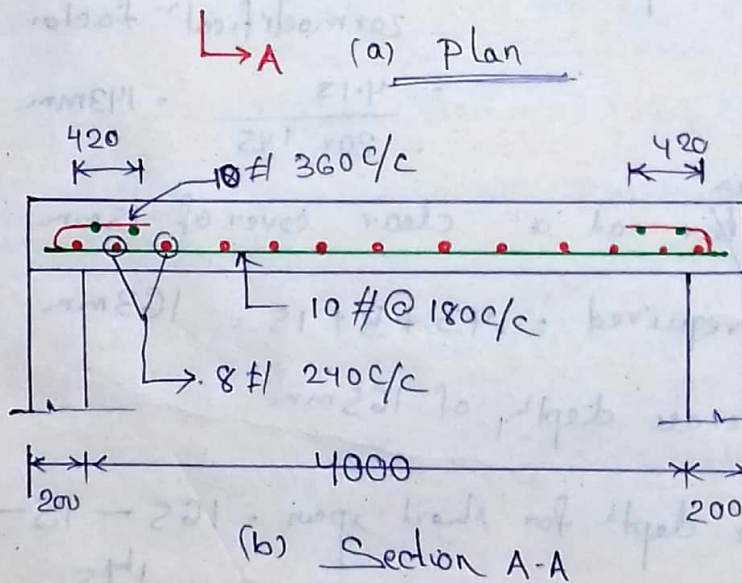
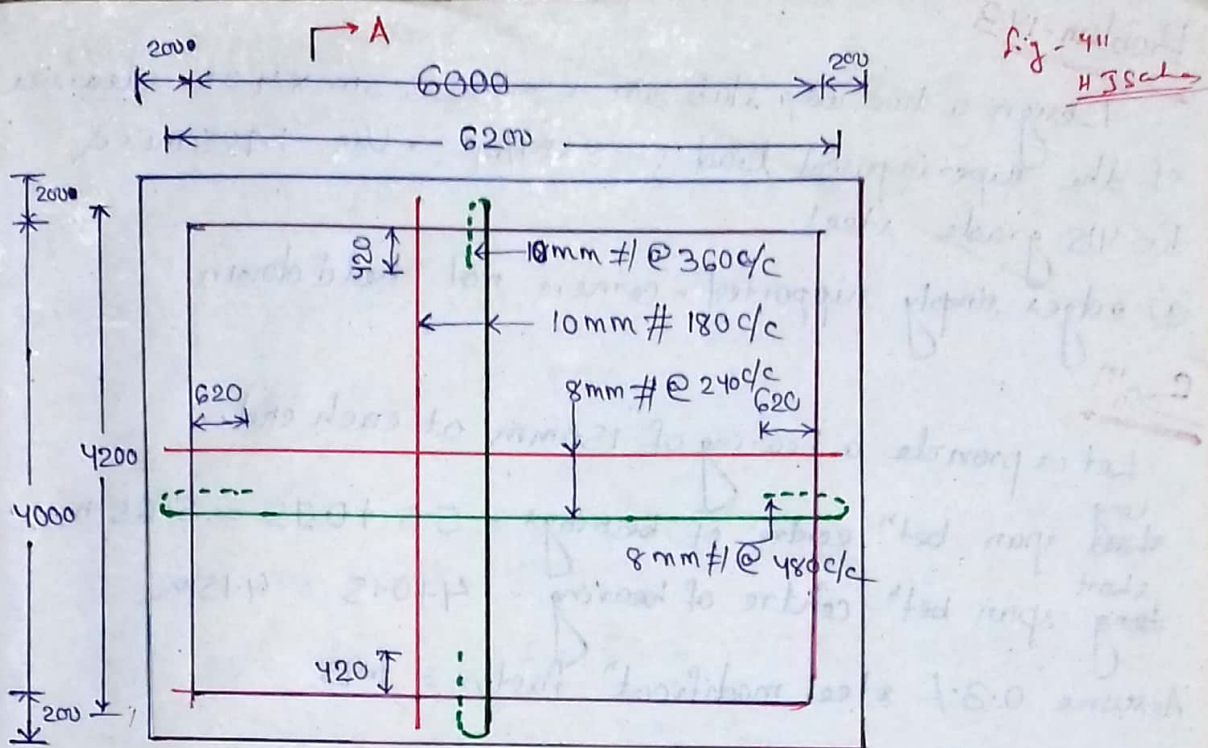
modification factor: 1.15

actual

$$\text{permissible span ratio} = 20 \times 1.15 = 30$$

$$\text{actual span ratio} = \frac{4150}{150} = 27.67 < 30$$

- (ok)



Q Design a cantilever slab to carry a live load of 3000 N/m^2 . The overhang of the slab is 1.25 m . Use M20 concrete & Fe415 steel.

Solⁿ Calcuⁿ Depth of slab
Assume 0.2% steel, modification factor = 1.65

$$\frac{\text{Basic span}}{\text{effective depth}} = 7$$

$$\text{Effective depth required} = \frac{\text{span}}{7 \times \text{modification factor}}$$

$$= \frac{1250}{7 \times 1.65} = 108.23 \text{ mm}$$

Providing $10 \text{ mm } \phi$ bars at a clear cover of 15 mm .

Effective cover to reinforcement = $15 + 5 = 20 \text{ mm}$

Overall depth required = $108.23 + 20 = 128.23 \text{ mm}$

Provide an overall depth of 130 mm & Reduce the depth to 100 mm

Actual effective depth (d) = $130 - 20 = 110 \text{ mm}$

Design load (W_u) & Factored Bending moment (M_u)

$$\text{DL of slab} = \left(\frac{0.13 + 0.1}{2} \right) \times 1 \times 25 = 2.875 \text{ kN/m}$$

$$\text{finishing load} = 1 + 1 = 2 \text{ kN/m}^2 \quad \left(\begin{array}{l} \text{Assume} \\ \text{finishing load} \\ = 1 \text{ kN/m}^2 \end{array} \right)$$

$$\text{Live load} = 3 \times 1 = 3 \text{ kN/m}^2$$

$$\text{Total load on the slab} = 5.875 \text{ kN/m}$$

$$\text{factored load (W_u)} = 1.5 \times 5.875 = 8.8125 \text{ kN/m}$$

$$\text{Max^m factored B.M (M_u) } = \frac{W_u l^2}{2} = \frac{8.8125 \times 1.25^2}{2} = 6.885 \text{ kN-m}$$

Min^m effective depth required.

Equating $M_{u,lim}$ to M_u

$$0.138 f_{ck} b d^2 = 6.885 \times 10^6$$

$$\Rightarrow 0.138 \times 20 \times 1000 \times d^2 = 6.885 \times 10^6$$

$$\Rightarrow d^2 = 49.95 \text{ mm} < d_{provided} = 110 \text{ mm} \quad (OK)$$

Calculatⁿ of Area of steel

Area of steel is given by

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$\Rightarrow 6.885 \times 10^6 = 0.87 \times 415 \times A_{st} \times 110 \left(1 - \frac{A_{st} \times 415}{1000 \times 110 \times 20} \right)$$

$$\Rightarrow 1.886 \times 10^{-4} A_{st}^2 - A_{st} + 173.36 = 0$$

$$\Rightarrow A_{st} = 179.43 \text{ mm}^2$$

~~calculate spacing for steel~~

Let us take 10mm ϕ bars for main reinforcement

$$\text{spacing of main bars} = \frac{\frac{\pi}{4} \times 10^2}{179.43} \times 1000 = 437.72 \text{ mm}$$

check for spacing

Max^m spacing of bars shall not exceed

$$(i) 3d = 3 \times 110 = 330 \text{ mm}$$

$$(ii) 300 \text{ mm}$$

Hence provide 10mm ϕ bars @ 300 mm c/c.

$$\text{Actual Area of main bars provided} = \frac{\frac{\pi}{4} \times 10^2}{300} \times 1000 = 261.80 \text{ mm}^2$$

calculate distribution area of steel.

provide 0.12% temperature reinforcement in the transverse dirⁿ, that is,

$$= \frac{0.12}{100} \times \frac{(130 + 110)}{2} \times 1000 = 144 \text{ mm}^2$$

Let us take 8mm ϕ bars for distribⁿ reinforcement

$$\text{spacing of distribⁿ bars} = \frac{\frac{\pi}{4} \times 8^2 \times (1000)}{144} = 349.06 \text{ mm}$$

Check for spacing

The spacing of the distribution bars shall not exceed

(i) $S_d = 5 \times 110 = 550 \text{ mm}$

(ii) 300 mm

Hence provide $8 \text{ mm } \phi$ bars @ 300 mm c/c .

Actual area of distribⁿ bars provided = $\frac{11 \times 8^2}{300} \times 1000 = 167.55 \text{ mm}^2$

Check for shear

S.F. at the support (V_u) = $w_u l = 88125 \times 1.25 = 11.02 \text{ kN}$

Nominal shear stress (τ_v) = $\frac{V_u}{bd} = \frac{11.02 \times 10^3}{1000 \times 110} = 0.10 \text{ N/mm}^2$

percentage of steel = $\frac{100 \times A_{st}}{bd} = \frac{100 \times 261.8}{1000 \times 110} = 0.23\%$

Shear strength of M20 concrete (τ_c) = 0.35 N/mm^2

but shear strength of slab (τ_c') = $k \times \tau_c$
 $= 1.3 \times 0.35 = 0.455 \text{ N/mm}^2$
 $> \tau_v$ (OK)

The cantilever slab is safe in shear and no shear reinforcement is necessary.

Check for development length

development length (l_d) is given by $\frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{0.87 f_y \phi}{4 \tau_{bd}}$

$= \frac{0.87 \times 415 \times \phi}{4 \times 1.2 \times 1.6}$

$= 47.01 \phi$

$= 47.01 \times 10 = 470.01 \text{ mm}$

$\approx 470 \text{ mm}$

The main bars must be embedded into the beam/ until upto a distance $l_d = 470 \text{ mm}$ from the fixed edge.

Check for deflection

$$\text{Basic } \frac{\text{span}}{d} \text{ ratio} = 7$$

$$\text{Percentage of steel} = \frac{100 \times A_{st}}{bd} = \frac{100 \times 261.8}{1000 \times 110} = 0.24$$

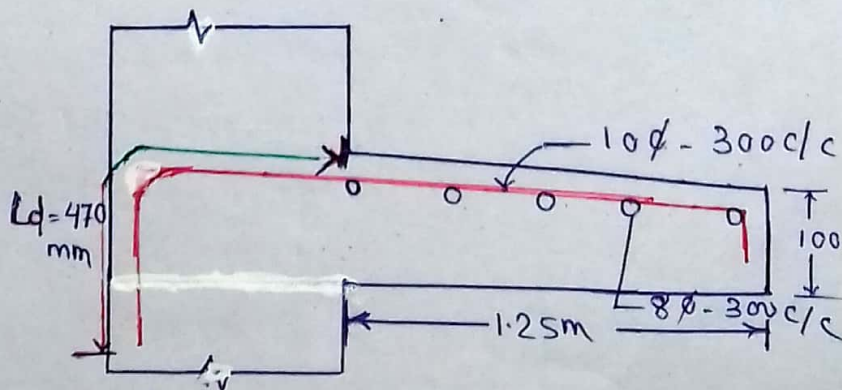
$$\begin{aligned} \text{service stress } (f_s) &= 0.58 f_y \times \frac{A_{st, \text{required}}}{A_{st, \text{provided}}} \\ &= 0.58 \times 415 \times \left(\frac{179.43}{261.8} \right) \\ &= 164.96 \quad (\text{say } 190) \end{aligned}$$

$$\text{So modification factor} = 2.0$$

$$\text{permissible } \frac{\text{span}}{d} \text{ ratio} = 7 \times 2.0 = 14.0$$

$$\text{actual } \frac{\text{span}}{d} \text{ ratio} = \frac{1250}{110} = 11.36 < 14.0 \quad (\text{OK})$$

Hence the cantilever slab is satisfactory from deflection consideration.



Reinforcement in cantilever slab

Column:

Assumptions in limit state of collapse : compression

Assumptions cl. 39.1

column may be

* A defined as an element used primarily to support axial compressive loads and with a height of at least 3 times its least lateral dimensions.

* A compression member whose effective length exceeds three times its least lateral dimension is termed as column.

* If the effective length is less than 3 times its lateral dimension, it is termed as pedestal.

* The pedestal may be designed as an unreinforced compression block, however, minimum reinforcement according to the code requirement shall be provided.

* The columns in a building usually carry axial compressive loads. In addition to the axial loads, columns usually carry some moments also.

* A column with only axial load is practically impossible. The moment in column with only axial load may be due to gravity loads, wind loads or earthquake loads.

* A column may be classified based on different criteria such as

(a) Shape of c/s

(b) Slenderness ratio

(c) type of loading

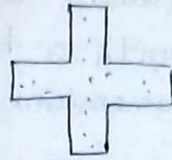
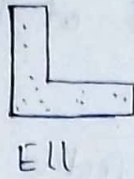
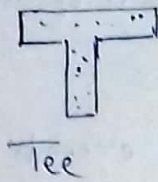
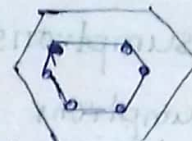
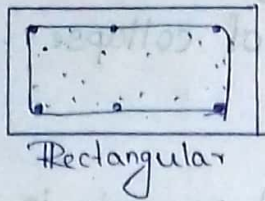
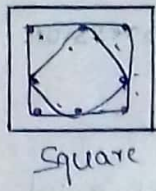
(d) pattern of lateral reinforcement

* The shape of a column may be square, rectangular, circular or any other shape depending on architectural requirement.

polygon $\leftarrow$ Hexagonal
octagonal

* Tee shape, ell shape or swastik columns are used to match with the architectural requirement.

* Some of the popular shapes of column



* A column may be classified as short or long column depending on its effective slenderness ratio

* The ratio of effective length of column to least lateral dimension is referred to as effective slenderness ratio.

* Short column are those which fail primarily due to material failure. Short columns do have buckling effect, but of a limited value.

* Long columns are those which fail due to material failure and also due to buckling.

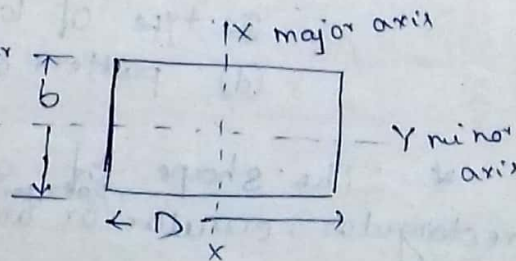
* According to IS: 456, a column may be considered as short when both slenderness ratios $\frac{L_{ex}}{D}$ and $\frac{L_{ey}}{b}$ are less than 12.

L_{ex} = effective length in respect of major axis

L_{ey} = " " " " " minor "

D = depth in respect of the major axis

b = width of the member.



* It shall be otherwise considered as a slender or a long column. However max^m slenderness ratio of a column should not exceed 50.

* Effective length of a column is defined as the length betⁿ the point of contraflexure of a buckled column.

$$\frac{lex}{D} > 12, \frac{ley}{b} < 12$$

$$\frac{lex}{D} < 12, \frac{ley}{b} > 12$$

$$\frac{lex}{D} > 12, \frac{ley}{b} > 12$$

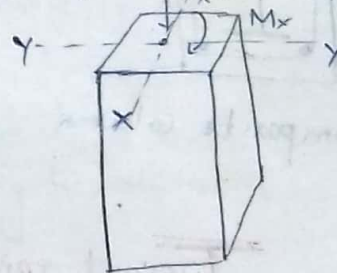
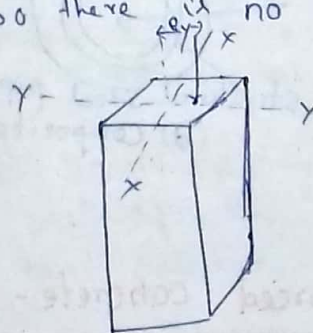
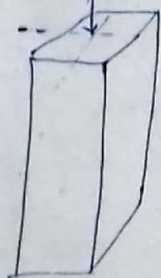


* A column may be classified as follows based on types of loading

- axially loaded column
- a column subjected to axial load and uniaxial bending
- a column subjected to axial load & biaxial bending

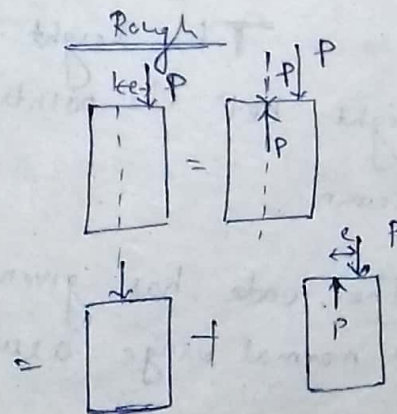
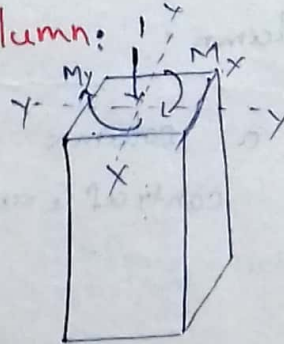
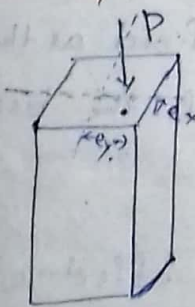
* Axially loaded columns are those in which the line of action of resultant thrust of the loads supported by a column coincides with the C.G. of the column c/c.

* As per IS:456-2000 there is no column which is truly axially loaded.



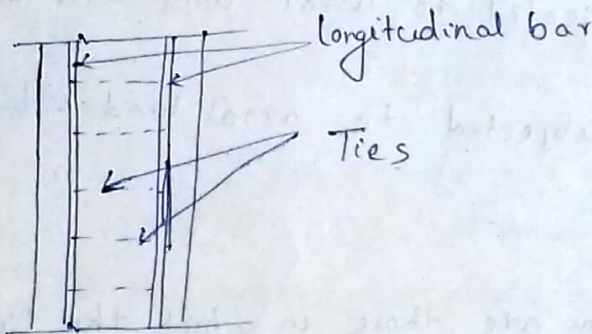
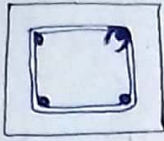
uniaxially eccentrically loaded column

Axially loaded column:

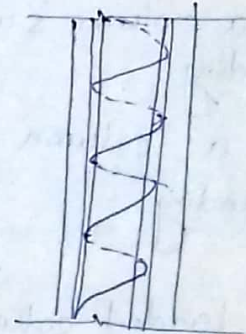
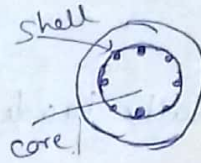


* A reinforced concrete column can also be classified according to the manner in which the longitudinal bars are laterally supported that is

- a) tied column
- b) spiral "
- c) Composite "

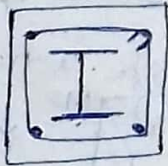


Tied column

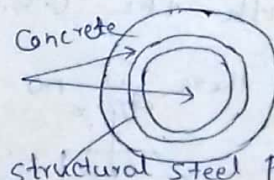


Spiral column

(c)



Composite column



(d) Composite column

Typical reinforced concrete - column

Effective height of a column:

The height of a column is defined as the height betⁿ the points of contraflexure of the buckled column.

* The code has given certain values of the effective height for normal usage assuming idealized end conditions :

~~at no~~ - Table No - 28.

These are briefly discussed as follows:

→ Tied Column:

* Fig shows a typical tied column where the transverse ties are used as lateral reinforcement restraint.

The function of the ^{longitudinal &} transverse ties are as follows:

(i) When the load commences on the column, they support the longitudinal bars so

sen
g
28.12

* RC columns concrete has a high compressive strength & a low tensile strength. Hence theoretically concrete should need no reinforcement when it is subjected to compression.

* Though a column is mainly a compression member, it is liable to some moment due to eccentricity of loads or transverse loads or due to its slenderness ratio.

* Such moments may occur in any dirⁿ and so it is necessary to maintain the posⁿ of the longitudinal reinforcement near all faces of the column.

* These reinforcements provide form the longitudinal steel.

* In order to maintain the position of the longitudinal reinforcement in the form of lateral ties or spirals at close pitch.

* The transverse reinforcement also exists in confining the concrete.

Specification for min^m & max^m reinforcement

26.5.3

* As per IS 456 a reinforced concrete column shall have min^m longitudinal reinforcement & the c/s area of such reinforcement shall not be less than 0.8% nor more than 6% of the c/s area of the column required to transmit all the loading.

* The object of stipulating a max^m percentage of steel is to provide re. make provision to prevent buckling of the column due to any accidental eccentricity of the load on it.

* The object of stipulating a max^m percentage of steel is to provide reinforcement within such a limit to avoid congestion of reinforcements which would make it very difficult to place concrete & consolidate it.

Clear cover

for slab = 15 mm

for beam = 25 mm

for column = 40 mm

" foundⁿ = 40-60 mm

for lintel = 20 mm

If the size of the column ≤ 200 mm provide : 25 mm & bars ≥ 25 mm

If the size of the column > 200 mm : 40 mm

IS Recommendations Regarding Longitudinal Reinforcement

cl - 26.5.3.1

Min^m Number of bars

1) Square column (4)



2) Rectangular " (4)



3) Circular " (6)

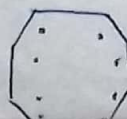


4) polygonal column

1) Hexagonal column (6)



2) Octagonal " (8)



Transverse reinforcement

26.5.3.2

- * The longitudinal reinforcement should be laterally tied by transverse links to provide a restraint against outward buckling of each of the longitudinal bars.

↓
← 12mm
transverse ← $c_m - d_s$

Unsupported length of column: 25.1.3

- * The unsupported length of a column shall be taken as the clear distance betⁿ slabs, beams or other members that provide lateral support to the column.

- * If the haunches in the beams at the columns or capital/drop panel in flat slab constructⁿ are present, the unsupported length shall be measured from the bottom of the haunch or capital/drop

- * It is the distance from the above floor to below the beam/slab.

- * The unsupported length, l of a compression member shall be taken as the clear distance betⁿ end restraints

Slenderness limits for column 25.3

- * The unsupported length betⁿ end restraints shall not exceed 60 times the least lateral dimension of a column.

- * If, in any given plane, one end of a column is unrestrained, its unsupported length, l shall not exceed $\frac{100b^2}{D}$

where b = width of that c/s.

D = depth of the c/s measured in the plane under consideration

Min^m eccentricity

c1-25.4

All columns shall be designed for min^m eccentricity, equal to the unsupported length of column/sav plus lateral dimension, subjected to a min^m of 20mm.

Short Axially loaded members in compression

c1-39.8

when min^m eccentricity $\nless 0.05 \times$ lateral dimension
the members may be designed by the following eqn

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

where P_u = axial load on the member

f_{ck} = characteristic compressive strength of the concrete.

A_c = Area of concrete

f_y = characteristic strength of the compression reinforcement.

A_{sc} = area of longitudinal reinforcement for column

Problem-1

A short column of size ϕ 230 mm $\times$ 350 mm is subjected to a factored load of 1500 kN. If the unsupported length of the column is 3.2 m, find out design moment due to min^m eccentricity. Can we apply simplified formula of design in this case?

Solⁿ

Min^m eccentricity about x-axis e_x , and about y-axis e_y are calculated and compared with respective sides D and b of the column as follows.

$$e_x = \frac{L}{500} + \frac{D}{30} = \frac{3200}{500} + \frac{350}{30} = 18.06 \text{ mm} < 20 \text{ mm}$$

$$e_x = 20 \text{ mm}$$

$$0.05 D = 0.05 \times 350 = 17.5 \text{ mm} < e_x$$

$$e_y = \frac{L}{500} + \frac{b}{30} = \frac{3200}{500} + \frac{230}{30} = 14.06 \text{ mm} < 20 \text{ mm}$$

$$e_y = 20 \text{ mm}$$

$$0.05 b = 0.05 \times 230 = 11.5 \text{ mm} < e_y$$

$$M_{ux} = M_{uy} = 1500 \times 0.02 = 30 \text{ kN-m}$$

Design the column for

$$P_u = 1500 \text{ kN}$$

$$M_{ux} = 30 \text{ kN-m}$$

$$M_{uy} = 30 \text{ kN-m}$$

As $e_x > 0.05 D$ and $e_y > 0.05 b$, simplified formula of design cannot be applied.

Problem: 2

Design the reinforcement in a column of size $400\text{mm} \times 600\text{mm}$ subjected to an axial load of 2000 kN under service load ^{dead and live}. The column has an unsupported length of 3m and is effectively held in position & restrained against rotation at both ends. Use M20 grade concrete & Fe 415 grade steel ^{page-91}

Solⁿ

Size of the column = $400 \times 600\text{mm}$

$b = 400\text{mm}$ $D = 600\text{mm}$

factored load (P_u) = $1.5 \times 2000\text{ kN} = 3000\text{ kN}$

unsupported length of the column (l) = 3m

effective length with respect to major / x-axis (l_{ex}) = $0.65l$
= $0.65 \times 3000 = 1.95$

" " " " minor / y-axis (l_{ey}) = $0.65l$
= 1.95m

slenderness ratio with respect to major axis (x-axis)

$$= \frac{l_{ex}}{D} = \frac{1.95}{0.6} = 3.25 < 12$$

slenderness ratio with respect to minor axis (y-axis)

$$= \frac{l_{ey}}{b} = \frac{1.95}{0.4} = 4.875 < 12$$

So the column may be considered as short.

In the dirⁿ of the longer dimension, min eccentricity is greater than the following

$$e_{x,\min} = \frac{L}{500} + \frac{D}{30} = \frac{2000}{500} + \frac{600}{30}$$

$$= 26\text{mm} > 20\text{mm}$$

$$\therefore e_{x,\min} = 26\text{mm}$$

In the dirⁿ of the shorter dimension, min eccentricity is greater than the following

$$e_{y,\min} = \frac{L}{500} + \frac{b}{30} = \frac{3000}{500} + \frac{400}{30}$$

$$= 19.33$$

$$\therefore e_{y, \min} = 20 \text{ mm}$$

$$0.05 b = 0.05 \times 400 = 20 \text{ mm}$$

$$0.05 D = 0.05 \times 600 = 30 \text{ mm}$$

$$A_x \quad e_{x, \min} \neq 0.05 D \quad \& \quad e_{y, \min} \neq 0.05 b$$

The column can be designed as axially loaded column by using the eqn

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$\Rightarrow P_u = 0.4 f_{ck} (A_g - A_{sc}) + 0.67 f_y A_{sc}$$

$$\Rightarrow 3000 \times 10^3 = 0.4 \times 20 (240000 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$\Rightarrow A_{sc} = 2238.39 \text{ mm}^2$$

provide 6 bars of 20 mm dia & 2 bars of 16 mm dia

$$(A_{sc \text{ provided}} \neq 2238.39 \text{ mm}^2)$$

Lateral ties

a) Dia of tie shall not be less than

$$(i) \frac{1}{4} \times \text{dia of largest longitudinal bar} = \frac{1}{4} \times 20 = 5 \text{ mm}$$

$$(ii) 6 \text{ mm}$$

provide 6 mm dia ties

b) Pitch / spacing of ties shall not exceed,

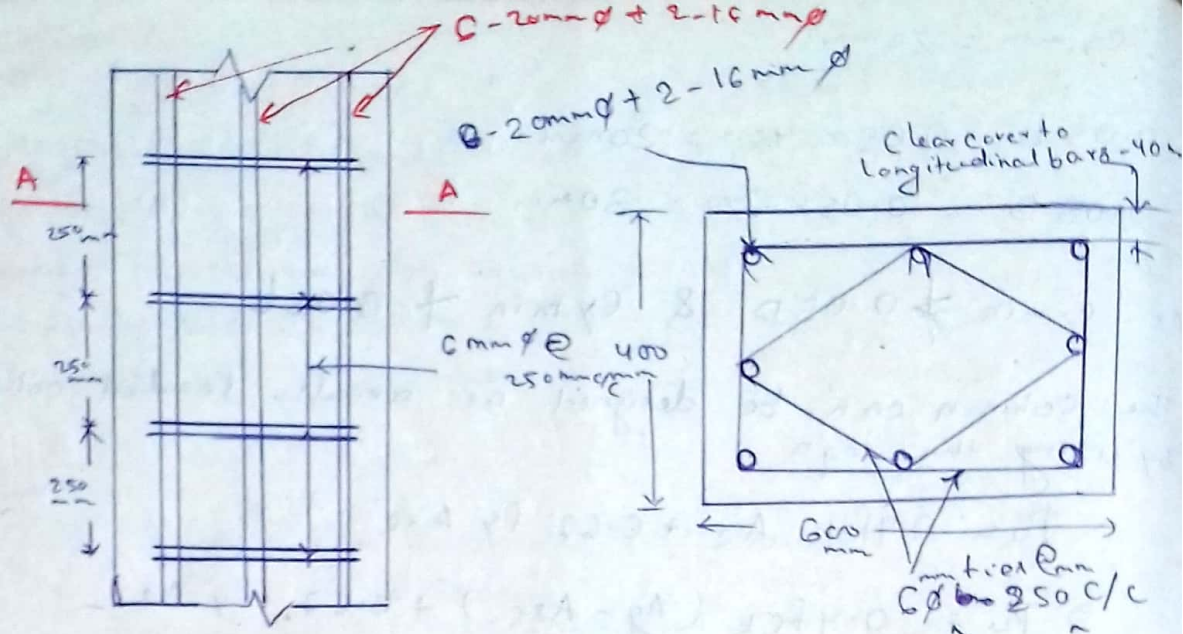
i) least lateral dimension of the column = 400 mm

$$(ii) 16 \times \text{dia of the longitudinal reinforcement}$$

$$= 16 \times 16 = 256 \text{ mm}$$

$$(iii) 300 \text{ mm}$$

let us provide 6 mm dia bar @ 250 mm c/c spacing



Details of reinforcement

Section AA

Problem - 3

A reinforced concrete ^{short} column $400\text{mm} \times 400\text{mm}$ has to carry an axial load of 1200kN . Find the area of steel required. Use M20 concrete and Fe 415 steel.

Solⁿ

Size of the column = $400\text{mm} \times 400\text{mm}$
 $\therefore b = D = 400\text{mm}$

Characteristic compressive strength of the concrete (f_{ck}) = 20N/mm^2

Characteristic strength of compression reinforcement = 415N/mm^2 (f_y)

axial load acting on the column = 1200kN

factored / ultimate load (P_u) = $1.5 \times 1200 = 1800\text{kN}$

ultimate load on the column given by the eqn

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$\Rightarrow 1800 \times 10^3 = 0.4 \times 20 \times (160000 - A_{sc}) + 0.67 \times 415 \times A_{sc}$$

$$\Rightarrow 1800 \times 10^3 = 64000 - 8 A_{sc} + 278.05 A_{sc}$$

$$\Rightarrow 1800 \times 10^3 = 128000 - 8 A_{sc} + 278.05 A_{sc}$$

$$\Rightarrow A_{sc} = 1925.57 \text{ mm}^2$$

provide 4 bars of 20mm ϕ & 4 bars of 16mm ϕ

$$(A_{sc, \text{provided}} = 2060.88 \text{ mm}^2)$$

As per code IS:456:2000

$$A_{sc, \text{min}} = 0.8\% \times A_g = 0.8\% \times (400 \times 400) = 1280 \text{ mm}^2$$

$$A_{sc, \text{max}} = 6\% \times A_g = 6\% \times (400 \times 400) = 9600 \text{ mm}^2$$

$$A_{sc, \text{min}} < A_{sc, \text{provided}} < A_{sc, \text{max}} \quad (\text{OK})$$

Lateral ties

a) Dia of tie shall not be less than
i) $\frac{1}{4}$ of dia of largest longitudinal bar = $\frac{1}{4} \times 20 = 5 \text{ mm}$

ii) 6mm

provide 6mm ϕ ties

b) Pitch/spacing of ties shall not exceed,

i) least lateral dimension of the column = 400mm

ii) 16 $\times$ smallest dia of the longitudinal reinforcement = $16 \times 16 = 256 \text{ mm}$

iii) 300mm

let us provide 6mm dia bar @ 250mm c/c spacing

Diagram: similar

Problem

A short RCC column is to carry a factored load of 1900 kN. If the column is to be square, design the column. Assume $e_{min} < 0.05D$. The materials are M20 grade concrete & mild steel.

Solⁿ

Here $e_{min} < 0.05D$. But $e_{min} = 20 \text{ mm}$. Therefore the side of the column shall be $\min^m \frac{20}{0.05} = 400 \text{ mm}$ i.e. $400 \text{ mm} \times 400 \text{ mm}$

As there is no restriction on size of the column, we may assume $\min^m$ percentage of steel to have economy in sectⁿ. Assume 0.8 percent steel.

$$\text{Then } A_{sc} = 0.008 A_g$$

$$A_c = A_g - A_{sc} = 0.992 A_g$$

ultimate load on the column given by

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$\Rightarrow 1900 \times 10^3 = 0.4 \times 20 \times 0.992 A_g + 0.67 \times 250 \times 0.008 A_g$$

$$\Rightarrow 1900 \times 10^3 = 7.936 A_g + 1.34 A_g$$

$$\Rightarrow 1900 \times 10^3 = 9.276 A_g$$

$$\Rightarrow A_g = 204829.67 \text{ mm}^2$$

If the column is to be a square, the side of the column = 452.58 mm

Adopt ~~450 mm~~ ^{mm} 455 mm size column
455 mm $\times$ 455 mm

$$Gross \text{ area of concrete provided} = 455 \times 455 = 207025 \text{ mm}^2$$

$$\begin{aligned} \text{required} &= 452.8 \times 452.8 \\ &= 205027.84 \text{ mm}^2 \end{aligned}$$

Longitudinal area of steel required
 $= 0.8\% \times 205027.84$

$$= 1640.22 \text{ mm}^2$$

provide 6 no 20 mm dia bars giving $A_{sc} = 6 \times \frac{\pi}{4} \times 20^2$
 $= 1884.95 \text{ mm}^2$

The section is shown in fig. Note that the distance betⁿ the bars exceeds 300 mm on two parallel sides. Therefore, the arrangement of reinforcement should be changed.

Provide 4 no^s of 18 mm diameter bar plus 4 no. 16 mm dia bars giving

$$A_{sc, \text{ provided}} = 4 \left(\frac{\pi}{4} \times 18^2 + \frac{\pi}{4} \times 16^2 \right) = 1822.12 \text{ mm}^2$$

Lateral ties

a) Dia of tier shall not be less than

i) $\frac{1}{4} \times$ dia of largest longitudinal bar

$$= \frac{1}{4} \times 18 = 4.5 \text{ mm}$$

ii) 6 mm

provide 6 mm ϕ tier

(b) Pitch/spacing of tier shall not exceed,

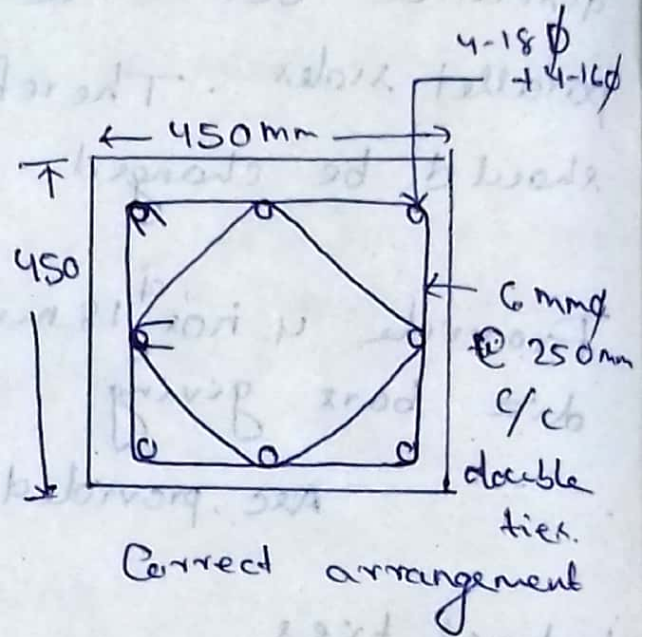
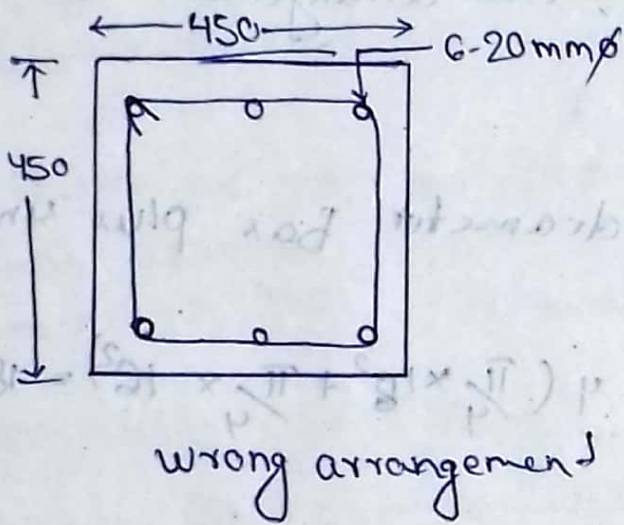
i) least lateral dimension of the column
 $= 455 \text{ mm}$

ii) $16 \times$ smallest dia of the longitudinal reinforcement $= 16 \times 16 = 256 \text{ mm}$

iii) 300 mm

Let us provide 6 mm ϕ tier about 250 mm c/c. This is shown in fig (b).

Note that distance betⁿ corner bars on one face equal to $455 - 80 - 18 = 357$. This is more than 48ϕ ($48 \times 6 = 288 \text{ mm}$). Therefore two sets of closed ties shall be used. Note that if this distance would be less than $(48 \times 6) \text{ mm}$ open ties for internal bars would be sufficient.



Q A 4m high column is effectively held in position at both ends and restrained against rotation at one end. Its size is restricted to 50cm. Calculate the reinforcement if it is required to carry a factored axial load of 1500 kN. Use M20 mix and Fe 415 grade steel.

Solⁿ

Effective height of a column effectively held in posⁿ at both ends and restrained against rotation at one end = $0.80L = 0.8 \times 4000 = 3200 \text{ mm}$

$$\text{slenderness ratio} = \frac{L_e}{D} = \frac{3200}{50} = 64 < 12$$

∴ It is a short column

Min^m eccentricity $e_{\min} = \frac{L}{500} + \frac{D}{30}$ is greater than the following

$$i) \frac{L}{500} + \frac{D}{30} = \frac{4000}{500} + \frac{500}{30} = 24.67 \text{ mm}$$

$$ii) 20 \text{ mm}$$

$$e_{\min} = 24.67 \text{ mm}$$

$$\text{Also } 0.05D = 0.05 \times 500 = 25 \text{ mm}$$

$$\text{As } e_{\min} > 0.05D$$

The strength of the column is given by

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$\Rightarrow 1500 \times 10^3 = 0.4 \times 20 ([400 \times 400] - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$\Rightarrow 1800 \times 10^3 = 8[160000 - A_{sc}] + 278.05 A_{sc}$$

$$\Rightarrow 1800 \times 10^3 = 1280000 - 8A_{sc} + 278.05 A_{sc}$$

$$\Rightarrow 520000 = 270.05 A_{sc}$$

$$\Rightarrow A_{sc} = 1925.56 \text{ mm}^2$$

As per provide

As per IS: 455 2000

$$A_{sc, min} = 0.8\% \times A_g = 1280 \text{ mm}^2$$

$$A_{sc, max} = 6\% \times A_g = 9600 \text{ mm}^2$$

provide $\phi = 22 \text{ mm}$ dia $A_{sc, provided} = 2280.80 \text{ mm}^2$

$$\therefore A_{sc, min} < A_{sc, provided} < A_{sc, max} \quad (OK)$$

Axially loaded spiral column:

The code gives the following expression

$$\frac{\text{volume of spiral in one loop}}{\text{volume of core for a length } p} \leq \frac{0.36 (A_g - 1)}{\frac{A_c}{f_y} \cdot \frac{f_{ck}}{f_y}}$$

$$= \frac{\pi (D_c - \phi_{sp}) a_{sp}}{\frac{\pi}{4} D_c^2 \times p}$$

$$= \frac{\pi D_c \times a_{sp}}{\frac{\pi}{4} \times D_c^2 \times p} \quad \text{neglecting the value of } \phi_{sp}$$

$$= \frac{4 a_{sp}}{p D_c}$$

$$A_c = \text{Area of the core} = \frac{\pi}{4} \times D_c^2$$

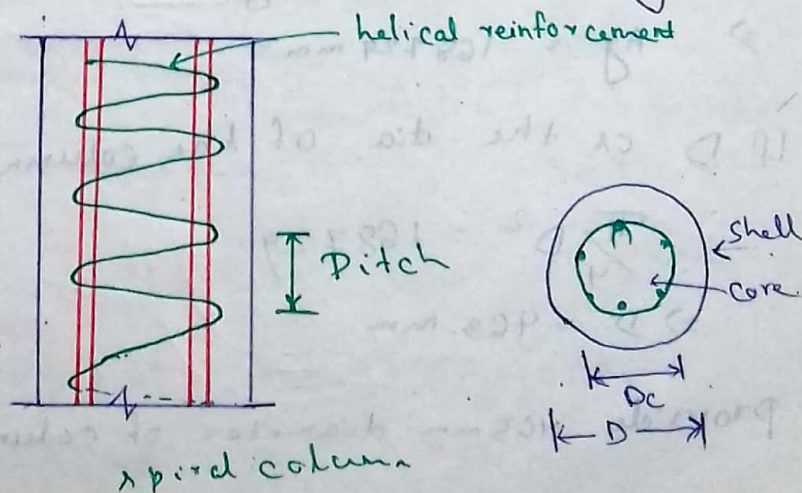
D_c = Diameter of the core, out to out of spiral

ϕ_{sp} = diameter of spiral wire

a_{sp} = area of spiral wire = $\frac{\pi}{4} \times \phi_{sp}^2$

p = pitch of helix

$$\frac{4 a_{sp}}{p D_c} > \frac{0.36 (A_g - 1)}{\frac{A_c}{f_y} \cdot \frac{f_{ck}}{f_y}}$$



Q Design a circular short column to carry an axial working load of 1200 kN. Assume $e_{min} < 0.05 D$

Use (a) lateral ties

(b) helical reinforcement

* The materials are M20 grade concrete & HYSD reinforcement of grade Fe 415.
For lateral reinforcement mild steel may be used

(b) Helical reinforcement

$$\text{Factored Load } (P_u) = 1.5 \times 2000 = 1800 \text{ kN}$$

The column with helical reinforcement can support 1.05 times the load of a similar member with lateral ties.

Therefore

$$P_u = 1.05 [0.4 f_{ck} A_c + 0.67 f_y A_{sc}]$$

$$\Rightarrow 1800 \times 10^3 = 1.05 [0.4 \times 20 \times$$

Assume min^m steel = 0.8% of gross sectⁿal area of the column

$$\text{Then } A_{sc} = 0.008 A_g$$

$$A_c = 0.992 A_g$$

substituting we have

$$1800 \times 10^3 = 1.05 [0.4 \times 20 \times 0.992 A_g + 0.67 \times 415 \times 0.008 A_g]$$

$$\Rightarrow 1800 \times 10^3 = 10.668 A_g$$

$$\Rightarrow A_g = 168729 \text{ mm}^2$$

If D is the dia of the column

$$\frac{\pi}{4} D^2 = 168729$$

$$\Rightarrow D = 463 \text{ mm}$$

provide 465 mm diameter of column

Area of longitudinal steel = $0.008 \times A_g = 0.008 \times 168729$
 $= 1349.83 \text{ mm}^2$

Use 6-18mm ^{diameter} bars giving

$A_{sc} = 1526.81 \text{ mm}^2$

Assume 6mm MS bars for helix at 40mm clear cover.

The code requires that

$$p_s = \frac{\text{volume of spiral in one loop}}{\text{volume of core for a length } p} \geq 0.36 \left(\frac{A_g}{A_c} - 1 \right) \frac{f_{ck}}{f_y}$$

$A_g = \text{gross concrete area} = \frac{\pi}{4} \times 465^2 = 168729 \text{ mm}^2$

Core Diameter $D_c = 465 - 2 \times 40 + 2 \times 6 = 397 \text{ mm}$

$A_c = \text{core area of the core} = \frac{\pi}{4} \times 397^2$

$\frac{A_g}{A_c} = \frac{465^2}{397^2} = 1.37$

$\therefore 0.36 \left(\frac{A_g}{A_c} - 1 \right) \frac{f_{ck}}{f_y} = 0.36 (1.37 - 1) \times \frac{20}{250}$
 $= 0.010656$

$p_s = \frac{\text{volume of spiral in one loop}}{\text{volume of core for a length } p} = \frac{\pi \times \phi_{sp} (D_c - \phi_{sp})}{\frac{\pi}{4} \times D_c^2 \times p}$
 $= \frac{4 \phi_{sp}}{D_c p}$

$\therefore p_s = \frac{4 \times \frac{\pi}{4} \times 6^2}{397 \times p} \geq 0.36 (1.37 - 1) \times \frac{20}{250}$
 $= \frac{4 \times \frac{\pi}{4} \times 6^2}{397 \times p}$

As per code

$\frac{4 \times \frac{\pi}{4} \times 6^2}{397 \times p} \geq 0.010656$

$\Rightarrow 26.73 \geq p \Rightarrow p \leq 26.73 \text{ mm}$

The code requires that pitch P

i) $\nless 75 \text{ mm}$

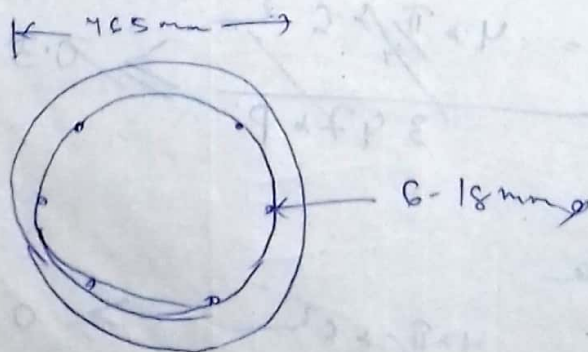
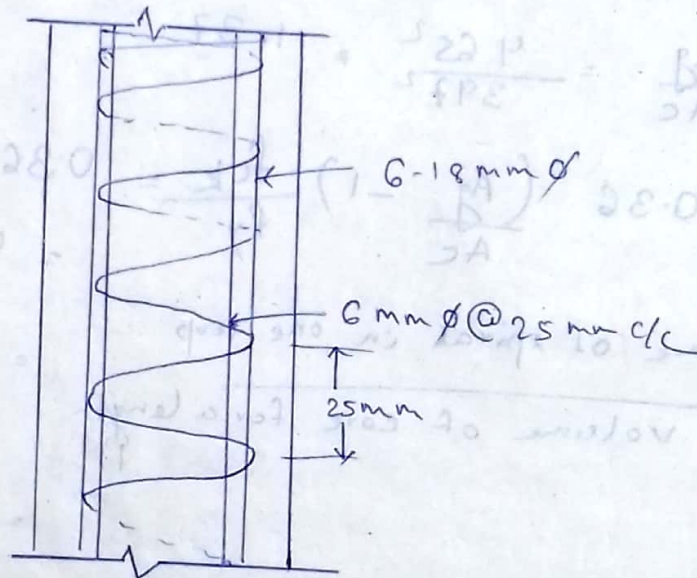
ii) $\nless \frac{1}{6} \times 397 = 66.16 \text{ mm}$

iii) $\nless 25 \text{ mm}$

iv) $\nless 3 \times 6 = 18 \text{ mm}$

Adopt 6mm helix with pitch @ 25mm c/c

* helical reinforced columns are very ductile as compared with the columns with lateral ties. Hence, columns with helical reinforcement are most desirable in highly seismic zone.



Footings

The loads from the supporting members of the structure like columns, walls, etc. known as super structure should be safely transmitted to the soil.

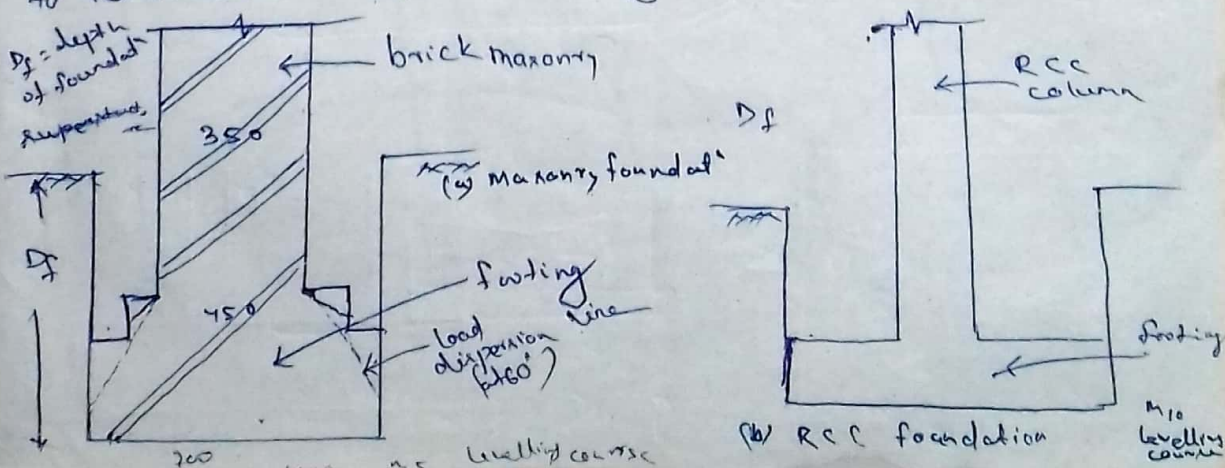
* The permissible pressure on soils is much less than that on the concrete column or a wall. Therefore it becomes necessary to spread the load over a sufficiently larger area.

* To transmit the load from the masonry wall for example, the area is increased slowly by providing a no of masonry steps so that the load dispersion lines remain within the steps as shown in fig (a).

* For the concrete column, the loaded area is suddenly increased by providing an interfacing element known as footing betⁿ the column & the soil. It is possible to increase the area suddenly like this because the bearing strength of concrete is sufficient & larger.

* In general a spread constructed in brickwork, stone masonry or concrete under the base of a wall or column for the purpose of distributing the load over a larger area is known as footing.

* Usually lean concrete (M5, M7.5, M10, PCC) is provided below the footing and above the soil to give more protection to reinforcement. & to properly level the ground.



Classification of Foundations:

The foundⁿ are classified as:

- 1) flexible & rigid
- 2) shallow & deep

(1) Flexible & rigid foundation:

This classification is based on the rigidity of the footings. Flexible footing is free to bend & follow the theoretical settlement profile such as rubber pad. Soil bearing pressure on such footings is always uniform. The rigid footing on the other hand does not bend and settles uniformly such as RCC footing.

(2) Shallow and Deep foundation

This classification is based on the ratio of depth of foundatⁿ D_f to the width of foundⁿ B .

* The foundation is classified as:

$$\text{shallow if } \frac{D_f}{B} \leq 2$$

$$\text{deep if } \frac{D_f}{B} > 2$$

In most practical cases for shallow foundatⁿ usually $\frac{D_f}{B} = 0.25 - 1.0$ where as for deep foundation this ratio is usually 5-20.

* Spread, isolated footings, combined footings, mat footing, raft etc are the examples of shallow foundⁿ, where as piles, piers, wells are the example of deep foundⁿ.

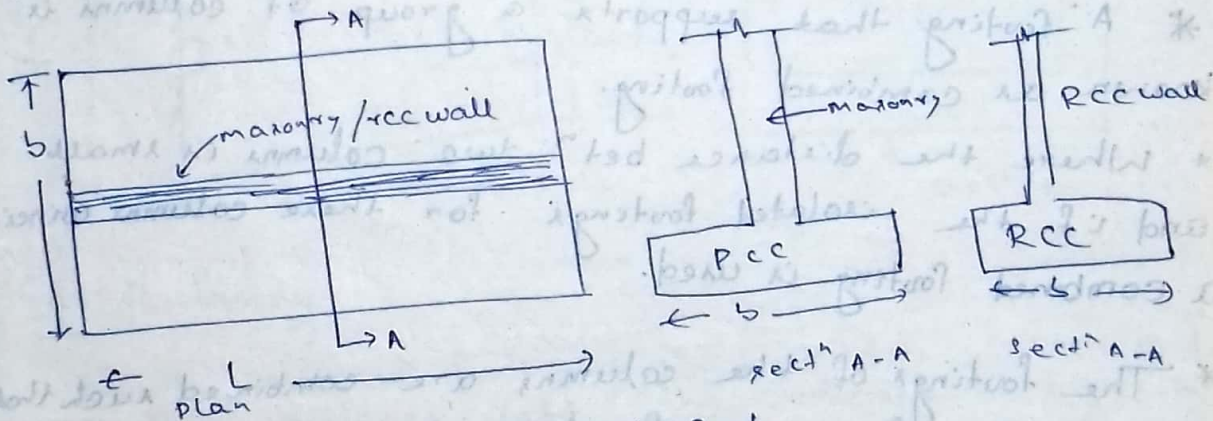
Types of Footings

Some of the common footings usual in general building construction are as follows:-

- (1) Continuous wall footing
- (2) Isolated footing
- (3) Combined "
- (4) Strap "
- (5) Strip footing
- (6) Raft foundation
- ✓ (7) pile foundation

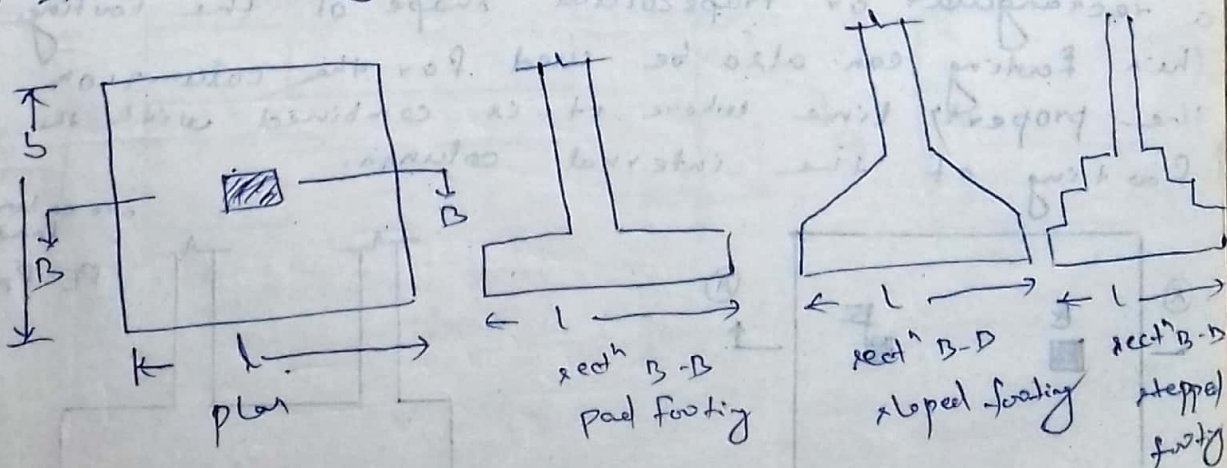
1) Continuous wall footing

A footing that supports a continuous long masonry or rcc wall is known as continuous footing. In this case, the width of the footing is very small than the length of the footing.



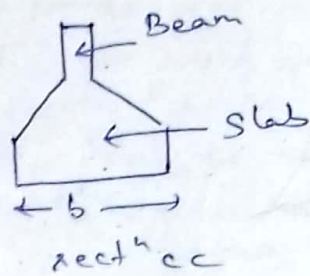
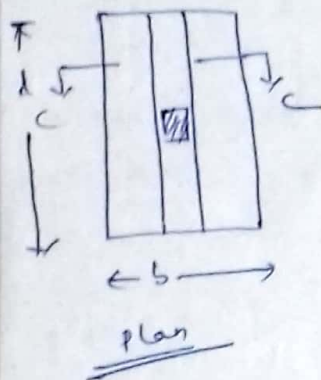
(a) Continuous wall footing

2) Isolated footing



An individual footing under a single column is known as an isolated footing.

* Where good soil & sufficient area is available, these footings are economical. These may be pad, slopped, steps or with isolated beam & slab type footings.

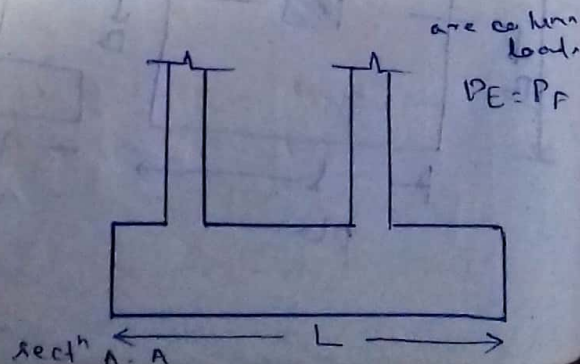
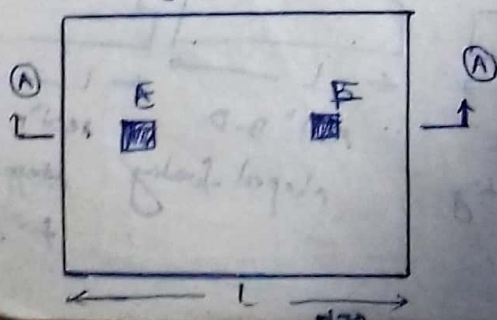


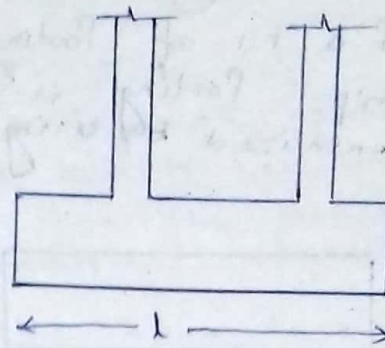
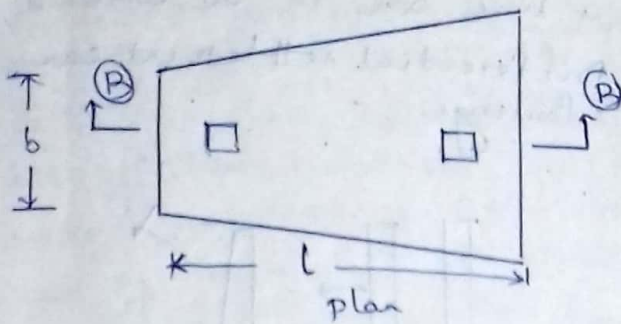
(b) Column footings

Isolated footing

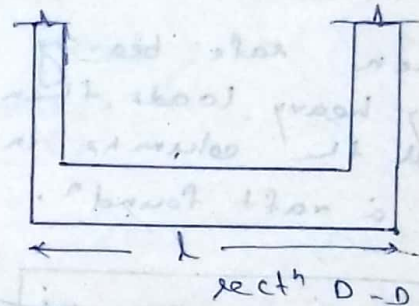
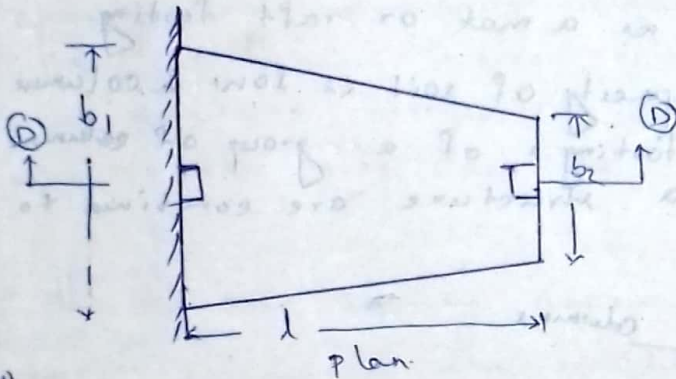
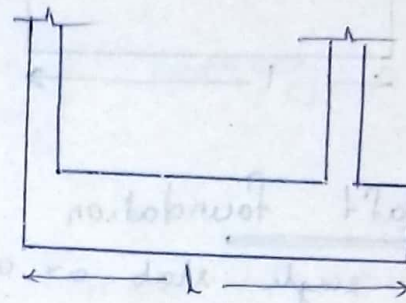
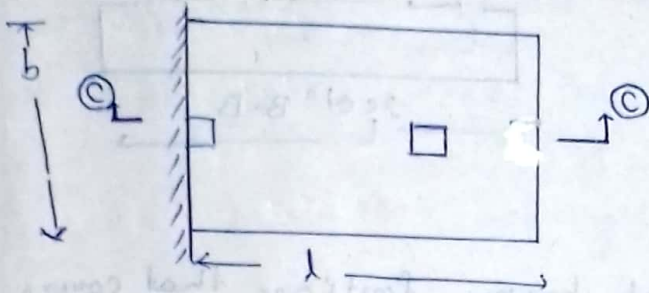
(3) Combined footing

- * A footing that supports a group of columns is known as combined footing.
- * Where the distance between two columns is small and if the isolated footings for these columns coincide a combined footing is used.
- * The footings of the columns are combined such that the centre of gravity of column loads and that of the combined footing coincide. This may result in a rectangular or trapezoidal shape of the footing. This footing can also be used for the column on the property line where it is combined with the footing of the internal column.



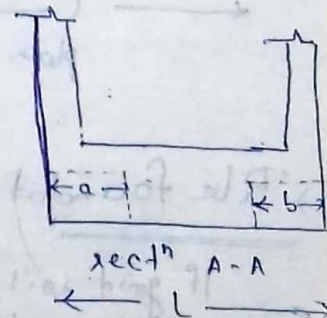
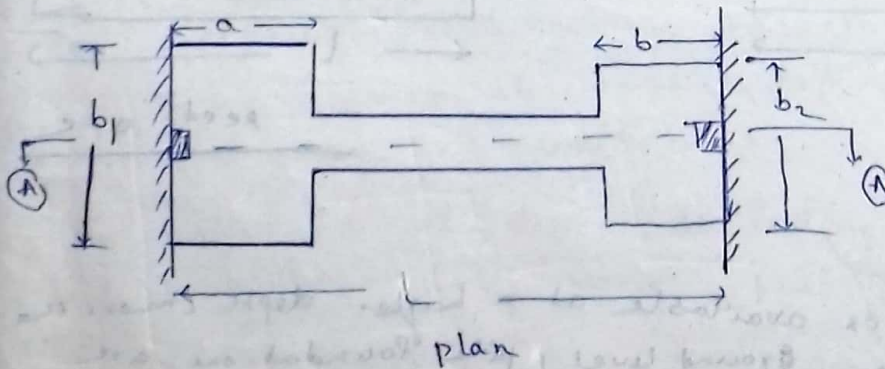


$$P_F > \frac{P_E}{2}$$



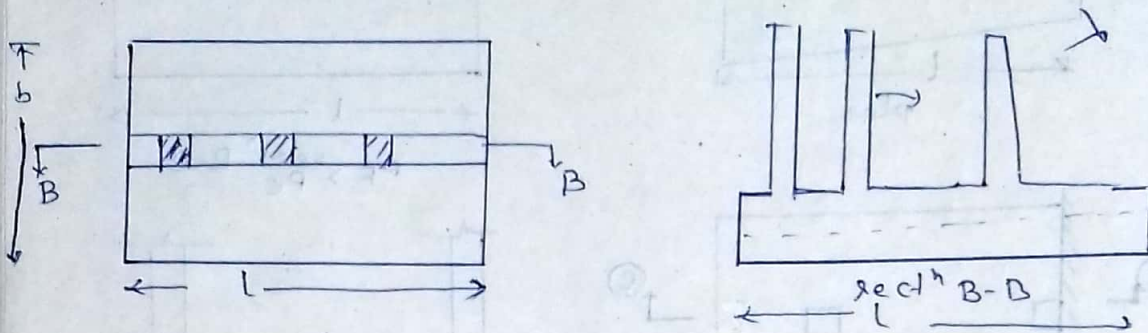
4) Strap footing

If a combined footing is required due to site condⁿ but the distance betⁿ the columns is large, a strap footing is used for economy.



5) Strip footing

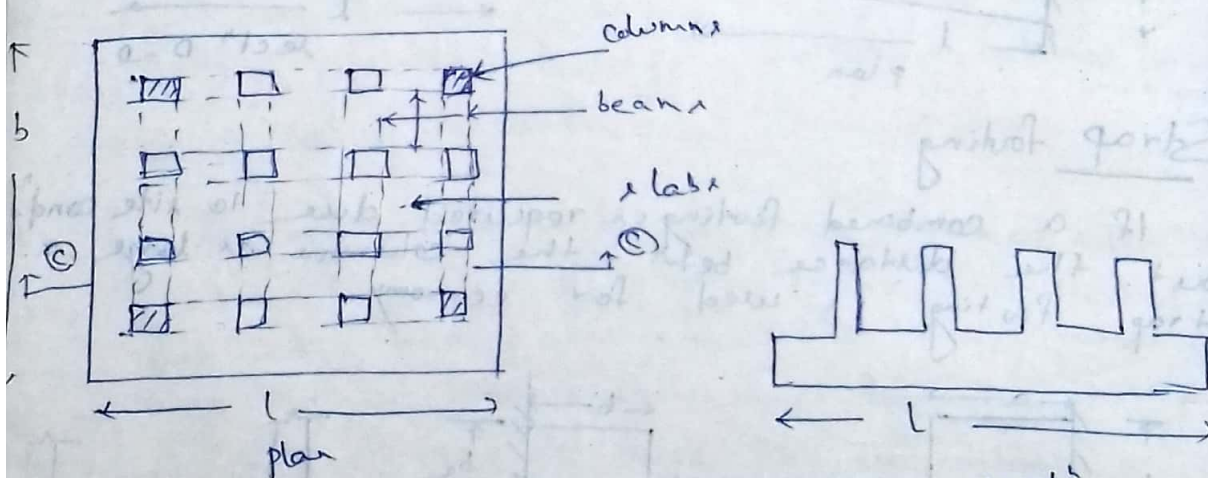
If a no of footings in a line are to be combined, a strip footing is used. Differential settlement can be minimized by using such footings.



6) Raft foundation

A single slab or a slab beam footing that covers the entire stratum beneath the entire area of the superstructure is known as a mat or raft footing.

When safe bearing capacity of soil is low & columns carry heavy loads, then footing of a group of columns or all the columns in a structure are combined to form a raft foundⁿ.



7) Pile foundation

If good soil is available at a higher depth (more than 3 m) below the ground level, pile foundations are economical.

* Pile transfer the loads from columns to the hard soil by ~~friction~~ and bearing and to the surrounding

Bearing Capacity of Soil.

* The size of foundation depends on the permissible bearing capacity of the soil. Total load per unit area under the footing must be less than the permissible bearing capacity of the soil to prevent excessive settlement. In general, foundations have to resist vertical load, horizontal load and moments.

* Two terms used for bearing capacity

- 1) Gross bearing capacity
- 2) Net bearing

* Gross bearing capacity is the total safe bearing capacity and net bearing capacity pressure at the bottom of the footing including the load of superstructure, weight of the footing and that of the earth lying over the footing.

* Net safe bearing capacity is the gross bearing capacity minus the weight per unit area dug out of trench.

Depth of Foundation

* IS: 1080-1985 requires that in all soils a min depth of 50 cm is necessary. However, if good rock is met at smaller depth, only removal of top soil may be sufficient.

* An estimate of depth of footing below ground level may be obtained by using the Rankine formula that is,

$$h = \frac{P}{\gamma} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2$$

h = minimum depth of found

p = gross bearing capacity

γ = density of soil

ϕ = angle of repose of soil

Problem-1

Determine the area and depth of foundation of a square column carrying 1000kN vertical load. The gross bearing capacity of soil is 100kN/m², density is 17kN/m³ and angle of repose is 29°.

Solution

Load on column $W = 1000\text{kN}$

$$\text{Aproximate area of footing} = \frac{W}{P} = \frac{1000}{100} = 10\text{m}^2$$

Minimum depth of foundation is given by

$$h = \frac{P}{\gamma} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2$$

$$= \frac{100}{17} \left(\frac{1 - \sin 29^\circ}{1 + \sin 29^\circ} \right)^2$$

$$= 0.71\text{m}$$

$$\text{Weight of foundation including earth} = 17 \times 10 \times 0.71 = 120.7\text{kN}$$

Total load on foundation = $1000 + 120.7 = 1120.7 \text{ kN}$

Area of foundation required = $\frac{1120.7}{100} = 11.21 \text{ m}^2$

60

Let us revise the area of foundation due to increased self weight of foundation and earth

weight of foundation and earth = $17 \times 11.21 \times 0.71 = 135.07 \text{ kN}$

Total load on foundation = $1000 + 135.07 = 1135.07 \text{ kN}$

Area of foundation required = $\frac{1135.07}{100} = 11.35 \text{ m}^2$

Adopt a foundation having an area of $3.4 \text{ m} \times 3.4 \text{ m}$ (11.56 m^2) at a depth of 0.71 m from the ground level.

Isolated Footings

* Isolated footings are most commonly used footings for R.C.C. columns because of the simplicity and economy.

* For any column footings, if provision of isolated footing is not possible due to geometry or any other reason, only then other types of footings may be thought.

* The isolated footings for the column may be a pad footing (constant thickness) or a sloped footing.

* It may be axially loaded or eccentrically loaded.

* Sometimes a pedestal is used to achieve economical design.

Problem: 2

Design a square spread footing to carry a column load of 1000 kN from a 40 cm square tied column containing 20 mm bars as the longitudinal steel. The bearing capacity of soil is 100 kN/m^2 . Consider base of footing at 1 m below the ground level. The unit weight of earth is 20 kN/m^3 . Use $f_{ck} = 15 \text{ N/mm}^2$, $f_y = 415 \text{ N/mm}^2$ and load factor of 1.5.

Solution

Size of footing

$$\text{Axial load} = 1000 \text{ kN}$$

$$\text{Approximate area of footing} = \frac{1000}{100} = 10 \text{ m}^2$$

$$\text{weight of footing including earth} = 10 \times 1 \times 20 = 200 \text{ kN}$$

$$\text{Total load on foundation} = 1000 + 200 = 1200 \text{ kN}$$

$$\text{Actual area of the footing} = \frac{1200}{100} = 12 \text{ m}^2$$

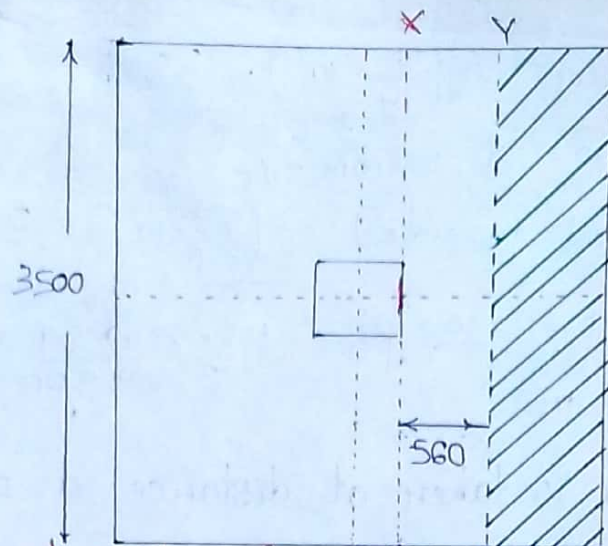
$$\text{provide } 3.5 \text{ m} \times 3.5 \text{ m square footing giving total area} = 12.25 \text{ m}^2 > 12 \text{ m}^2$$

Bending moment

The net earth pressure acting upward due to factored loads is

$$p = \frac{1000 \times 1.5}{12.25} = 122.45 \text{ kN/m}^2$$

(where 1.5 is the partial safety factor)



(61)

34.2.3

Fig (1) Critical section for moment and one-way shear.
Bending moment about an axis x-x passing through the face of the column as shown in fig (1)

$$= 122.45 \times 3.5 \times \left(\frac{35 - 0.4}{2} \right)^2 \times \frac{1}{2}$$

$$= 514.83 \text{ KN-m}$$

The effective depth required is

$$BM = 0.138 f_{ck} b d^2$$

$$\Rightarrow 514.83 \times 10^6 = 0.138 \times 15 \times 3500 \times d^2$$

$$\Rightarrow d = 266.57 \text{ mm}$$

provide D = 60mm
d = 550mm

Adopt 560mm effective depth and ~~600mm~~⁶¹⁰ overall depth. Increase depth is taken due to shear consideration.

Area of tension steel is given by

$$B.M = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$\Rightarrow 514.83 \times 10^6 = 0.87 \times 415 \times A_{st} \times 560 \left(1 - \frac{A_{st} \times 415}{3500 \times 560 \times 15} \right)$$

$$\Rightarrow 2546.29 = A_{st} - 14.11 \times 10^{-5} A_{st}^2$$

$$\Rightarrow 14.11 \times 10^{-5} A_{st}^2 - A_{st} + 2546.29 = 0$$

$$\text{or } A_{st} = 2645 \text{ mm}^2$$

$$\text{spacing of } 10 \text{ mm } \phi \text{ bars} = \frac{\left(\frac{\pi}{4} \times 10^2\right) \times 3500}{2645} = 103.92 \text{ mm}$$

provide 10mm ϕ bars @ 100 mm c/c

$$\text{Actual area of steel provided} = \frac{\left(\frac{\pi}{4} \times 10^2\right) \times 3500}{100} = 2748.89 \text{ mm}^2$$

$$\text{percentage of steel } p = \frac{100 A_{st}}{bd} = \frac{100 \times 2748.89}{3500 \times 560} = 0.14\% > 0.14\% \quad (\text{OK})$$

Shear one-way action.

The critical section is taken at distance d away from the face of column as shown in fig. 1

Shear force (V_u)

$$= 122.45 \times 3.5 \left[\left(\frac{3.5 - 0.4}{2} \right) - 0.56 \right]$$

$$= 424.29 \text{ kN}$$

$$\text{Nominal shear stress } (\tau_v) = \frac{V_u}{bd}$$

$$= \frac{424.29 \times 10^3}{3500 \times 560}$$

$$= 0.216 \text{ N/mm}^2$$

$$\text{percentage of tensile steel} = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 2645}{3500 \times 560}$$

$$= 0.135$$

So design shear strength of M15 concrete with 0.14% steel

$$\tau_c = 0.28 \text{ N/mm}^2$$

$$> \tau_v$$

OK

Shear two-way action

The critical section is taken at a distance $0.5d$ away from the face of column as shown in fig (2)

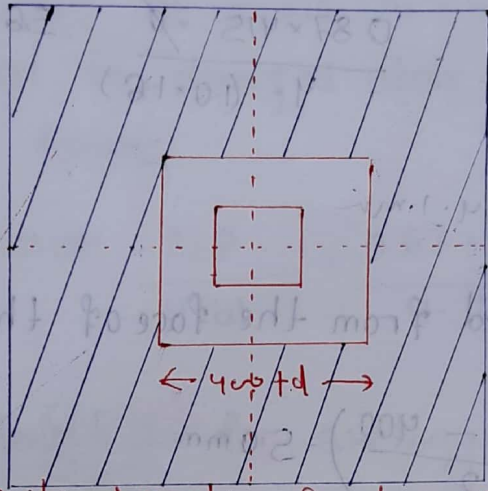


Fig (2) Critical section for two-way shear.

$$\text{Shear force } (V_u) = 122.45 \left[12.25 - (0.40 + 0.56)^2 \right]$$
$$= 1387.17 \text{ kN}$$

$$\text{Nominal shear stress } (\tau_v) = \frac{V_u}{b \cdot d}$$
$$= \frac{1387.17 \times 10^3}{4(400 + 560) \times 560}$$
$$= 0.645 \text{ N/mm}^2$$

Design

Shear strength of M15 concrete

$$\tau'_{vc} = k_s \tau_c$$

$$k_s = (0.5 + B_c)$$

$$B_c = \frac{\text{length of shorter side of column}}{\text{length of longer side of column}} = 1$$

$$k_s = 0.5 + 1 = 1.5$$

$$k_s = 1.5$$

$$\therefore k_s = 1.0$$

$$\tau'_{vc} = 1 \times 0.25 \sqrt{f_{ck}}$$
$$= 1 \times 0.25 \sqrt{15} = 0.97 \text{ N/mm}^2 > 0.645 \text{ N/mm}^2$$

OK

This shows that a footing having an effective depth 266 mm would not be safe in shear.

Development of reinforcement

Development length for 10mm bars

$$L_d = \frac{\sigma_s \phi}{4\tau_{bd}} = \frac{0.87 \times 415 \times \phi}{4 \times (1.0 \times 1.6)} = 56.41 \phi$$

$$\therefore L_d = 560 \text{ mm } 564.1 \text{ mm}$$

Actual embedment provided from the face of the column is

$$= \left(\frac{3500 - 400}{2} \right) - 50 \text{ mm}$$
$$= 1500 \text{ mm} > L_d \quad (\text{OK})$$

Load transfer from column to footing

Nominal bearing stress in the column concrete

$$\sigma_{br} = \frac{P_u}{A_c} = \frac{1.5 \times 1000 \times 10^3}{400 \times 400} = 9.38 \text{ N/mm}^2$$

where 1.5 is the partial safety factor

$$\text{Allowable bearing stress} = 0.45 f_{ck} = 0.45 \times 15 = 6.75 \text{ N/mm}^2$$
$$< 9.4 \text{ N/mm}^2 \quad (\text{NG})$$

Thus the column load cannot be transferred by bearing alone

$$\text{To carry the excess load } P = 9.4 \times 400 \times 400 - 6.75 \times 400 \times 400$$
$$= 424000 \text{ N}$$

$$\text{Required area of steel } A_s = \frac{424000}{0.87 \times 415} = 1525 \text{ mm}^2$$

Minimum $A_s = 0.5\%$ of column area

$$= 0.5\% \times 400 \times 400$$

$$= 800 \text{ mm}^2$$

$$< 1525 \text{ mm}^2$$

OK.

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provide 5-20mm bars as dowels. $A_s = 1570 \text{ mm}^2$. The stress in 20mm dowels must be developed above and below the junction of column and footing.

$$L_d \text{ for compression} = \frac{\sigma_s \phi}{476d} = \frac{0.67 \times 415 \phi}{4 \times (1.6 \times 1.25 \times 1)} = 34.75 \phi$$
$$= 34.75 \times 20 = 695 \text{ mm}$$

The available length L_1 for anchorage is

$$L_1 = 600 - 35 - 2 \times 10 - 20 = 520 \text{ mm} < 695 \text{ mm}$$
$$= 525 \text{ mm} < 695 \text{ mm} \rightarrow 510 \text{ mm} < 695 \text{ mm}$$

NG.

Let us provide smaller diameter bars as dowels so that the available vertical length in the footing is sufficient for anchorage.

Use 16mm bars as dowels

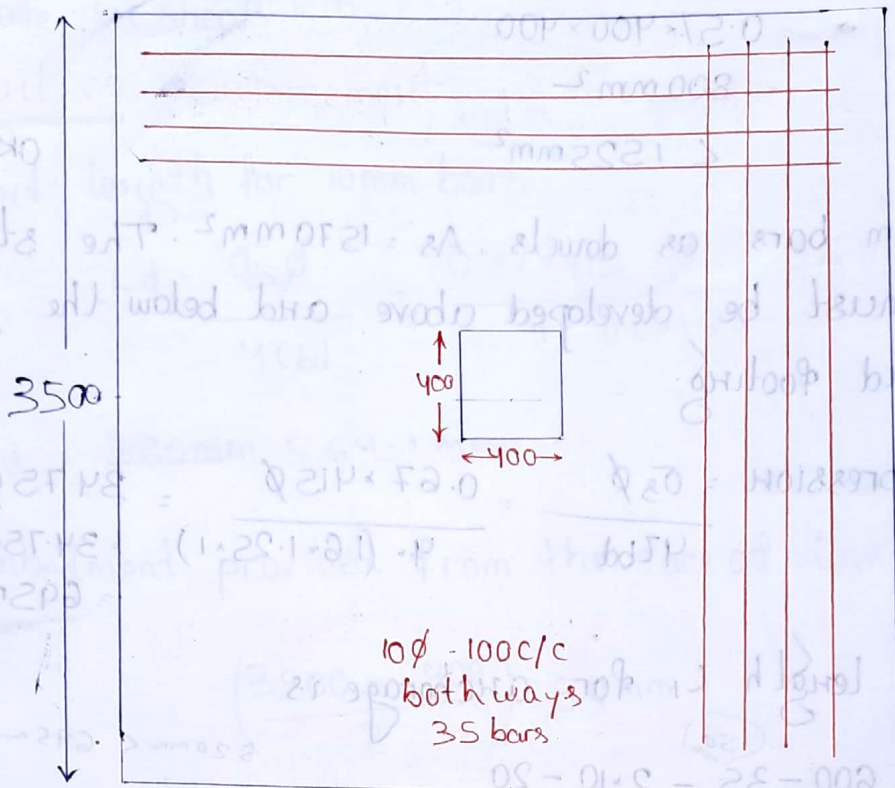
$$\text{Development length} = 34.75 \times 16 = 560 \text{ mm}$$
$$\approx L_1$$

OK

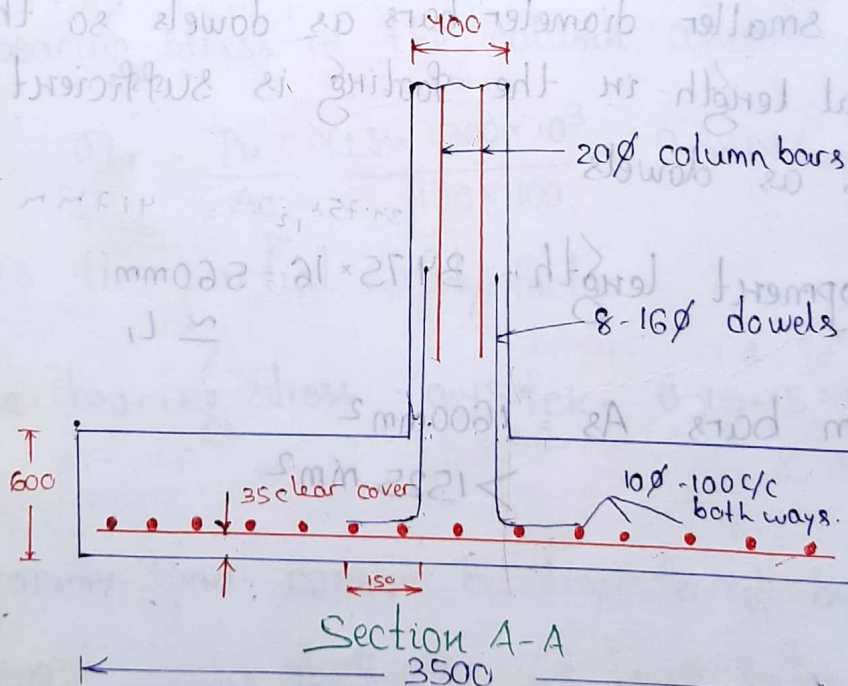
provide 8-16mm bars $A_s = 1600 \text{ mm}^2$

$$> 1525 \text{ mm}^2$$

OK



Plan of reinforcement



Sectional elevation showing reinforcement