CHAPTER - 1

Fundamental & Engineering Mechanics

Dt. 11.01.19
Friday

MECHANICS :-

Mechanics may be defind as the science which described and cradit the condition of rest or motion of the body under the action of force.

- Mechanics
  - Physical Mechanics
    - Partial Mechanics
    - Celestical Mechanics
    - Quantum Mechanics
    - Relative Mechanics
  - Engineering Mechanics
    - Solid Mechanics
      - Solid Body
      - Flexible body
    - Fluid Mechanics

- Fluid Mechanics
  - Ideal fluid
  - Viscus fluid
  - Compressed fluid

- Solid Body
  - Statics
  - Dynamics
    - Kinetics
    - Kinematics

Flexible body

- Strength of Materials
- Mechanics of Elastic bodies
- Mechanics of Plastic bodies

STATICS:-
It is the study of the force and condition of equilibrium of the body subjected to action of force.

DYNAMICS:-
It deals with the analysis of body in motion.

KINETICS:-
The study of motion of the body reference to the force causing motion.

KINEMATICS:-
It is the study of geometry of motion of the body without ~~for~~ reference of the force causing motion.
Ex:- Distance, displacement, velocity, speed, acceleration.

DISTANCE:-
The actual path covered by the body time interval 't'.
Unit is 'm'.

DISPLACEMENT:-
The shortest path covered by the body time interval 't'.

PARTICLE:-

It is defind as the material point without dimensioning but occupies on space.

MOTION:-

A body is said to be motion if it changes it position w.r.t reference

SPEED:-

The rate of change of distance w.r.t to time is called speed.

- Unit is m/s.

VELOCITY:-

$$V = \frac{\text{Distance}}{\text{time}} = m/s$$

ACCELERATION:-

The rate of change of velocity w.r.t time is called acceleration.

- Unit is m/sec^2

Dt 12.01.17
Saturday

PRIMARY DIMENSION :-
1. Length
2. time
3. Force or mass
4. Temperature
5. Electric charge.

FORCE :-
The action which produces or tends to produces change in the state of rest or uniform motion in a straight line.

CLASSIFICATION OF SYSTEM OF FORCES :-
1. Co-planner forces
2. collinear
3. concurrent.

1. CO-PLANNER FORCES :-
The forces acts on the same plane of the body is called coplanner forces.

2 COLLINEAR FORCES :-
The forces acts on the same line of the body is called collinear forces.

3. **CONCURRENT FORCES:-**
The forces acts at one point of the body is called concurrent forces.

4. **CONCURRENT COLLINEAR FORCES:-**
The forces acts at one point and lies on the same line is called concurrent collinear forces.

5. **CONCURRENT NON-COLLINEAR FORCES:-**
The forces acts at one point and not on the same line of the body.

6. **COPLANNER CONCURRENT FORCE:-**
The forces acts at one point and lies ~~on th~~ on the same plane of the body.

7. **COPLANNER NONCONCURRENT FORCES:-**
The forces do not acts at one point and lies on the same plane of the body.

8. **NON-COPLANNER CONCURRENT FORCES:-**
The forces acts at one point and do not lie on the same plane of the body.

9. NON-COPLANNER NON-CONCURRENT FORCES:

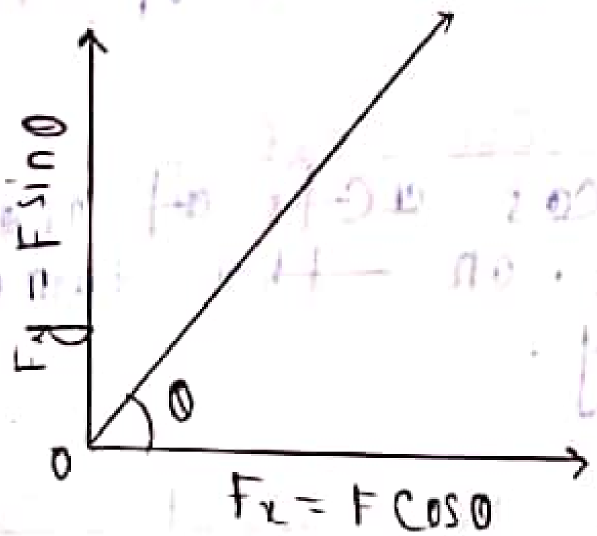
The forces do not acts at one point and do not lie on the same plane of the body.

Dt. 14 01 19
Monday

RESOLUTION OF FORCES:-

Disolving the given forces in two mutually perpendicular directions into two components has called resolution.

RESOLUTION OF SINGLE FORCE:-



Let,
a force 'F' is acting on a body at a point 'o' making angle 'θ' with horizontal.

The force act can be resurt into two components into two mutually perpendicular direction.

The component along x-x is $\boxed{F_x = F\cos\theta}$.

The component along ~~x~~ ~~ax~~ y-y is

$$\boxed{F_y = F\sin\theta}$$

Resultant $\boxed{R = \sqrt{F_x^2 + F_y^2}}$

The angle of the resultant with the horizontal is given by.

$$\boxed{\tan\theta = \frac{F_y}{F_x}}$$

$$\Rightarrow \boxed{\theta = \tan^{-1}\frac{F_y}{F_x}}$$

PROBLEM - 1

A force of 100N is acting at 30° with the horizontal find its horizontal & vertical components.

Ans:- Given data,

$F = 100N$

$\theta = 30°$

$F_x = ?$

$F_y = ?$

$$F_x = 100 \cos 30$$
$$= 50\sqrt{3}$$
$$= 86.60N$$

$$F_y = 100 \sin 30$$
$$= 50N$$

PROBLEM - 2

A force as a horizontal component of 100N and vertical component of 60N, find magnitude and direction of a given force.

Given data,

$$F_x = 100N$$
$$F_y = 60N$$

$$R = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{100^2 + 60^2}$$
$$= \sqrt{10000 + 3600}$$
$$= \sqrt{13600}$$

$$\theta = \tan^{-1} = \frac{F_y}{F_x}$$
$$\tan^{-1} = \frac{60}{100}$$
$$= \frac{6}{10}$$

PROBLEM - 3

A force of 200N acting at 60° with horizontal find its magnitude & direction of force.

Given data,

$F = 200N$

$\theta = 60°$

$F_x = ?$

$F_x = F\cos\theta$

$= 200\cos 60°$

$= 100N$

$F_y = F\sin\theta$

$= 200\sin 60°$

$= 173.20N$

$R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{100^2 + 173.20^2} = \sqrt{10000 + 29998.24}$

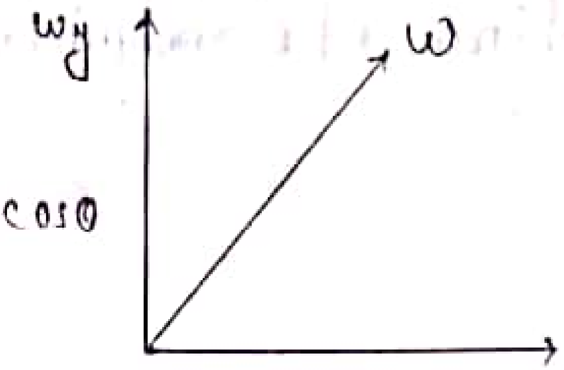
$= \cancel{199.99}$ 199.99

$\theta = \tan^{-1} = \frac{F_y}{F_x}$

$\tan^{-1} = \frac{173.20}{100}$

$= 59.99$

Dt. 14.01.19
Monday



Consider a body on a inclined plane the weight is acting vertically downward then, it is result into two component one is horizontal component.

$w_x = w\sin\theta$
$w_y = w\cos\theta$

PROBLEM - 4

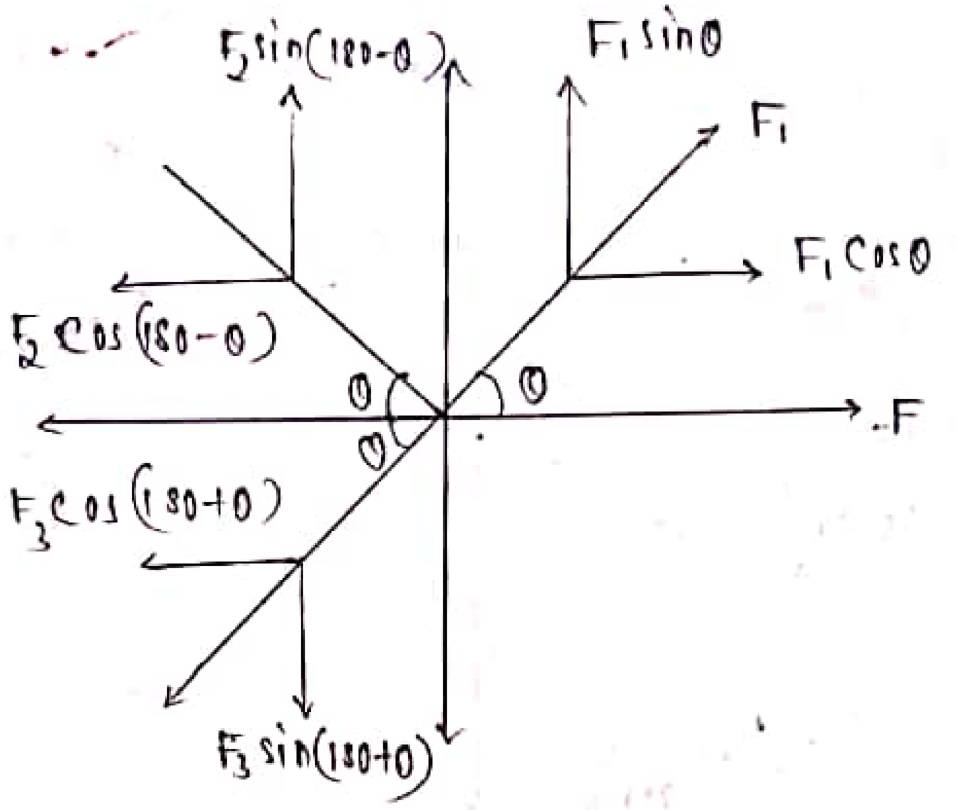
A weight of 400N placed on a inclined plane, the plane is a horizontal angle is 60°. What are the component of weight

Ans:- Given data,

$w = 400N$
$\theta = 60°$
$w_x = ?$, $w_y = ?$

$w_x = w\sin\theta$
$= 400\sin 60°$
$= 400 \times \frac{\sqrt{3}}{2} = 364.4\ N$

$w_y = w\cos\theta$
$= 400.$



$$\Sigma H = F_1 \cos\theta + F_2 \cos(180-\theta) + F_3 \cos(180+\theta)$$
$$\Sigma V = F_1 \sin\theta + F_2 \sin(180-\theta) + F_3 \sin(180+\theta)$$

PROBLEM-5

The following forces acts at a point

a. 30 KN inclined at 35° towards N of East.

b. 22 KN towards North.

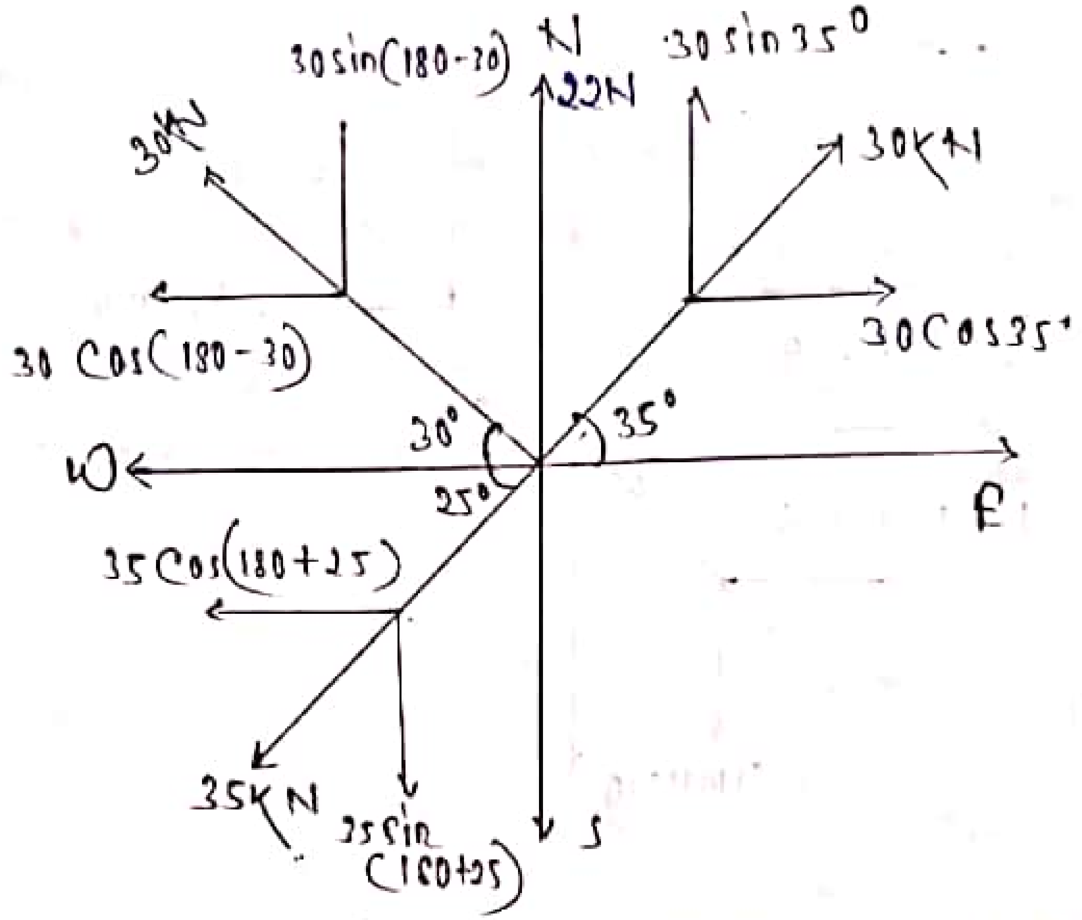
c. 30 KN inclined at 20° towards North of West.

d. 35 KN inclined at 25° towards south of West.

Find the magnitude and direction of resultant force,

$\theta = ?$, $R = ?$

Ans:-



$\Sigma H = 30\cos 35 + 30\cos(180-30) + 35\cos(180+25)$
$= -33.12 KN$

$\Sigma V = 22 + 30\sin 35 + 30\sin(180-30) + 35\sin(180+25)$
$= 39.41$

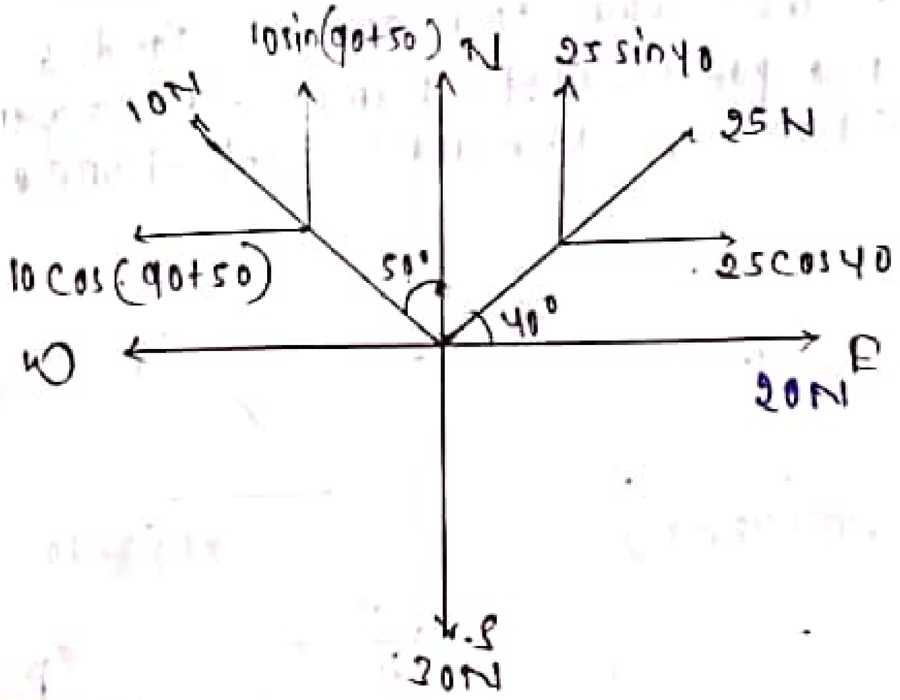
$R = \sqrt{\Sigma H^2 + \Sigma V^2}$
$= \sqrt{(-33.12)^2 + (39.41)^2}$
$= 51.48 KN$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$
$= \tan^{-1}\frac{39.41}{-33.12} = +49.94 = -50$

Dt. 25.01.19
Friday

PROBLEM:- 1

Resolving the forces horizontally.



Resolving the forces horizontally

$\Sigma H = 20 + 25\cos 40 + 10\cos(90+50)$

$= 31.49N$

Resolving the forces vertically,

$\Sigma V = 25\sin 40 + 10\sin(90+50) + 30\sin 270$

$= -7.50N$

$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$= \sqrt{(31.49)^2 + (-7.50)^2}$

$= 31.49N$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$= \tan^{-1}\left(\frac{-7.50}{31.49}\right)$

$= -13.39$

$360 - \theta$

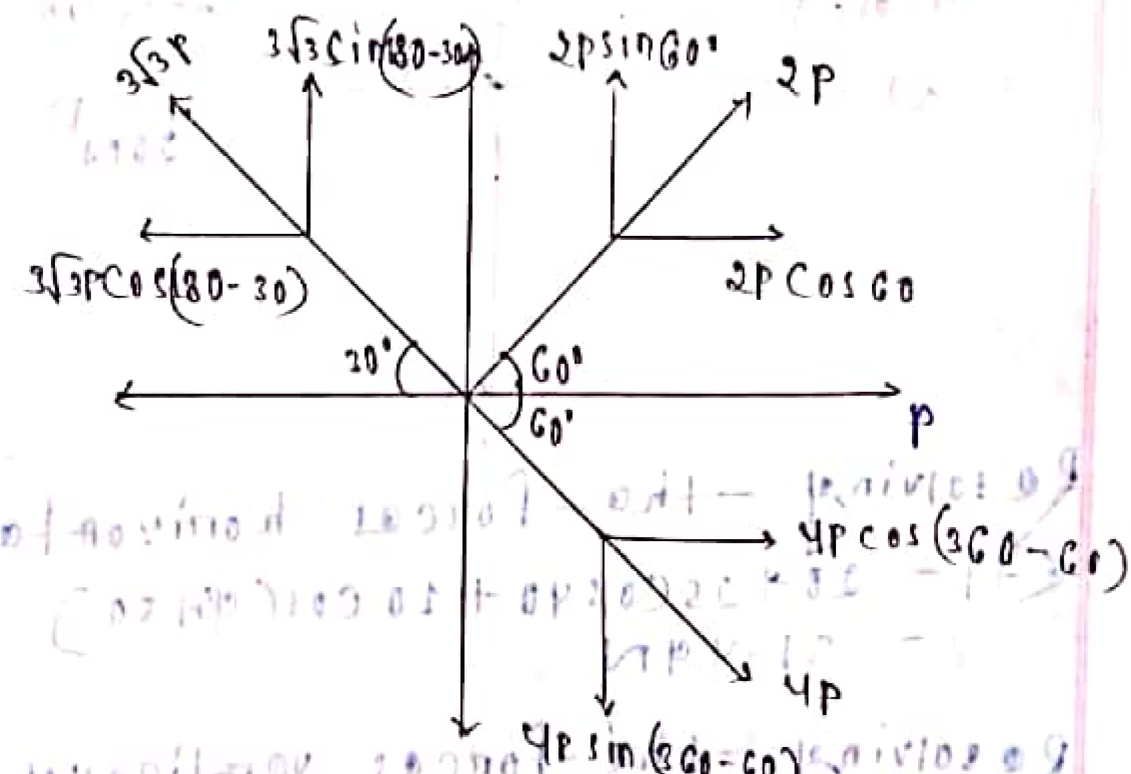
$= 360 - (-13.39)$

$= 373.39$

PROBLEM - 2

particle is acted by force 'p'; 2p, $3\sqrt{3}p$, 4p, the angle between 1st & 2nd is 60° and 3rd 90° & 3rd & 4th is 150° respectively. Find the magnitude & direction of the resultant force.

Ans:-



Resolving the forces horizontal,

$\Sigma H = p + 2p\cos 60 + 3\sqrt{3}\cos(180-30) + 4p\cos(360-60)$

$= -1.5p$

Resolving the forces vertical

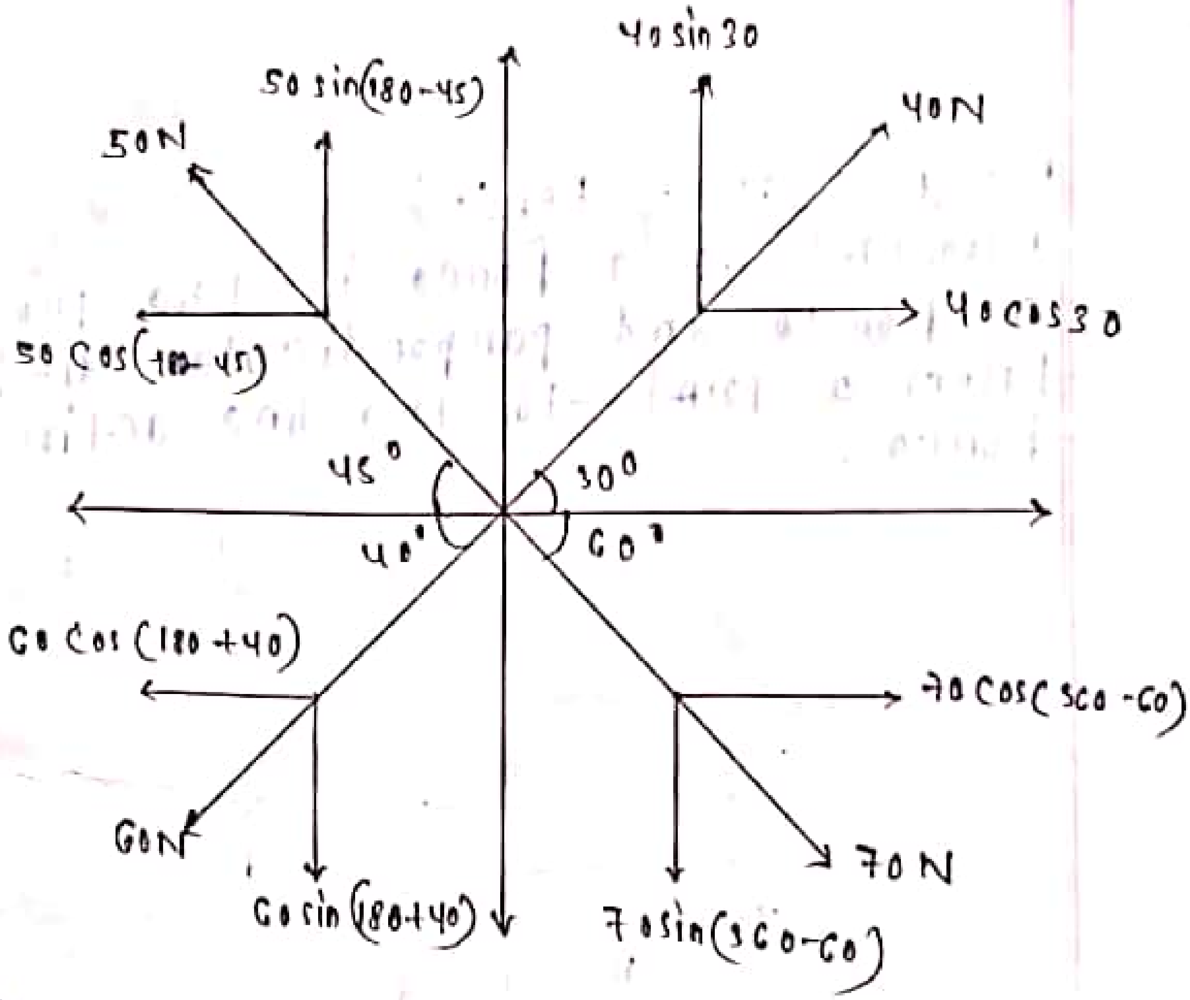
$\Sigma V = 2p\sin 60 + 3\sqrt{3}\sin(180-30) + 4p\sin(360-60)$

$= 0.86p$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$
$$= \sqrt{(-1.5)^2 + (0.86)^2}$$
$$= 1.729$$

$$\theta = -\tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$
$$= -\tan^{-1} \frac{0.86}{-1.5}$$
$$= \cancel{-0.57}\ -29.82$$

Dt. 28.01.19
Monday



$$\Sigma H = 40\cos 30 + 50\cos(180-45) + 60\cos(180+40) + 70\cos(360-60)$$
$$= -46.176$$

$$\Sigma V = 40\sin 30 + 50\sin(180-45) + 60\sin(180+40) + 70\sin(360-60)$$
$$= -43.833$$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{(-46.17)^2 + (-43.83)^2}$$

$$= 63.66.$$

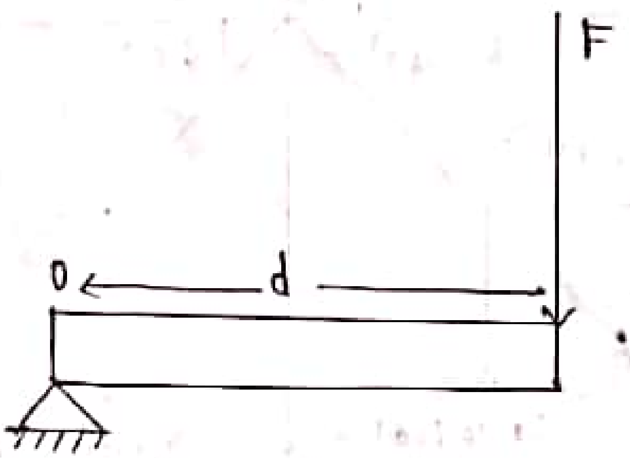
$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$= \tan^{-1} \frac{-43.83}{-46.17} = 0.949$$

$$= 43.51$$

Moment of a Force:-

Moment of a force is the product of a force and perpendicular distance from a point to the line action of the force.



$$m = F \times d$$

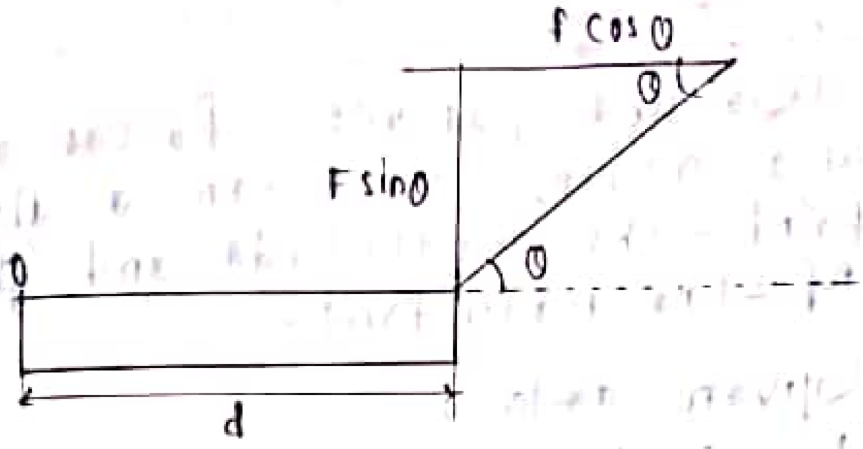
Where,

F = Force acting on the body

O = Fixed point of the body.

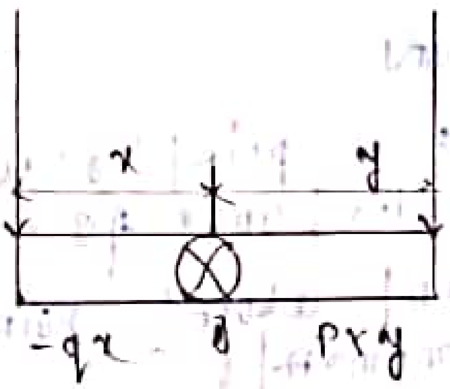
d = perpendicular distance from point 'O'

Moment of the force 'F' about point 'O'
$F \times d$.



$m = F\sin\theta \times d$

Law of Moments:-
The algebric sum of moments of all the coplannar forces about a point in that plane is zero.



The condition of law of moments is that all the coplanner forces should be equilibrium.

According to law of moment,

$P \times y + (-q \times x) = 0$

$P \times y - q \times x = 0$

$\Rightarrow P \times y = q \times x$

clockwise moment = Anticlockwise moment.

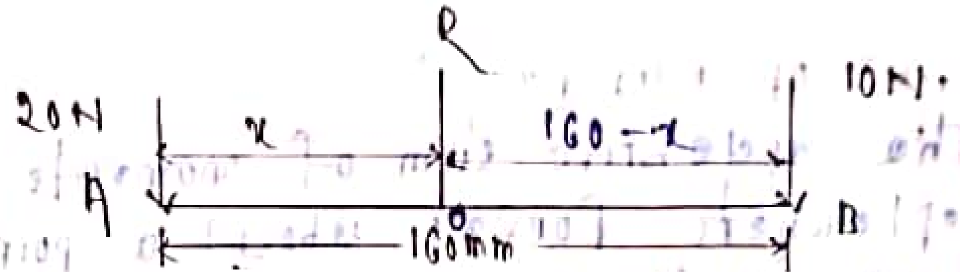
Dt 04.01.19
Tuesday

PROBLEM-1

Two like parallel forces of 20N & 10N are acting between a distance of 160mm. Find the magnitude and point of action of the resultant.

Ans:- Given data,

$F_1 = 20N$, $F_2 = 10N$



Let, R is the resultant with given force.

$R = F_1 + F_2$

$= 20+10 = 30N$

Let, th. take a point 'O' where resultant is act. the distance $AO = x$, $OB = (160-x)$

Taking moment about point 'O', according to law of moments.

clockwise moment = Anticlockwise moment

$10(160-x) = 20x$

$\Rightarrow 1600 - 10x = 20x$

$\Rightarrow 20x + 10x = 1600$

$\Rightarrow 30x = 1600$

$\Rightarrow x = \frac{1600}{30} = 53.30$

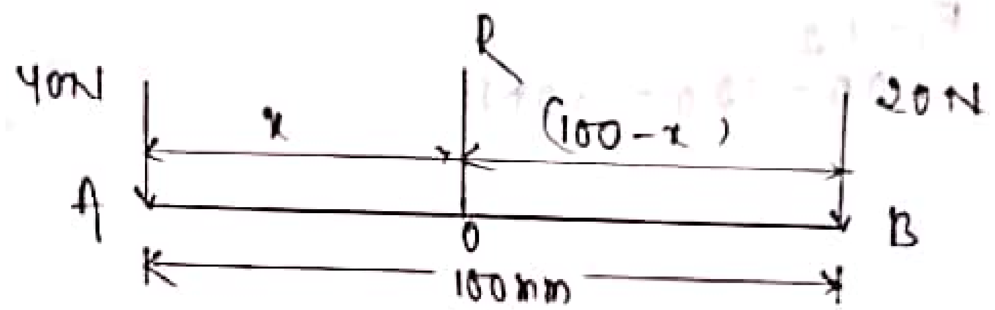
The point 'O' is 53.33 distance from 'A'.

PROBLEM - 2.

Two like parallel forces of 40N and 20N are acting between a distance of 100mm. Find the magnitude and point of action of the resultant.

Ans:- Given data,

$F_1 = 40N$, $F_2 = 20N$



$R = F_1 + F_2$
$= 40 + 20 = 60N$

Let, take a point 'O' where resultant is act, the distance $AO = x$, $OB = 100 - x$

Taking moment about point 'O', according to law of moments.

Clockwise moment = Anticlockwise moment

$20(100 - x) = 40x$

$\Rightarrow 2000 - 20x = 40x$

$\Rightarrow 40x + 20x = 2000$

$\Rightarrow 60x = 2000$

$\Rightarrow x = \frac{2000}{60} = 33.33$.

The point 'O' is 33.33 distance from 'A'.

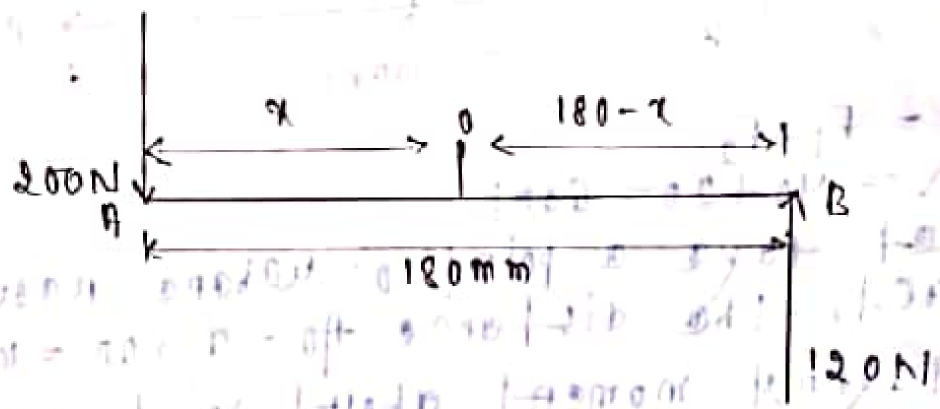
PROBLEM-3

Two unlike parallel forces of 200N and 120N are acting between a distance of 180 mm. Find the magnitude and point of action of the resultant.

Ans:- Given data,

$F_1 = 200N$, $F_2 = 120N$

$R = F_1 - F_2$
$= 200 - 120 = 80N$



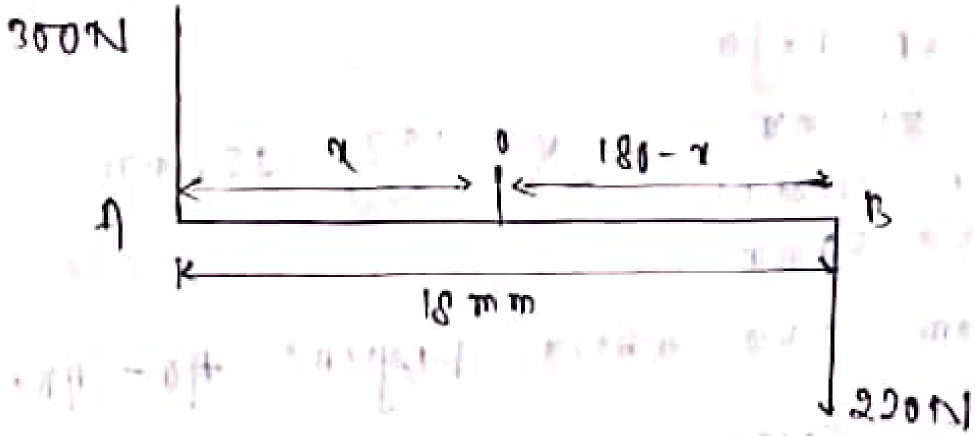
Consider that the resultant 'R' is acting 'x' distance from 'A'.

Both the forces are acting in anticlockwise direction there is no clockwise forces acting.

According to law of moments the algebric sum of moment meeting at a point is zero.

$= 200 \times x + 120(180 - x) = 0$
$= 200x + 21600 - 120x = 0$
$= 80x = -21600$
$= x = \frac{-21600}{80} = -270mm$

Negative sign indicate has the point 'O' in the left side of the 'A'.



$F_1 = 300N$, $F_2 = 220N$

$R = 300 - 220$
$= 80$

$300 \times x + 220(180 - x) = 0$

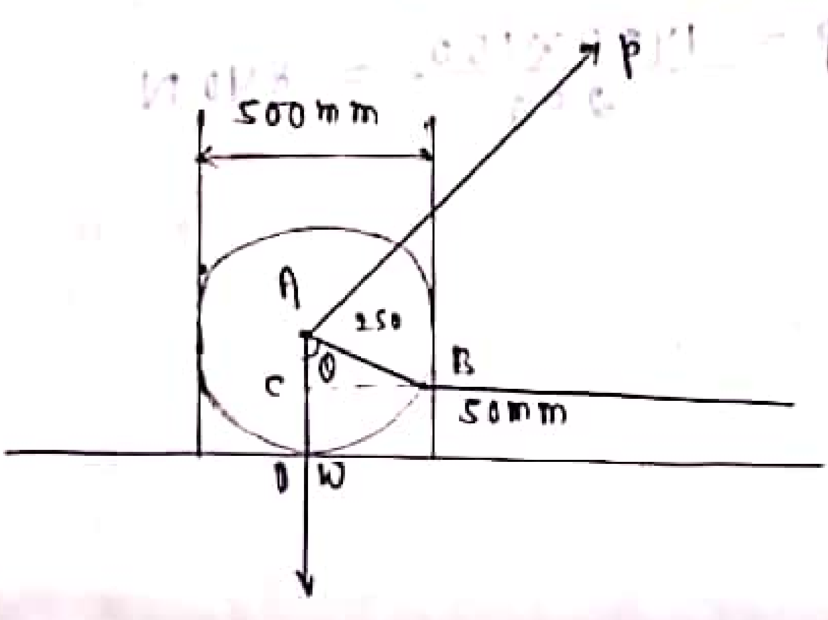
$\Rightarrow 300x + 39600 - 220x = 0$

$\Rightarrow 80x = -39600$

$\Rightarrow x = \frac{-39600}{80} = -495$

PROBLEM-4

A roller of diameter 500mm and weight 1400kg is to be taken up a ~~step~~ up 500mm high, Find the magnitude and direction of the maximum pull required on the handle to pull the roller of a step.



Ans: Given data,

$D = 500mm$ $\quad R = \frac{500}{2} = 250\ mm$

$W = 1400\ N$

$x = 50mm$

From the above figure $AO = AB = R$ (Radius of Roller)

$= 250mm$

$AC = AO - CO$

$= 250 - 50 = 200mm$

From the triangle ABC,

$BC = \sqrt{(AB)^2 - (AC)^2}$

$= \sqrt{(250)^2 - (200)^2}$

$= 150mm$

Pull will be minimum when the perpendicular distance is maximum, the maximum perpendicular distance is radius of the roller. distance is AB,

Taking moment about B,

$W \times BC = P \times AB$

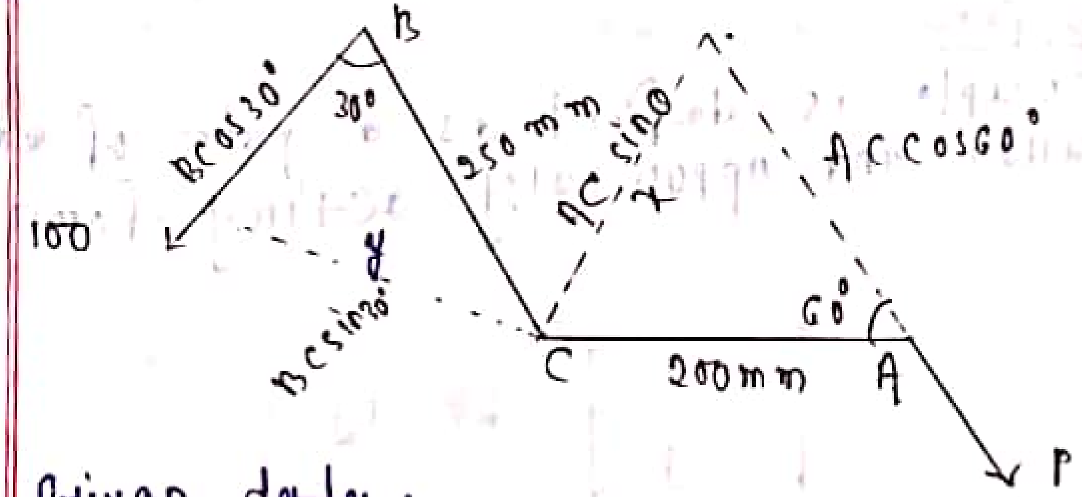
$1400 \times 150 = P \times 250$

$\Rightarrow P = \frac{1400 \times 150}{250} = 840\ N$

Dt. 1 . 02 . 19
Friday.

PROBLEM :-

A bell cran liver is hing at 'c' as shown in figure, a force of 100N is appiled at 'B'. at an angle of 30° to 'BC'. AC = 200 mm, BC = 250 mm, A force 'p' is acting 'A' at an angle of 60° with 'AC'. Find the force 'p'.



Ans:- Given data,

AC = 200 mm, BC = 250 mm, F = 100N

θ of F = 30°, θ of P = 60°.

parpendicular distance between the line. of action of force 'p' and point 'c'

$x = AC \sin 60$

$= 200 \times 0.866$

$= 173.20 mm$

parpendicular distance between the line of the action of force 'F' at the point 'c'.

$y = BC \sin 30$

$= 250 \times 0.5$

$= 125 mm$

Taking moment about 'c'.
clockwise moment = Anticlockwise moment

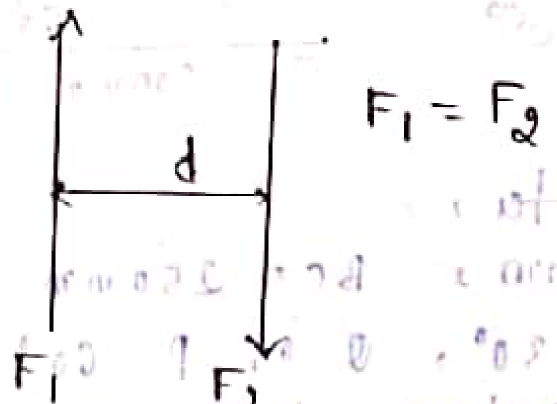
$P \times x = 100 \times y$

$\Rightarrow P \times 173.20 = 100 \times 125$

$\Rightarrow P = \frac{100 \times 125}{173.20} = 72.17\,N$

Couple:-

A couple is defind as a pair of equal parallel and oppositely acting force.



Couple always produces rotary motion of the body on which it acts about an axis perpendicular to the plane of the couple.

d = Moment arc of the couple.

Force:-

Force is the action which produces and tends produces, change in the state of rest or of uniform motion in a straight line of body.

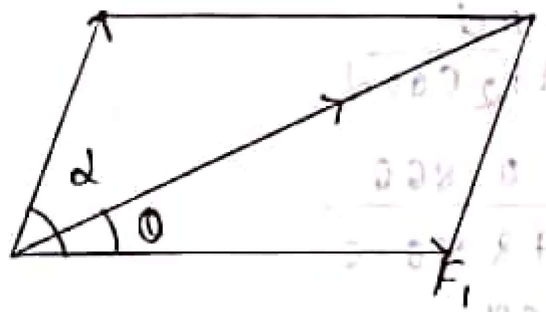
RESULTANT FORCE:-

If the combine effect of several forces like F_1, F_2, F_3, F_4 etc, acting on a body is the same as that of single force 'R', then 'R' is called resultant force F_1, F_2, F_3, & F_4.

PARALLELOGRAM LAW OF FORCES:-

(i) The parallelogram law of forces state that any two forces acting at 'a' point on the body are considered in magnitude and direction as the two adjacent sides of a parallelogram.
- there are resultant is considered in magnitude and direction as a diagonal of the parallelogram.

(ii) the resultant passes through the point of intersection of two force.



Let, F_1, F_2 are two forces acting on the body the angle between two force is α.

- the formula for calculation of resultant by parallelogram is given by.

$$\boxed{R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}}$$

$$\boxed{\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2\cos\alpha}}$$

PROBLEM-1

Two force 7N and 8N act simultaneously act at a point. Find the resultant force if angle between them is 60°. Find the resultant.

Ans:- Given data,

$F_1 = 7N$

$F_2 = 8N$

$\alpha = 60°$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$

$$= \sqrt{7^2 + 8^2 + 2\times 7\times 8\times \cos 60°}$$

$$= \sqrt{49 + 64 + 112\times 0.5} = \sqrt{169} = 13N$$

$$\tan\theta = \frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha}$$

$$= \frac{8\times 0.866}{7 + 8\times 0.5}$$

$$= 4.989$$

Dt. 2.02.19
saturday

PROBLEM :- 1

two forces of 70N and 80N acts simultaneously acting at a point, Find the magnitude and the direction of resultant force if the angle between them 150°.

Ans:- Given data,

$$R = \sqrt{70^2 + 80^2 + 2 \times 70 \times 80 \cos 150}$$

$$= 4900 + 6400 + 11200.$$

$$= 40N$$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}$$

$$= 55.571$$

$$\rightarrow \theta = \tan^{-1}(55.71) = 88.97$$

PROBLEM - 2

The resultant of two forces is 232N the angle between the two forces 45°, find the force, if the F_2 is 1.5 times of F_1.

Ans:- Given data,

$R = 2.32N$

Angle between two forces $= 45°$

$F_2 = 1.5 F_1$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$

$\Rightarrow 232 = \sqrt{F_1^2 + (1.5F_1)^2 + 2 \times F_1 \times 1.5F_1 \cdot \cos 45}$

$\Rightarrow 232 = \sqrt{F_1^2 + 2.25F_1^2 + 2.12F_1^2}$

$= \sqrt{5.37F_1^2}$

$= 2.31F_1$

$\Rightarrow F_1 = \frac{232}{2.31}$

$\Rightarrow F_1 = 100.43N$

$F_2 = 1.5F_1$

$= 1.5 \times 100.43N$

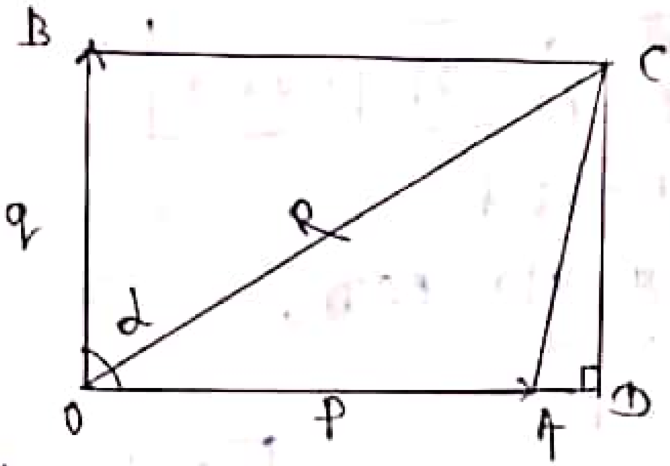
$= 150.645N$

PARALLELOGRAM LAW OF FORCES:-

(i) The parallelogram law of forces state that at any two forces acting at a point on the body are considered in magnitude and direction as the two adjacent sides of parallelogram. There resultant is considered in direction and magnitude as the diagonal of the parallelogram.

(ii) As the resultant passes through the point of interaction of two force.

PROVED:-



According to parallelogram law,

$$\boxed{R^2 = p^2 + q^2 + 2pq\cos\alpha}$$

considered triangle OCD,

$OC^2 = OD^2 + CD^2$

$\Rightarrow OC^2 = (OA + AD)^2 + CD^2$

$\Rightarrow R^2 = (p + AD)^2 + CD^2$ ------ eqn (1)

considered triangle ACD,

$\cos\alpha = \frac{AD}{AC}$

$\Rightarrow AD = AC\cos\alpha$

$\Rightarrow AD = q\cos\alpha$ ($\because OB = AC = q$) ---- eqn (2),

$\sin\alpha = CD/AC$

$\Rightarrow CD = AC\sin\alpha$

$\Rightarrow CD = q\sin\alpha$ ------ eqn (3)

putting eqn (2) & (3) in eqn (1),

$R^2 = (p + AD)^2 + CD^2$

$\Rightarrow R^2 = (p + q\cos\alpha)^2 + (q\sin\alpha)^2$

$\Rightarrow R^2 = p^2 + q^2\cos^2\alpha + 2\times p\times q\cos\alpha \; q^2\sin^2\alpha$

$\Rightarrow R^2 = p^2 + q^2(\sin^2\alpha + \cos^2\alpha) + 2pq\cos\alpha$

$$\Rightarrow \boxed{R^2 = P^2 + Q^2 + 2PQ\cos\alpha}$$

Hence proved

In triangle OCD,

$$\tan\theta = \frac{CD}{OD}$$

$$\Rightarrow \tan\theta = \frac{CD}{OA + AD} = \boxed{\frac{Q\sin\alpha}{P + Q\cos\alpha}}$$

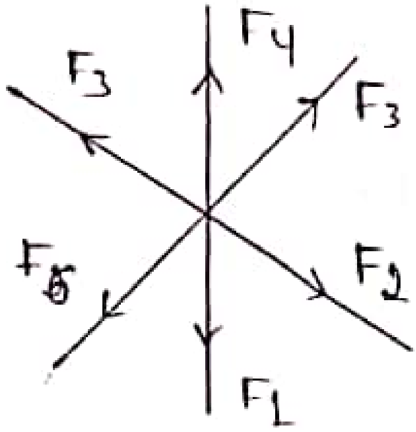
Dt: 4.02.19
Monday.

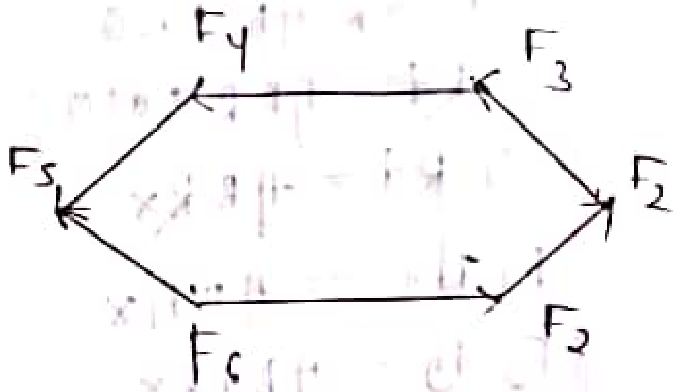
TRIANGLE LAW OF FORCES:-

The triangle law of forces state that any three forces acting at a point on the body are considered in magnitude and direction as the three sides of the triangle taken in order and the three forces are equilibrium.

POLYGON LAW OF FORCES:-

Polygon law of forces state that any no. of forces acting at a point on the body are considered in magnitude and direction as the sides of a polygon taken in order, then the resultant of these forces is considered in magnitude and direction as the closing sides of a polygon taken in opposite order.

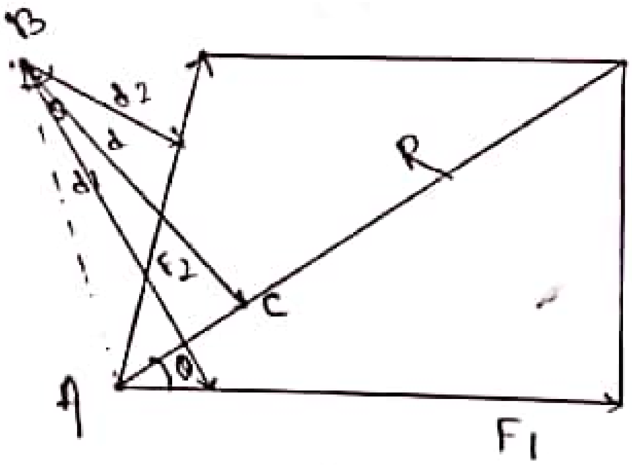




~~VERAG· VARIGNION'S~~ LOW

VARIGNON'S THEOREM :-

The algebric sum of the moments of a system of co-planner forces about a centre in there plane is equal to the moment of their resultant force about the same moment centre.



$$\boxed{Rd = F_1 d_1 + F_2 d_2}$$

Δ ACB ,

$\cos\theta = \frac{BC}{AB}$

$\Rightarrow \cos\theta = \frac{d}{AB}$

$\Rightarrow d = AB\cos\theta$

$$Rd = R \cdot AB \cos\theta$$

$$\Rightarrow Rd = AB \, R \cos\theta$$

$$\Rightarrow Rd = AB \, Rx$$

$$F_1 d_1 = AB \, F_1 x$$

$$F_2 d_2 = AB \, F_2 x$$

$$F_1 d_1 + F_2 d_2 = AB \, F_1 x + AB \, F_2 x$$

$$= AB(F_1 x + F_2 x)$$

$$= AB \, Rx = RD$$

CHAPTER - 2

Dt 05.02.19
Tuesday

EQUILIBRIUM

Equilibrium is the condition of system of forces acting on a body where the resultant is zero,

$R = 0$

EQUILIBRANT:-

Equilibrant is the force which brings the system of forces in equilibrium.

LAMI'S THEOREM:-

It states that if any three co-planner forces acting at a point on the body are in equilibrium, then each forces is proporsional to the signe of angle between the other two forces.

According to Lami's theorem,

$F_1 \propto \sin\alpha$

$F_2 \propto \sin\beta$

$F_3 \propto \sin\gamma$

$F_1 = K \sin\alpha$

$\Rightarrow K = \dfrac{F_1}{\sin\alpha}$

$F_2 = K \sin\beta$

$\Rightarrow K = \dfrac{F_2}{\sin\beta}$

$F_3 = K \sin\gamma$

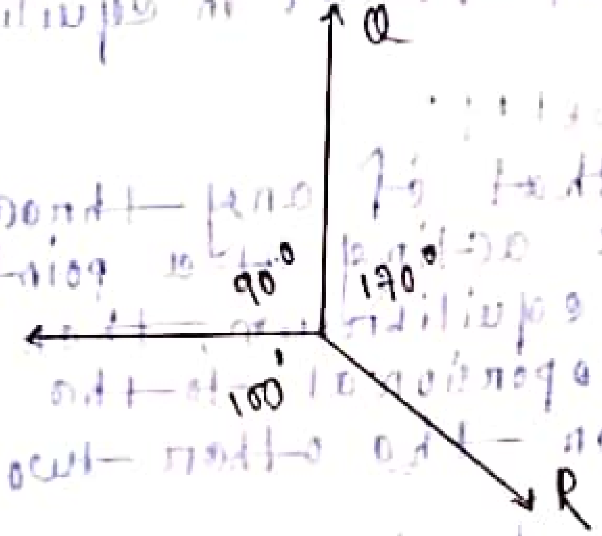
$\Rightarrow K = \dfrac{F_3}{\sin\gamma}$

Mathmatically Lami's theorem can be written as,

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

PROBLEM - 1

Find the ratio of the given forces :



Applying Lami's theorem is,

$$\frac{P}{\sin 90} = \frac{Q}{\sin 100} = \frac{R}{\sin 170} \text{ .... eqn (1)},$$

$$\frac{P}{\sin 90} = \frac{Q}{\sin 100}$$

$$\Rightarrow \frac{P}{Q} = \frac{\sin 90}{\sin 100} = 1.015$$

$$\Rightarrow Q = \frac{P}{1.015} \text{ .... eqn (2)},$$

$$\frac{Q}{\sin 100} = \frac{R}{\sin 170}$$

$$\Rightarrow \frac{Q}{R} = \frac{\sin 100}{\sin 170} = 5.671$$

$$\Rightarrow R = \frac{Q}{5.671} \text{ .... eqn (3)},$$

putting eqn (2) in eqn (3)

$$\Rightarrow R = \frac{\frac{P}{1.015}}{5.671}$$

$$\Rightarrow R = \frac{P}{1.015} \times \frac{1}{5.671}$$

$$\Rightarrow R = \frac{P}{5.756} \text{ .... eqn (4)}$$

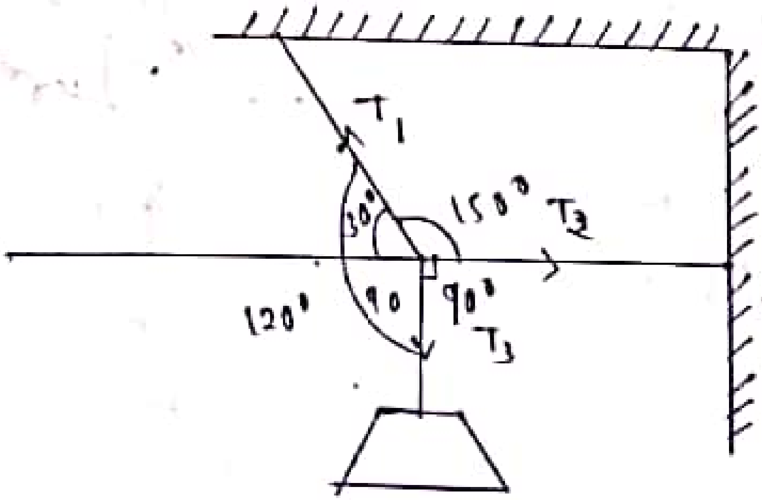
$P : q : R$

$$P : \frac{P}{1.015} : \frac{P}{5.671} = 1 : \frac{1}{1.015} : \frac{1}{5.756}$$

$$= 1 : 0.985 : 0.173$$

PROBLEM-2

A weight of 80 N is suspendent by two fine strings one of which is horizontal & the other is inclined at an angle of 30° to the horizontal, than what is tension in the inclined string.



T_1 = Tension in the inclined string

T_2 = Tension in the horizontal string

T_3 = Tension in the load string.

According to Lami's theorem

$$\frac{T_1}{\sin 90} = \frac{T_2}{\sin 120} = \frac{T_3}{\sin 150}$$

$$\Rightarrow \frac{T_1}{\sin 90} = \frac{T_2}{\sin 120} = \frac{80}{\sin 150}$$

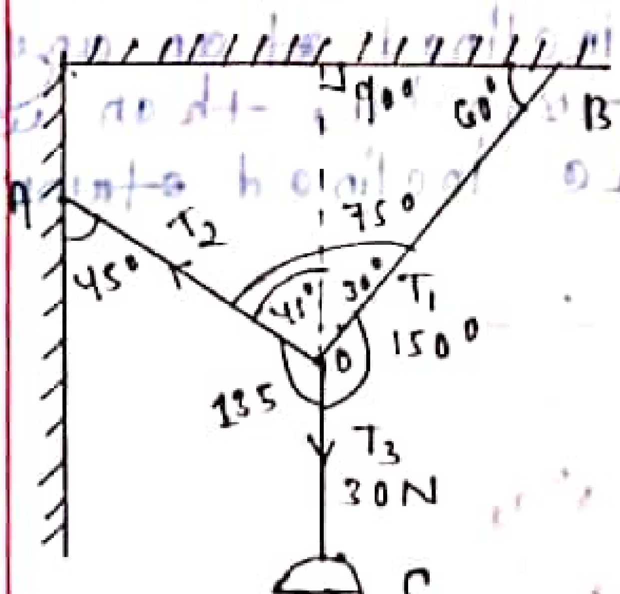
$$\Rightarrow \frac{T_1}{\sin 90} = \frac{80}{\sin 150}$$

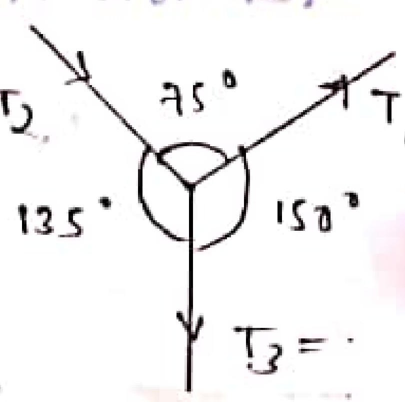
$$\Rightarrow T_1 = \frac{80}{\sin 150} \times \sin 90$$

$$\Rightarrow T_1 = 160 N$$

PROBLEM-3

A electric light of fixture weight 30N from a point 'O' by two strink OA & OB. As shown in figure. Determine the forces in the strink.





Applying Lami's theorem:-

$$\frac{T_1}{\sin 135} = \frac{T_2}{\sin 150} = \frac{T_3}{\sin 75}$$

$$\Rightarrow \frac{T_1}{\sin 135} = \frac{T_2}{\sin 150} = \frac{30}{\sin 75}$$

$$\Rightarrow \frac{T_1}{\sin 135} = \frac{30}{\sin 75}$$

$$\Rightarrow T_1 = \frac{30}{\sin 75} \times \sin 135 = 21.961 N$$

$$\frac{T_2}{\sin 150} = \frac{30}{\sin 75}$$

$$\Rightarrow T_2 = \frac{30}{\sin 75} \times \sin 150 = 15.52 N$$

Dt. 08.02.19
Friday

PROVED:

LAMI'S THEORY:-

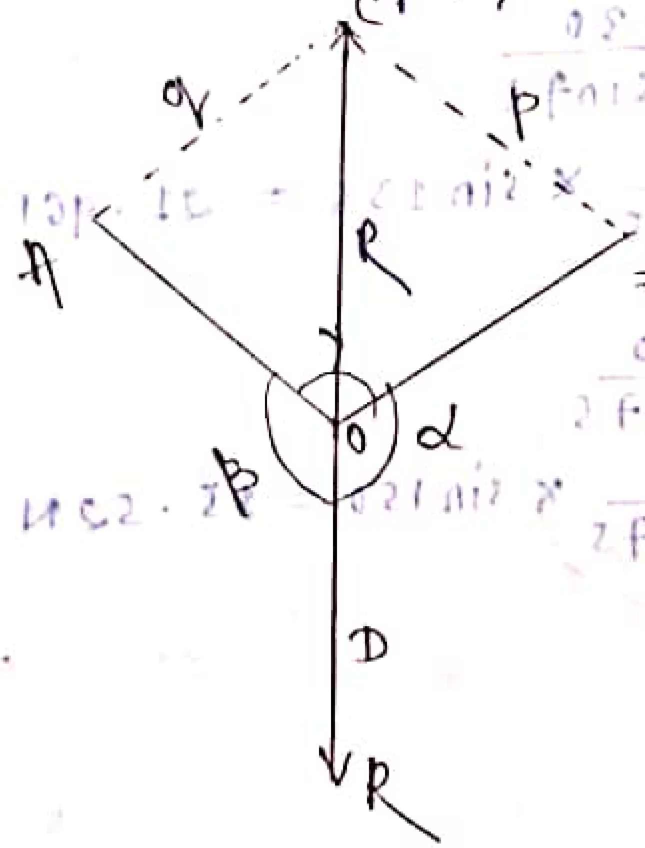
It states that if three co-planner forces acting at a point be in equilibrium then each forces is proporsional to the signe of the angle both other two forces.

Mathmatically,

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

PROOF :-

Consider three co-planner forces P, q, R acting at a point 'o'.
Let the opposite angle to three forces be α, β, γ respectively.



Now let us complete the parallelogram OACB, with OA & OB at adjacent side as shown in the figure.
We know that the resultant of two force P & q will be given by the diagonal OC both in magnitude and direction.
From the given geometry

$BC = P$

$AC = q$

$\angle AOC = (180° - \beta)$

$\angle ACO = \angle BOC = (180° - \alpha)$

$\angle CAO = 180° - (AOC + ACO)$

$\Rightarrow \angle CAO = 180° - ((180° - \beta) + (180° - \alpha))$

$\angle CAO = 180° - 180° + \beta - 180° + \alpha$

$\Rightarrow \angle CAO = \alpha + \beta - 180°$ ———— eqn ①

$\alpha + \beta + \gamma = 360$

$\Rightarrow \alpha + \beta - 180 + \gamma = 360° - 180°$ (According to eqn ①)

$\Rightarrow \angle CAO + \gamma = 180$

$\Rightarrow \angle CAO = 180 - \gamma$

In triangle AOC,

$$\frac{OA}{\sin(ACO)} = \frac{AC}{\sin(AOC)} = \frac{OC}{\sin(CAO)}$$

$$\frac{OA}{\sin(180° - \alpha)} = \frac{AC}{\sin(180° - \beta)} = \frac{OC}{\sin(180 - \gamma)}$$

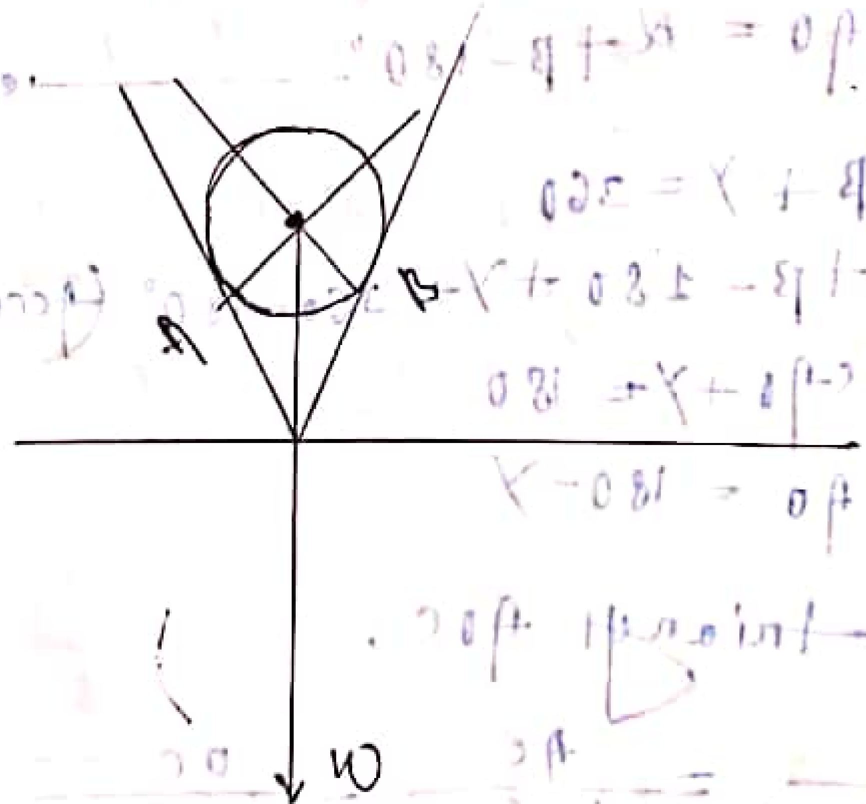
$$\frac{OA}{\sin\alpha} = \frac{AC}{\sin\beta} = \frac{OC}{\sin\gamma}$$

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

21.09.2019
Saturday

FREE BODY DIAGRAM

A free body diagram is a skech of the particle or body which represents it being isolated from its surrounding system.



CHAPTER - 3
FRICTION

Friction is a retarding force always acting opposite to the motion or the tendency to move the body.

TYPES OF FRICTION :-
There are two type of friction.
1. static friction.
2. kinetic friction.

STATIC FRICTION :-
(i) If the applyed force on the body is less than limiting friction, the body remain at rest & the friction is ~~th~~ called static friction.

(ii) The value of static friction betn zero & limiting friction.

KINETIC FRICTION :-
If the applyed force on the body is more than the limiting friction, the body moves (slides & ~~ro~~ roles) on the other surfaces the friction is called kinetic friction.

LIMITING FRICTION:-

Limiting friction is a maximum amount of frictional force that comes into play when a body just begin to move over the other surface.

COEFFICIENT OF FRICTION:-

(i) The ratio between the limiting friction and the normal reaction of the two contact surface is called coefficient of friction.

$F \propto N$

$\Rightarrow F = \mu N$

$$\boxed{\mu = F/N}$$

(ii) At limiting friction the angle of friction is equal to the angle of inclination of the plane.

Angle of friction

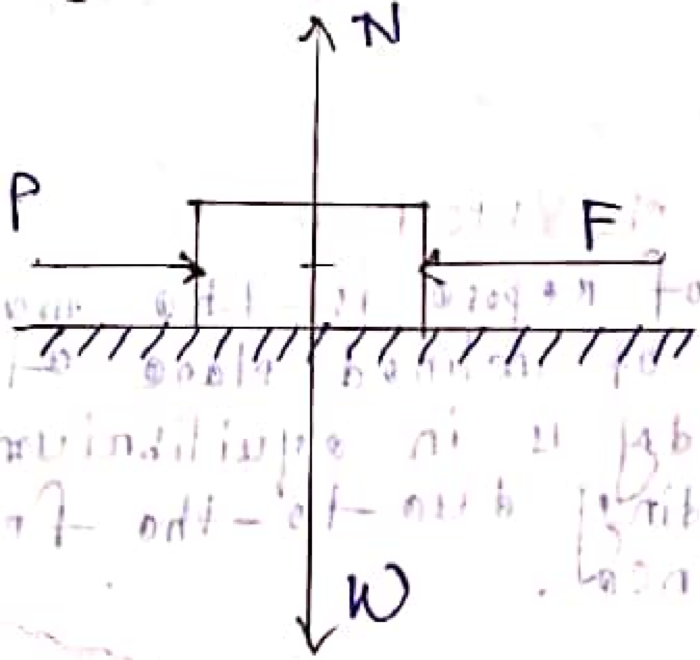
$$\boxed{\mu = \tan\theta = F/N}$$

ANGLE OF FRICTION:—

The angle of friction is the angle betn normal reaction & the resultant of resultant force and reaction.

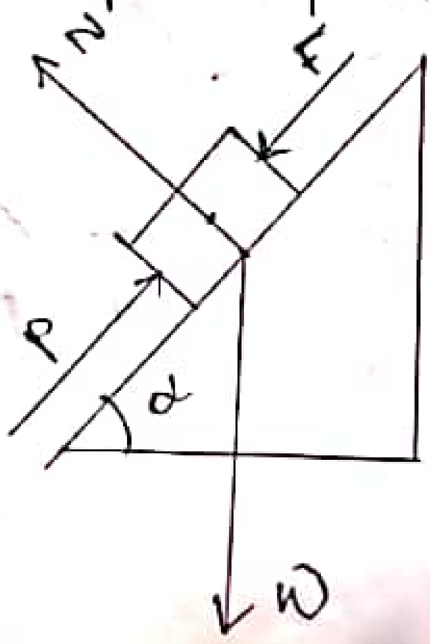
CASE 1:—

Pushing on horizontal plane:—



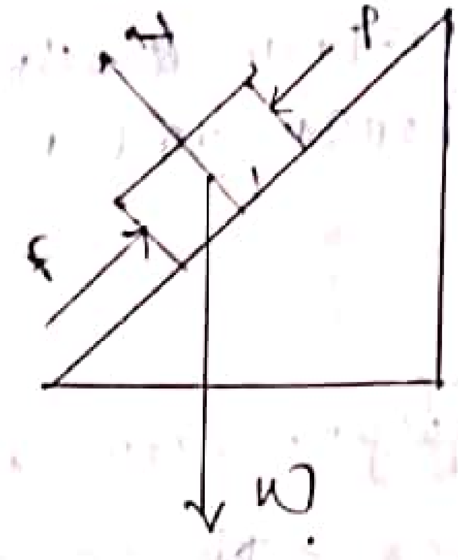
CASE 2:—

Pushing up on it inclined plane:—



CASE 3 :-

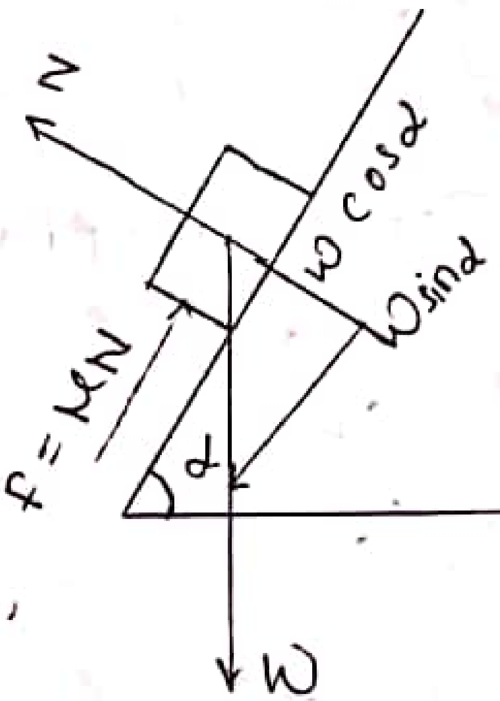
sliding down the inclined plane:-



11.02.19
Monday

ANGLE OF REPOSE:-

Angle of repose is the maximum angle of inclined plane at which the body is in equilibrium without sliding due to the frictional resistance.



α = Angle of inclination of the plane
W = Weight of the body
F = Frictional force of the body
N = Normal reaction acting perpendicular to the plane.

Horizontally,
$F = W\sin\alpha$
Vertically,
$N = W\cos\alpha$
$F = \mu N$
$\Rightarrow \mu = F/N$
$\Rightarrow \mu = \dfrac{W\sin\alpha}{W\cos\alpha} = \tan\alpha = \tan\phi$

LAWS OF FRICTION :-

(i) The force of friction always acts in opposite direction to that in which the body is moving.

(ii) If the force applied is less than the force of friction, then the body is at rest.

(iii) Limiting friction is always at constant ratio ~~of~~ with normal reaction.

(iv) The force of friction depends open roughness of the contact surface.

(v) If the force applied is equal to the force of friction, then the body is ready to the move.

(vi) If the force applied is more than the force of friction, then the body is move.

PROBLEM:-

If the coefficient of friction is 0.2 find the angle of friction.

Ans:- Given data:-

$\mu = 0.2$

$\phi = ?$

We know that

$\mu = \tan\phi$

$\Rightarrow \phi = \tan^{-1}\mu$

$\Rightarrow \phi = 11.30$

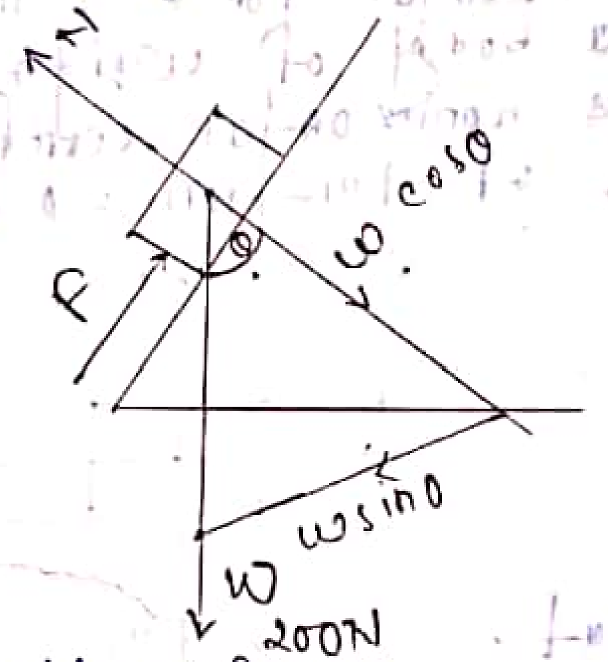
PROBLEM:-

A block weighing 200N rest on a inclined plane, if the coefficient is friction is 0.35, find the angle of repose & the greatest force of friction.

Ans:- Given data:-

$W = 200N$

$\mu = 0.35$



At limiting friction.

Angle of friction = Angle of repose.

At limiting friction

$\phi = \theta$

$\mu = \tan\phi$

$\Rightarrow \phi = \tan^{-1}\mu$

$\Rightarrow \phi = \tan^{-1} 0.35$

$\Rightarrow \phi = 19.29$

Greatest force of friction =

$F_{max} = W\sin\theta$

$= 200\sin 19.29$

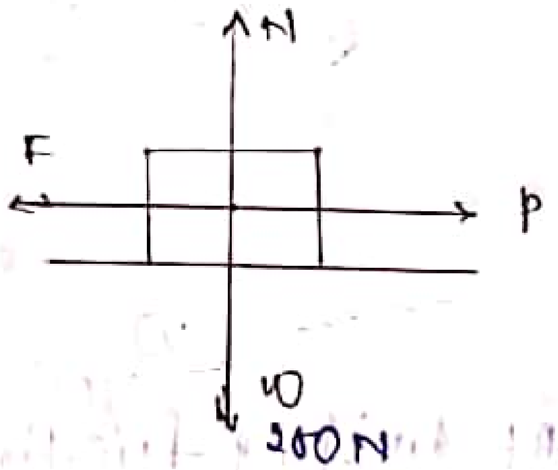
$= 66.06$ N

D/ . 15.02.19
Friday

PROBLEM

What horizontal force is required to pull a body of weight 200N along the horizontal surface when coefficient of friction is 0.2 .

Ans:-



Given that,

$W = 200N$

$\mu = 0.2$

Resolving the force horizontally,

$P = F$

$\Rightarrow P = F = \mu N$ - - - - - eqn (1)

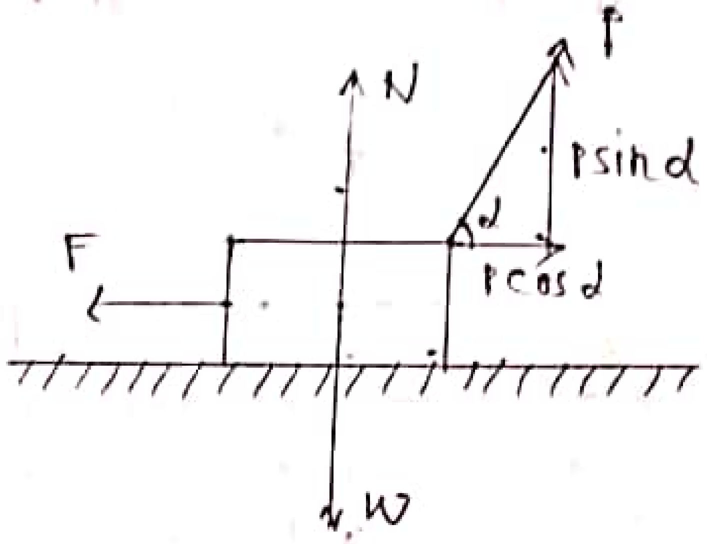
Resolving the force vertically,

$N = W$

$\Rightarrow N = 200N$

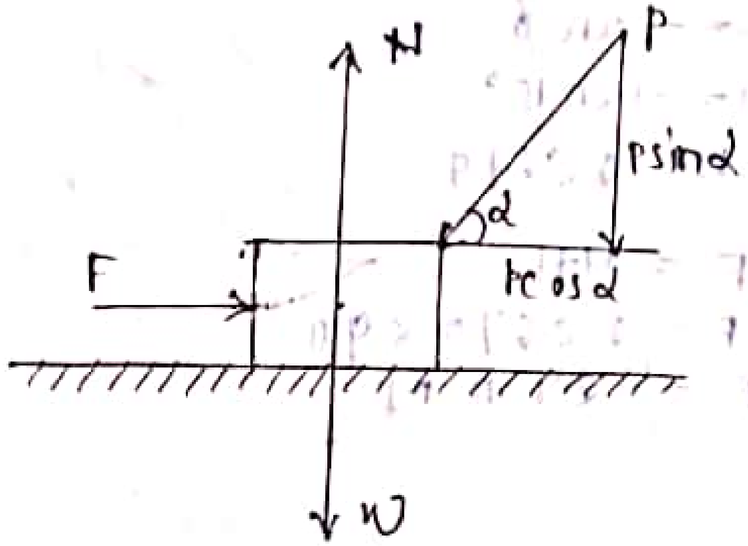
$F = \mu N$
$= 0.2 \times 200\,N$
$= 40\,N$ (Ans)

PULL :-



$F = P\cos\alpha$
$W = N + P\sin\alpha$

PUSH :-

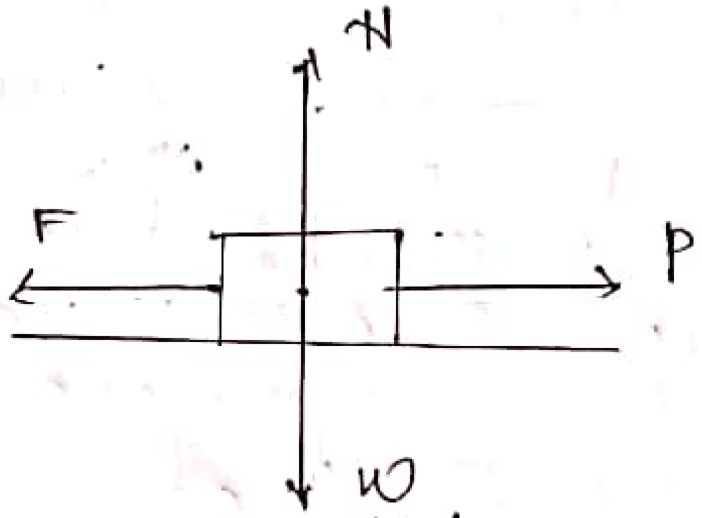


$F = P\cos\alpha$
$N = W + P\sin\alpha$
or $W = N - P\sin\alpha$

PROBLEM:-

Find the horizontal effort from move the body w=90 N along a horizontal plane, the plane is such that if it is gradually 15 degree the body is slide down.

Ans:-



Given data,

$d = 15°$

$\mu = F/N$

$\mu = \tan \phi$

$\Rightarrow \mu = \tan 15°$

$\Rightarrow \mu = 0.2679$

$\Rightarrow F = \mu N$

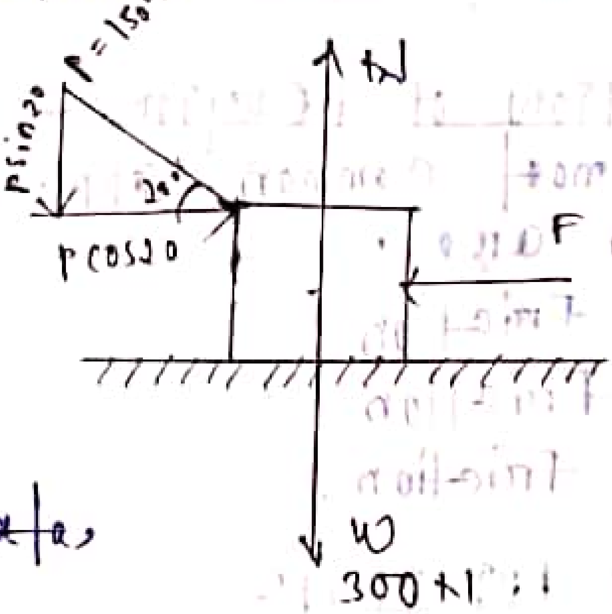
$\Rightarrow F = 0.2679 \times 90$

$\Rightarrow F = 24.11\ N$

PROBLEM:-

A body of weight 300 N is placed on a ~~rough~~ horizontal plane to move the body on horizontal plane, a push of 150 N inclined at 20° to the horizontal plane is required. Find the coefficient of friction.

ts:-



Given data,

Resolving the forces horizontally,

$F = P\cos 20$

$\Rightarrow F = 150 \times \cos 20$

$\Rightarrow F = 140.95$

Resolving the force vertically,

$N = W + P\sin 20$

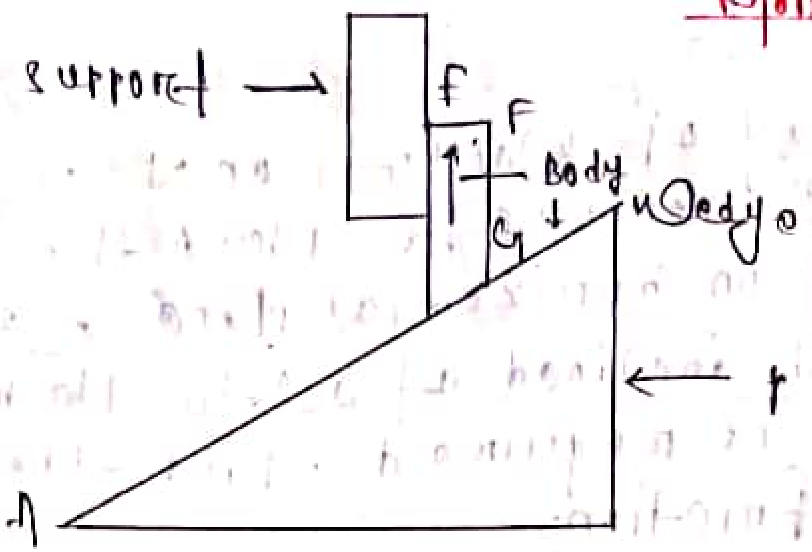
$\Rightarrow N = 300 + 150 \times \sin 20$

$= 351.30$

The coefficient of,

$\mu = \frac{F}{N} = \frac{140.95}{351.30} = 0.401$

Dt · 18·12·17
Monday.



APPLICATION OF FRICTION:-

The most common applications of friction are :

1- Wedge friction
2- Ladder friction
3- Screw friction.

WEDGE FRICTION:-

(i) A wedge is a piece of metal or wood in the shape of ~~fr~~ trunkted, prism whose cross-section is usually triangle or trapizium.

(ii) It is used for lifting wedge, for fitaning file, a key for shaft.

(iii) The problem of the wedge are usually solved by tritening wedge as inclined plane.

(iv) In the above figure it is clear that the force P is pushes the wedge to-words left.

(v) Due to the support, the body moves of along the inclined plane of the wedge.

(vi) We can solved the problem taking the contact of a ground & the wedge.

Dt. 19.02.19
Tuesday

PROBLEM 1

A mass of 100kg if dragg horizontal surface by an effort of 0.2 kN acting at a angle of 20° with the horizontal. Determine coefficient of friction.

Ans: Given data,

$m = 100 \, kg$

$w = mg$

$P = 0.2 \, kN$

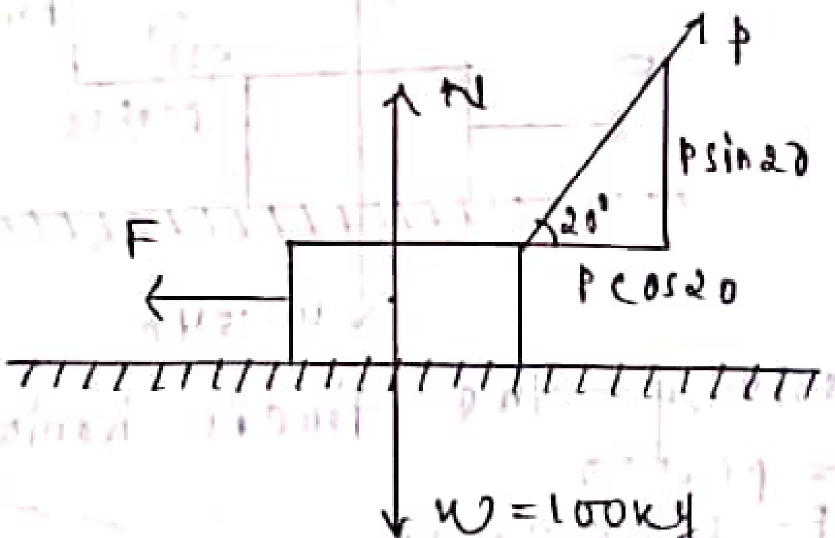
$= 0.2 \times 1000 \, N$

$= 200 \, N$

$w = mg$

$= 100 \times 9.81$

$=$



Resolving the forces horizontally,

$F = P \cos 20$

$\Rightarrow F = 200 \times \cos 20$

$= 187.93 \, N$

$\mu = \frac{F}{N}$

Resolving the force vertically,

$W = N + P\sin 20$

$\Rightarrow N = 981 - 200 \sin 20$

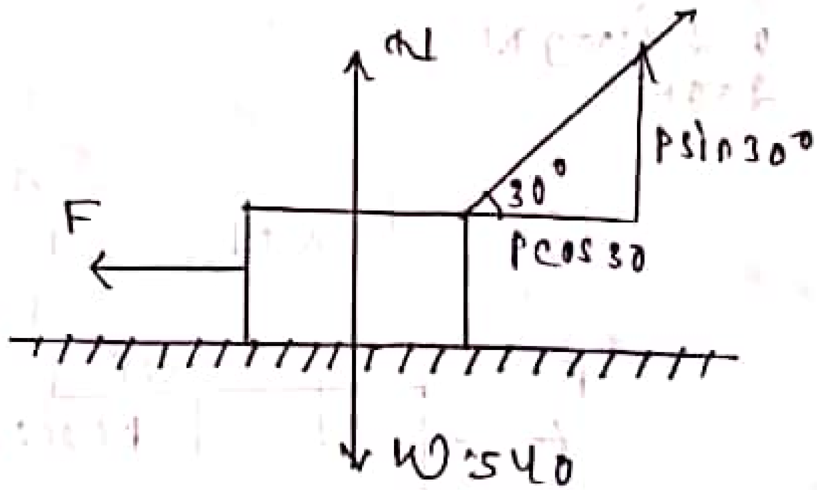
$\Rightarrow N = 912.59$

$\mu = \frac{F}{N}$

$= \frac{187.93}{912.59} = 0.205$

PROBLEM-2.

A body of weight 540N pulling force 180N horizontal angle 180N. Find the coefficient of friction.



Resolving the forces horizontally,

$F = P\cos 30$

$\Rightarrow F = 180 \times \cos 30$

$\Rightarrow 155.88N$

$\mu = \frac{F}{N}$

Resolving the force vertically.

$W = N + P\sin 30$

$\Rightarrow N = W - 180\sin 30$

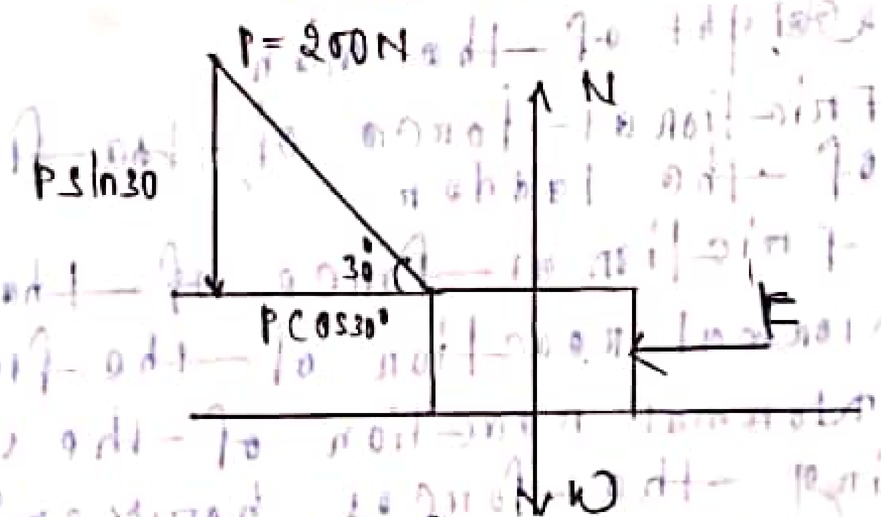
$\Rightarrow N = 540 - 180\sin 30$

$\Rightarrow N = 450$

$$\mu = \frac{F}{N} = \frac{155.88}{450} = 0.3464\,N$$

PROBLEM-3:

A body of weight 500N is placed on a rough horizontal plane to move the body on the horizontal a push of 200N inclined at 30° to the horizontal plane Find the coefficient of friction.



Resolving the forces horizontally,

$F = P\cos 30$

$\Rightarrow F = 200 \times \cos 30$

$\Rightarrow 173.20$

Resolving the forces vertically.

$N = W + P\sin 30$

$\Rightarrow N = 500 + 200\sin 30$

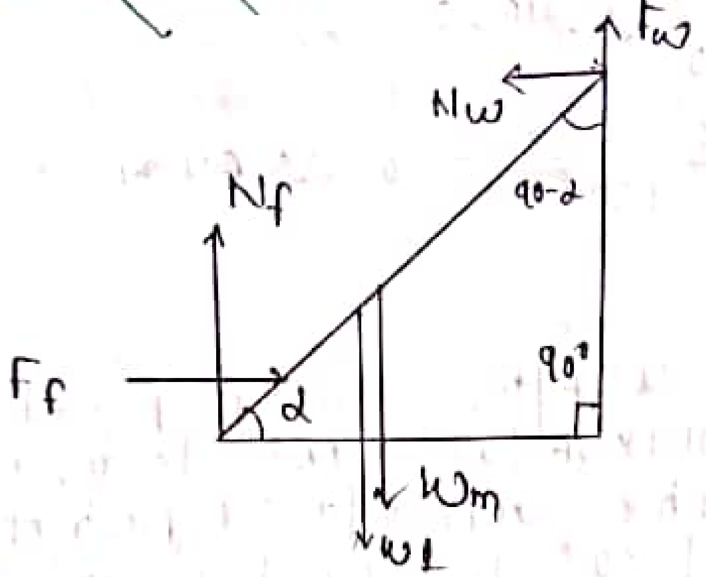
$= 600$

coefficient of friction.

$$\mu = \frac{F}{N} = \frac{173.20}{600} = 0.288\,N$$

Dt. 21.02.19
Thursday

LADDER FRICTION:-



W_l = Weight of the ladder.
l = Length of the ladder
W_m = Weight of the man
F_f = Frictional force of the floor of the ladder
F_w = Frictional force of the wall
N_f = Normal reaction of the floor.
N_w = Normal reaction of the wall
Resolving the forces horizontally,

$F_f = N_w$ —— eqn (1)

Resolving the forces vertically,

$N_f + F_w = W_l + W_m$ —— eqn (2)

PROBLEM:-

A uniform ladder of 7m rest against a vertical wall with which it make an angle 45°, the coefficient friction between ladder & in the wall is 0.4 & that between ladder in the floor is 0.5, If a man whose weight is half of that of the ladder, how thi will be the ladder is sleep.

Ans:- Given data,

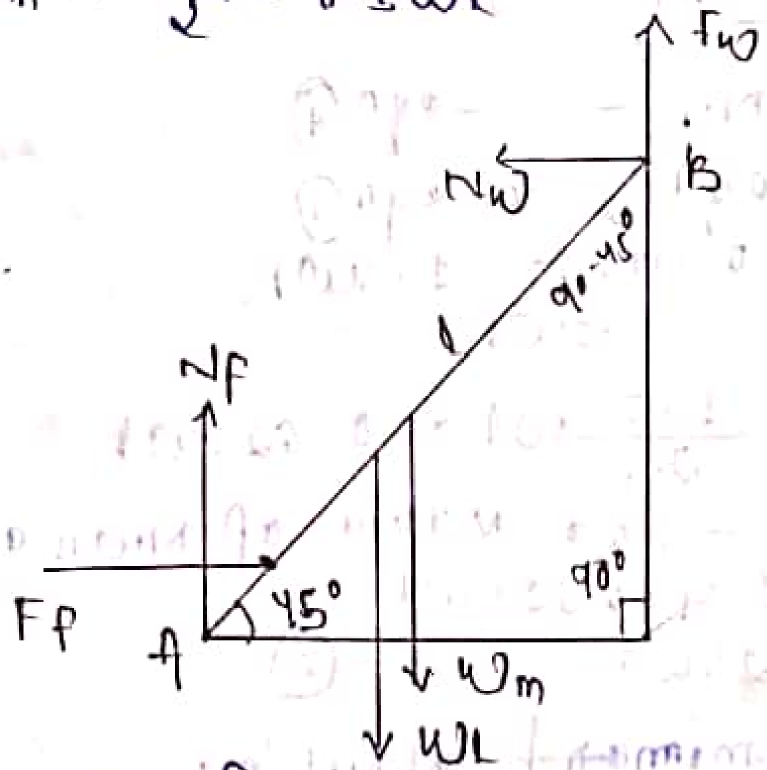
$l = 7\,m$

$\alpha = 45°$

$\mu_w = 0.4$

$\mu_f = 0.5$

$W_m = \frac{W_l}{2} = 0.5W_l$



$F_f = \mu N_f$

$\Rightarrow F_f = 0.5N_f$ —— eqn (1)

Frictional force of the wall,

$F_w = \mu_w \times N_w$

$\Rightarrow F_w = 0.4N_w$ —— (2)

Resolving the forces vertically,

$N_f + N_w = W_l + W_m$

$\Rightarrow N_f + N_w = W_l + 0.5W_l$

putting the value of F_w

$\Rightarrow N_f + 0.4 N_w = 1.5 W_l$ ——— eqn (3)

Resolving the forces horizontally,

$N_w = F_f$

putting the forces horizontally,

$N_w = F_f$

putting the value of F_f

$\Rightarrow N_w = 0.5 N_f$

$\Rightarrow N_f = \frac{N_w}{0.5}$

$\Rightarrow N_f = 2 N_w$ ——— eqn (4)

putting eqn (4) in eqn (3)

$\Rightarrow 2N_w + 0.4 N_w = 1.5 W_l$

$\Rightarrow 2.4 N_w = 1.5 W_l$

$\Rightarrow N_w = \frac{1.5}{2.4} W_l = 0.625 W_l$

putting the value of N_w is eqn (2)

$\Rightarrow F_w = 0.4 \times 0.625 W_l$

$\Rightarrow F_w = 0.25 W_l$ ——— (5)

Taking moment about A.

$(W_l \times 3.5 \cos 45) + (0.5 W \times x \cos 45) = (N_w \times 7 \sin 45) + (F_w \times 7 \cos 45)$

$\Rightarrow (W_l \times 3.5 \cos 45) + (0.5 W_l \times x \cos 45) = (0.625 W_l \times 7 \sin 45) + (0.25 W_l \times 7 \cos 45)$

$= 2.47 W_l + 0.35 W_l x + 3.09 W_l + 1.23 W_l$

$= 2.47 + 0.35x = 4.32$

$= 0.35x = 4.32 - 2.47$

$= 0.35x = 1.85$

$\Rightarrow x = \frac{1.85}{0.35} = 5.28\,m$ (Ans)

Dt. 22.02.19 Friday.

CHAPTER - 04

CENTRE OF GRAVITY

The point through which the resultant force of gravity of the body acts is called centre of gravity.

CENTROID :-

(i) The centre of area of plane figure like rectangle, circle etc is know as centroid.

(ii) The method of finding the centroid is same as finding the centre of gravity of the plane body.

(iii) The term centre gravity is commonly used for centroid calculation also.

Dt. 23.02.19 Saturday.

CENTRE OF GRAVITY ON A PLANE AREA :-

The method of mass moment, for the areas also moments are taken and equate to total area moment.

$$A\bar{x} = a_1x_1 + a_2x_2 + a_3x_3 \ldots$$

$$\Rightarrow \bar{x} = \frac{a_1x_2 + a_2x_2 + a_3x_3}{A}$$

$$\Rightarrow \boxed{\bar{x} = \frac{a_1x_2 + a_2x_2 + a_3x_3 \ldots}{a_1 + a_2 + a_3 \ldots}}$$

$$\boxed{\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 \ldots}{a_1 + a_2 + a_3 \ldots}}$$

Procedure of finding centre of gravity or centroid :-

1- Identify the symmetrical axis of the composite section.

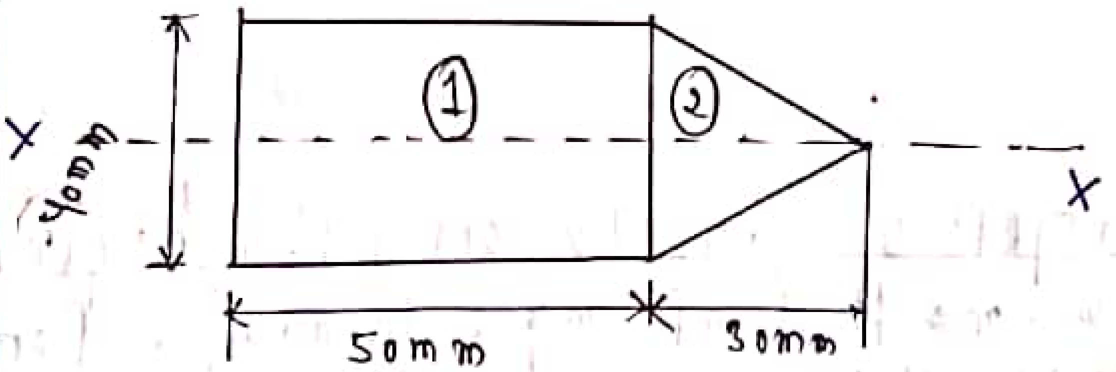
2- Divide the composes section into parts by identifying common geometrical set of the section.

3- Calculate the area and distance from the reference.

4- Calculate the centroid or c.g = $(\bar{x}, \bar{y})$

PROBLEM-1

Find the centroid $(\bar{x}, \bar{y})$ of the composed section shown in figure.



- The composed section is symmetrical about x-x axis,
- Hence the centroid is line.

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$\bar{y} = 0$$

split the composed section into two part no 1 is rectangle and 2nd is triangle.

section - 1

$a_1 = 50 \times 40 = 2000\ mm^2$

$x_1 = 50/2 = 25mm$

section - 2

$a_2 = \frac{1}{2} \times 30 \times 40$

$= 600mm^2$

$x_2 = 50 + \frac{h}{3}$

$= 50 + \frac{30}{3}$

$= 60$

$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{2000 \times 25 + 600 \times 60}{2000 + 600}$

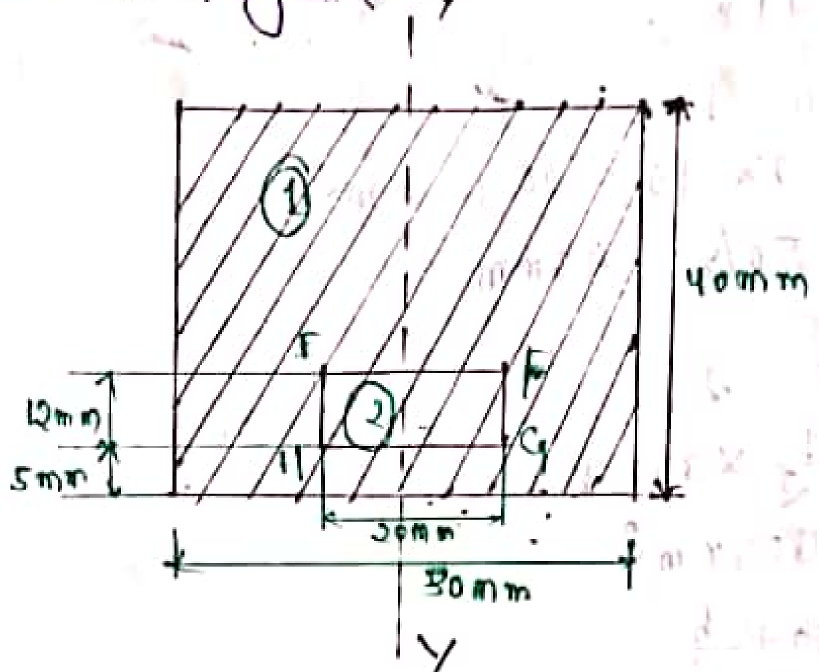
$= 33.07\ mm$

$C.G = (\bar{x}, \bar{y})$

$= (33, 0)$

PROBLEM · 2

Find the centroid of shaded area shown in figure



The section is symmetrical about Y - Y axis

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$\bar{x} = 0$

The section EFGH is a cut section.

section - 1

$a_1 = 40 \times 30$

$= 1200 mm^2$

$y_1 = \frac{40}{2} = 20 mm$

section - 2

$a_2 = 20 \times 12 = 240 mm^2$

$y_2 = 5 + 12/2$

$= 11 mm$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{1200 \times 20 - 240 \times 11}{1200 - 240}$$

$= 22.25$

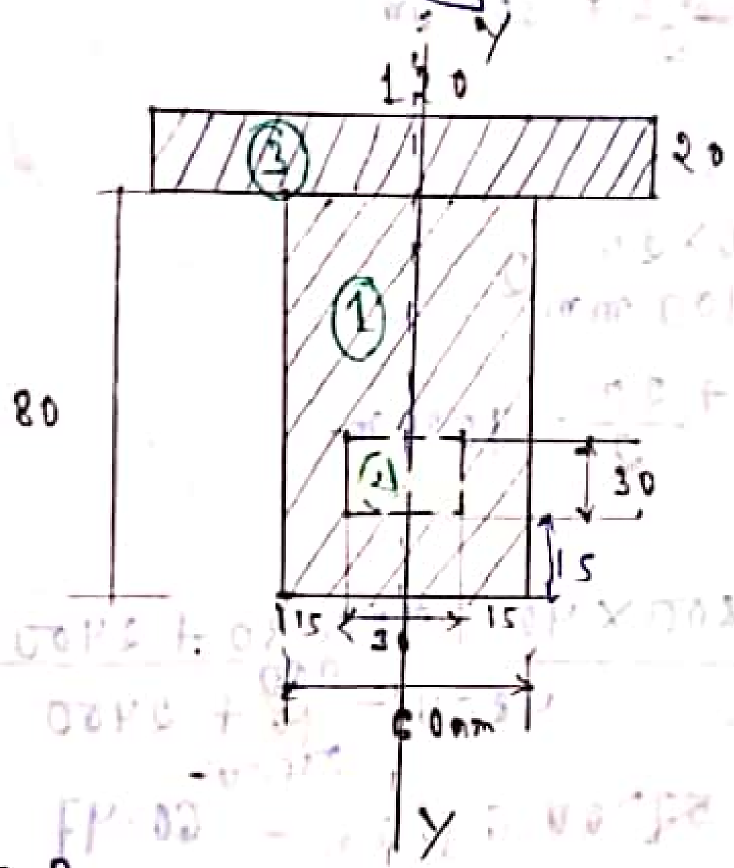
$c \cdot g = (\bar{x}, \bar{y})$

$= (0 \cdot 22).$

Dt. 25.02.19
Monday

PROBLEM-3

Find the Centroid of a shadded area as shown in figure.



$\bar{X} = 0$

$$\bar{Y} = \frac{a_1 y_1 - a_2 y_2 + a_3 y_3}{a_1 - a_2 + a_3}$$

The given section is symmetrical about Y-Y axis & section 2 is a cut section.

Sec-1

$a_1 = 60 \times 80$
$= 4800\ mm^2$

$y_1 = \frac{80}{2} = 40\ mm$

section - 2

$a_2 = 30 \times 30$
$= 900\ mm^2$

$y_2 = 15 + \frac{30}{2} = 30\ mm$

section - 3

$a_3 = 120 \times 20$
$= 2400\ mm^2$

$y_3 = 80 + \frac{20}{2} = 90\ mm$

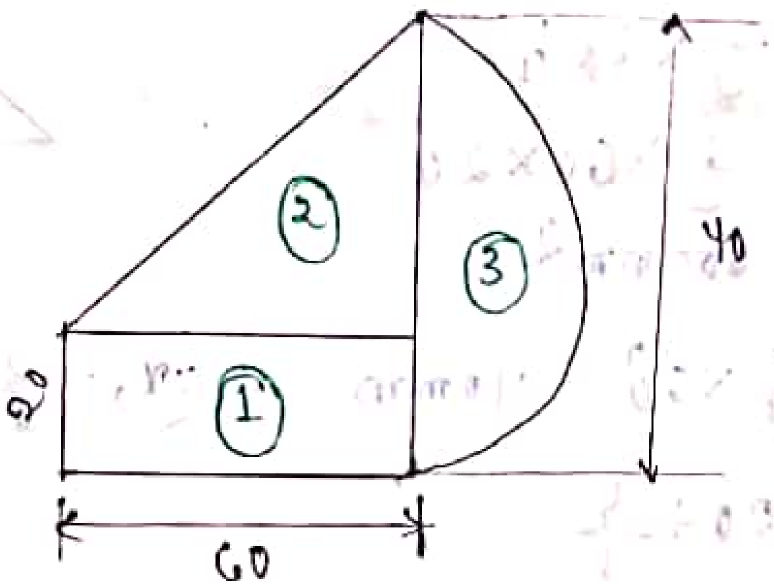
$$\bar{y} = \frac{4800 \times 40 - 900 \times 30 + 2400 \times 90}{4800 - 900 + 2400}$$

$= 60.47$

c.g. $(\bar{x}, \bar{y})$
$= (0, 60.47)$

PROBLEM - 4

Find the centroid of built section as shown in figure :



The given figure is not symmetrical about any axis.
Hence we have to calculate both $\bar{x}$ & $\bar{y}$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

Section - 1

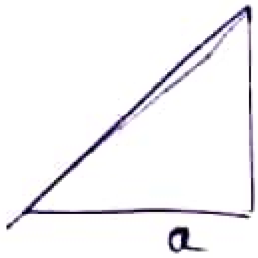
$a_1 = 60 \times 20$
$= 1200 \text{ mm}^2$

~~$y_1 = 10$ mm~~

$x_1 = 60/2 = 30$

$y_1 = 20/2 = 10$

section-2



$c.g = (\frac{2}{3}a,$

$$a_2 = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 60 \times 20$$
$$= 600\,mm^2$$

$$x_2 = \frac{2}{3} \times 60 = 40\,mm \Rightarrow (\because x_2 = \frac{2}{3}a)$$

$$y_2 = 20 + \frac{h}{3}$$
$$= 20 + \frac{20}{3}$$
$$= 26.66\,mm$$

section-3

$$a_3 = \frac{\pi r^2}{2} = \frac{\pi \times 20^2}{2}$$
$$= 628.31\,mm^2$$

$$x_3 = \cancel{\frac{4r}{3\pi}}$$
$$= 60 + \frac{4r}{3\pi} = 60 + \frac{4 \times 20}{3\pi}$$
$$= 68.48\,mm$$

$$y_3 = 40/2 = 20\,mm$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{1200 \times 30 + 600 \times 40 + 628.31 \times 68.48}{1200 + 600 + 628.31}$$

$$= 42.42$$

$$= \bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{1200 \times 10 + 600 \times 26.66 + 628.31 \times 20}{1200 + 600 + 628.31}$$

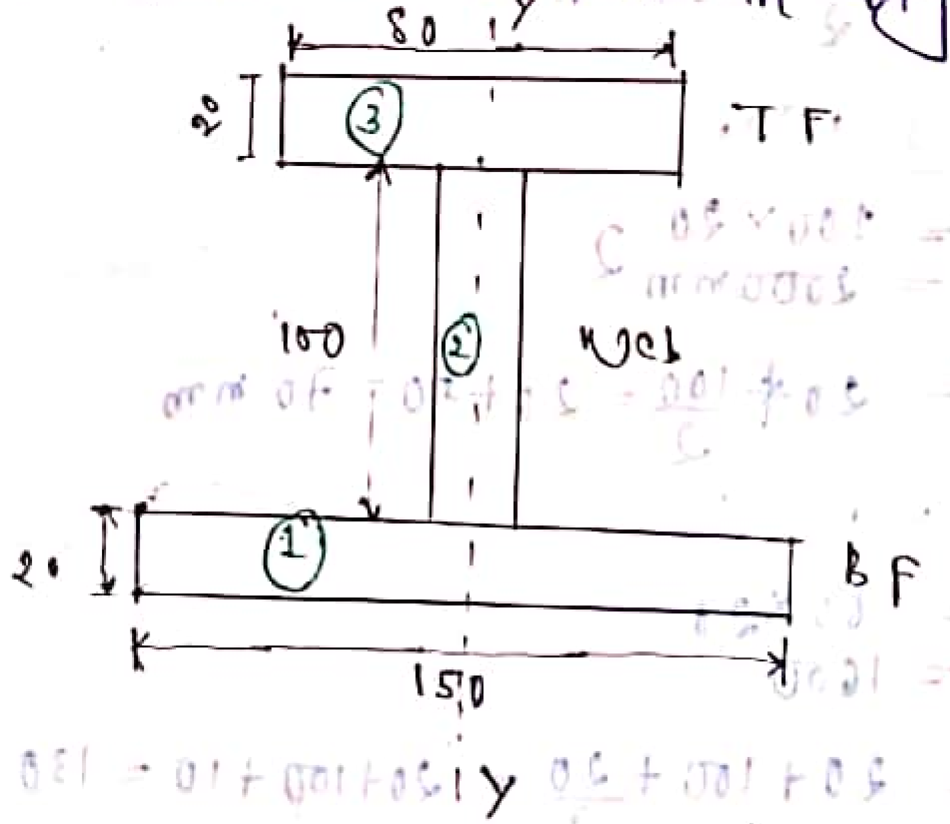
$= 16.70$

$C.G = (\bar{x}, \bar{y})$

$= (42.42, 16.70)$

CENTRE OF GRAVITY OF 'I' SECTION :-

Determine the centroid of 'I' section as shown in figure.



$\overline{X} = 0$

$$\overline{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

Let / ?

- The 'T' section is symmetrical about 'YY' axis.

Take the bottom of the bottom flange as the axis of refference

Rec-1 (BF)

$a_1 = 150 \times 20$
$= 3000 mm^2$

$y_1 = \frac{20}{2} = 10 mm$

Rec-2 (Web)

$a_2 = 100 \times 20$
$= 2000 mm^2$

$y_2 = 20 + \frac{100}{2} = 20 + 50 = 70 mm$

Rec-3 (T.F.) :-

$a_3 = 80 \times 20$
$= 1600$

$y_3 = 20 + 100 + \frac{20}{2} = 20 + 100 + 10 = 130$

$$\bar{Y} = \frac{3000\times10 + 2000\times70 + 1600\times130}{3000+2000+1600}$$

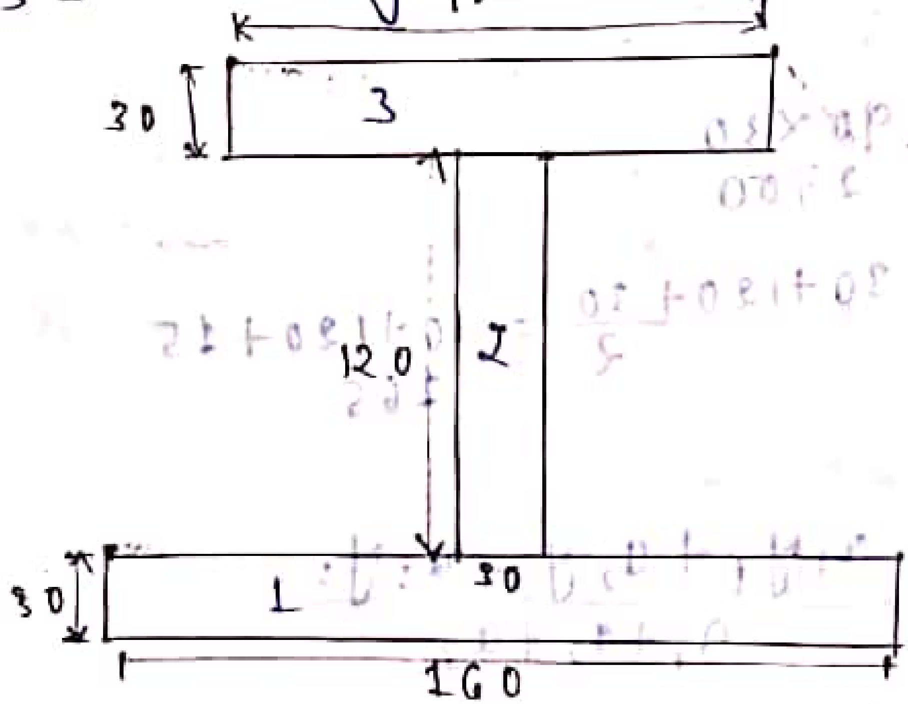
$$= 57.27$$

$$C.G = (\bar{x}, \bar{y})$$

$$= (0, 57.27)$$

PROBLEM-2

Determine the centroid of 'I' section as shown in fig.90



$$\bar{x} = 0$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1+a_2+a_3}$$

The 'I' section is symmetrical about 'YY' axis.

Rect - 1

$a_1 = 160 \times 30$
$= 4800$

$y_1 = \frac{30}{2} = 15$

Rect - 2

$a_2 = 120 \times 30$
$= 3600$

$y_2 = 30 + \frac{120}{2} = 30 + 60$
$= 90$

Rect - 3

$a_3 = 90 \times 30$
$= 2700$

$y_3 = 30 + 120 + \frac{30}{2} = 30 + 120 + 15$
$= 165$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{4800 \times 15 + 3600 \times 90 + 2700 \times 165}{4800 + 3600 + 2700}$$

$$= 75.81$$

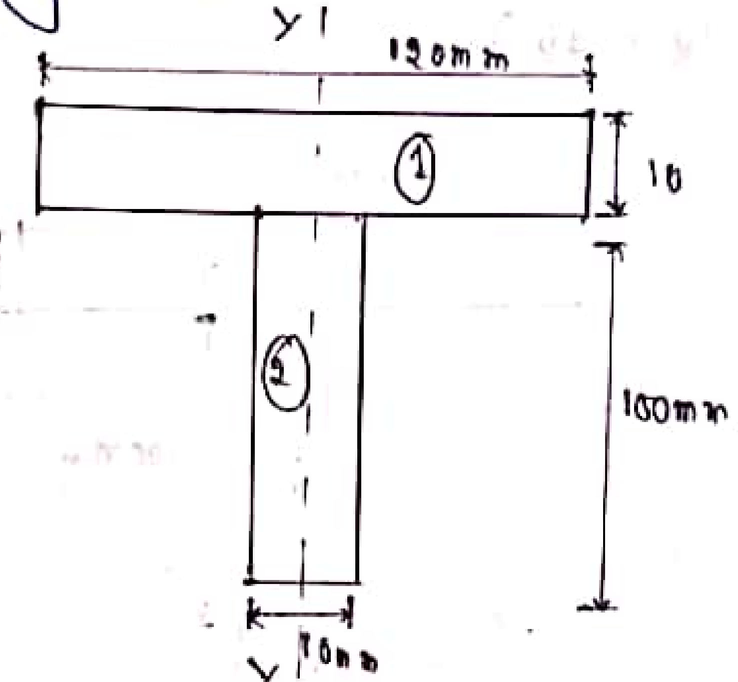
$c.g = (\bar{x}, \bar{y})$
$= (0, 75.81)$

Dt. 7.03.19
Tuesday

CG of the T-section:-

1. A 'T' cross-section is shown in the figure below, find the centroid of figure.

Given figure.



The section is symmetrical about YY axis.

$\bar{X} = 0$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

Rect-1

$a_1 = 120 \times 10$
$= 1200$

$y_1 = 100 + \frac{10}{2} = 105$

Rect-2

$a_2 = 100 \times 10 = 1000$

$y_2 = \frac{100}{2} = 50$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

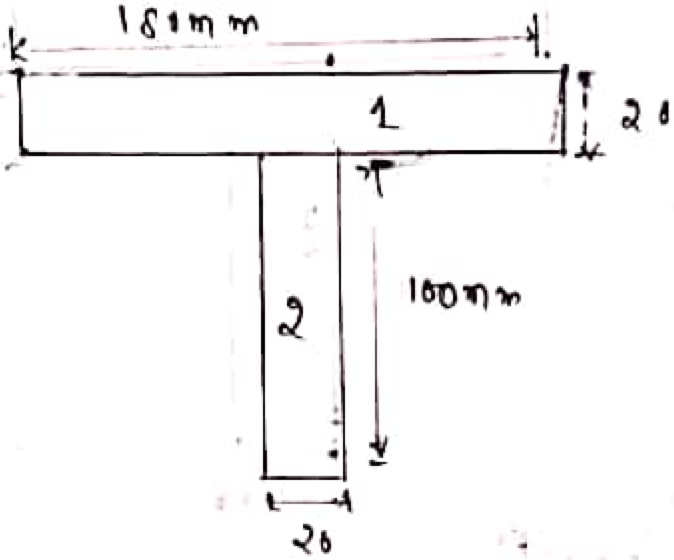
$$= \frac{1200 \times 105 + 1000 \times 50}{1200 + 1000}$$

$$= 80$$

$$C.G = (\bar{x}, \bar{y})$$

$$= (0, 80)$$

2.



The section is symmetrical about YY' axis.

$\bar{x} = 0$

Rect. 1

$a_1 = 180 \times 20$
$= 3600$

$y_1 = 100 + \frac{20}{2} = 100 + 10 = 110$

Rect - 2

$a_2 = 100 \times 20$
$= 2000$

$y_2 = \frac{100}{2} = 50$

$$\frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{3600 \times 110 + 2000 \times 50}{3600 + 2000}$$

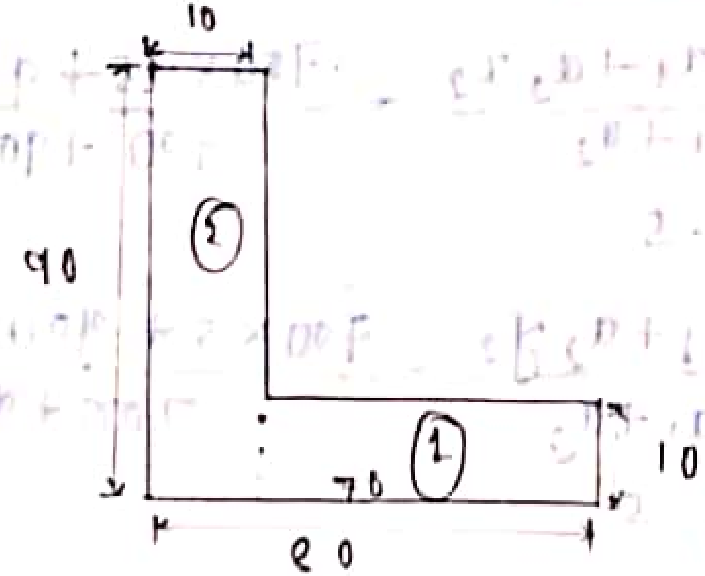
$$= 88.57$$

$$C.G = (\bar{x}, \bar{y})$$

$$= (0, 88.57)$$

Centre of gravity of 'L' section & Angle section.

Q. Determine the centroid at the angle section as shown in figure



The 'L' section is not symmetrical about any axis so we have found out both $\bar{x}$ & $\bar{y}$.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} \qquad \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_2 + a_2}$$

Rect - 1

$a_1 = 70 \times 10$
$= 700$

$y_1 = \frac{10}{2} = 5$

$x_1 = \frac{70}{2} + 10 = 45$

Rect - 2

$a_2 = 90 \times 10$
$= 900$

$y_2 = \frac{90}{2} = 45$

$x_2 = \frac{10}{2} = 5$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{700 \times 45 + 900 \times 5}{700 + 900}$$

$= 22.5$

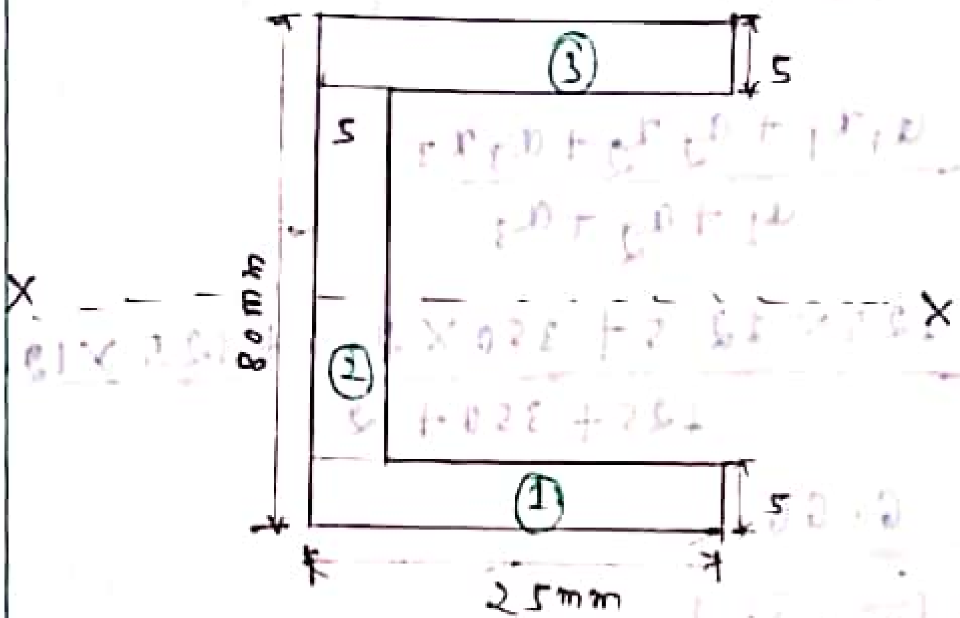
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{700 \times 5 + 900 \times 45}{700 + 900}$$

$= 27.5$

x_1, x_2 It is the distance between the centre of gravity & the refference axis.

Dt. 08.03.19
Friday

1. Determine the centroid of channel section as shown in figure.



$\bar{Y} = 0$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

The channel section is symmetrical about 'xx' axis, then $\bar{y} = 0$

And we have the found out the $\bar{x}$

Rect - 1

$a_1 = 25 \times 5$
$= 125$

$x_1 = 12.5$ mm

Rect - 2

$a_2 = 350$

$x_2 = 2.5$

Rect - 3

$a_3 = 25 \times 5$
$= 125$

$x_3 = 12.5$

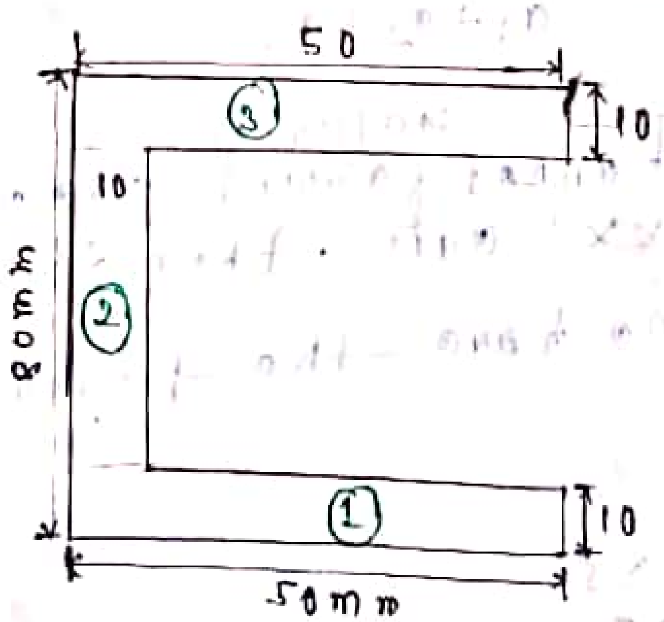
$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{125 \times 12.5 + 350 \times 2.5 + 125 \times 12.5}{125 + 350 + 125}$$

$= 6.66$.

$C.G = (\bar{x}, \bar{y})$
$= (6.66, 0)$

Q.



The channel is symmetrical about 'xx' axis

And we have the found out the $\bar{x}$

$\bar{Y} = 0$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

Rect - 1

$a_1 = 50 \times 10$
$= 500$

$x_1 = \frac{50}{2} = 25$

Rect - 2

$a_2 = \cancel{80\times} \; 60 \times 10$
$= 600$

$x_2 = \frac{10}{2} = 5$

Rect - 3

$a_3 = 50 \times 10$
$= 500$

$x_3 = \frac{50}{2} = 25$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{500 \times 25 + 600 \times 5 + 500 \times 25}{500 + 600 + 500}$$

$= 17.5$

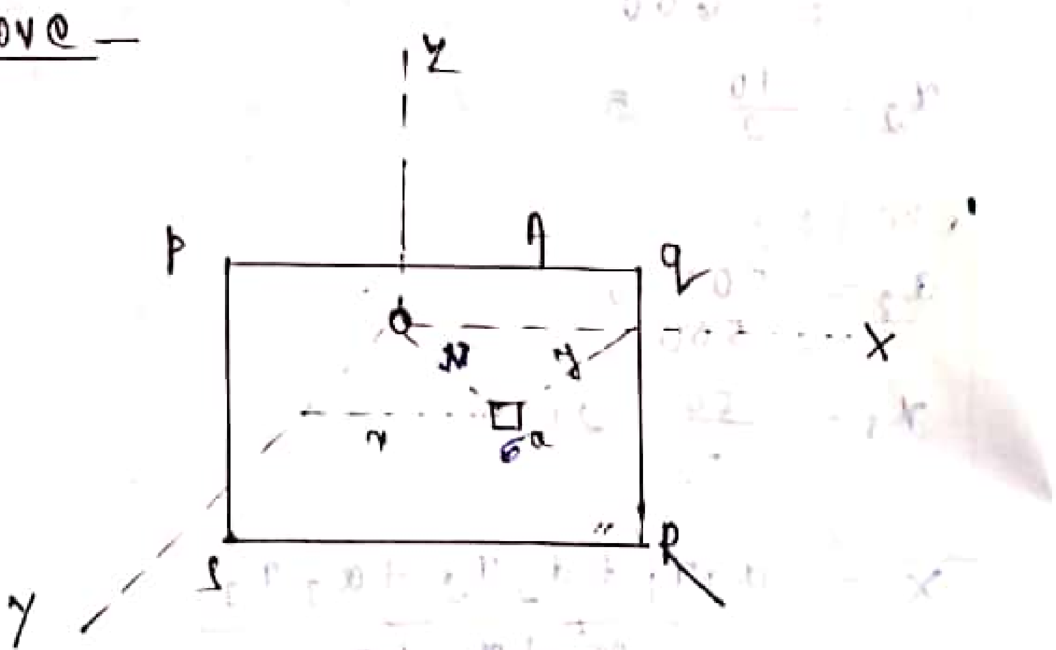
$C.G = (\bar{x}, \bar{y})$
$= (17.5, 0)$

Dt. 11.03.19
Monday

PERPENDICULAR AXIS THEOREM:-

perpendicular - It states that MI_{xx} & MI_{yy} be the moment of inertia of a plane area about a perpendicular axis x-x, & y-y, which are meeting at 'o', the moment of inertia MI_{zz} about zz axis perpendicular to the plane & passing through the point $MI_{zz} = MI_{xx} + MI_{yy}$

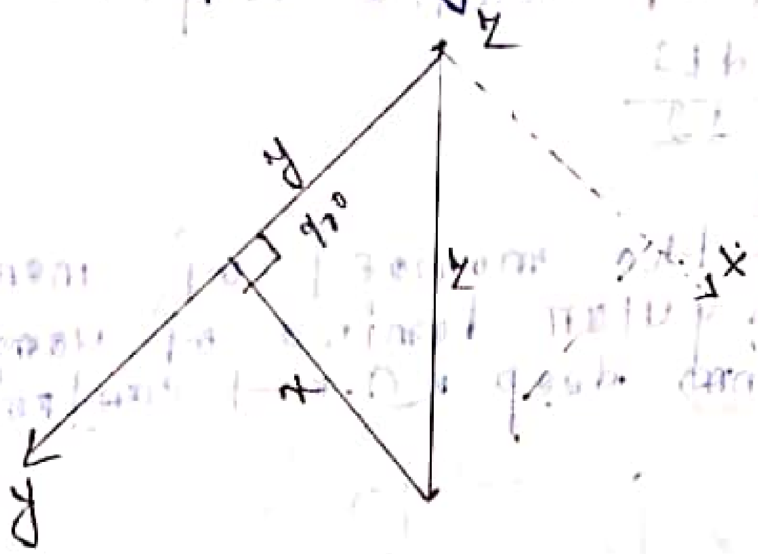
Proove -



Let us consider a plane area 'A' in the co-ordinate axis of xx, yy & zz as shown in figure.
Assume that the axis xx & yy are parallel to the plane area & zz axis is perpendicular to the plane area.

All the three perpendicular axis are meeting at a point 'o'.

Assume that a small strip area δa of the plane is at a distance 'x' y, z with refference to xx, yy, zz axis respectively.



From the above Geometry

$z^2 = x^2 + y^2$ ——— eqn (1)

MI of a strip area δa about xx axis is written as.

$MI_{xx} = \delta a y^2$ ——— eqn (2)

$MI_{yy} = \delta a x^2$ ——— eqn (3)

$MI_{zz} = \delta a z^2$ ——— eqn (4)

putting the value of eqn (1) & eqn (4)

$MI_{zz} = \delta a z^2$

$\Rightarrow MI_{zz} = \delta a(x^2 + y^2)$

$\Rightarrow MI_{zz} = \delta a x^2 + \delta a y^2$

$\Rightarrow MI_{zz} = MI_{yy} + MI_{xx}$

MOMENT OF INERTIA OF A RECTANGULAR SECTION :-

Moment of inertia about xx axis is

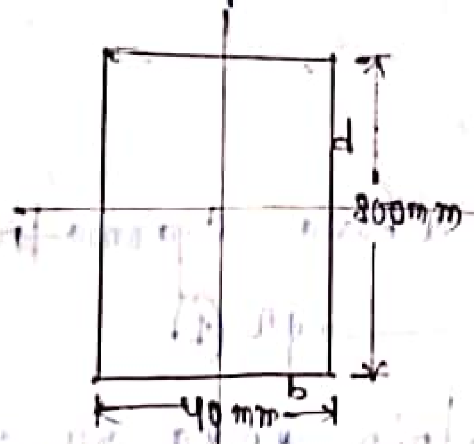
$$MI = \frac{bd^3}{12}$$

Moment of inertia about yy axis is

$$MI = \frac{db^3}{12}$$

Problem:

Find the moment of inertia rectangular lamina of 40mm wide & 800mm deep w.r.t centroid axis.



Given data,

$b = 40mm$

$d = 800mm$

Moment of inertia of rectangle about an axis passing through the centre of gravity & parallel (xx) axis

$$MI_{cg} = \frac{bd^3}{12}$$

$$= \frac{40 \times 800^3}{12} = 1706666667$$

$$= 17.07 \times 10^8$$

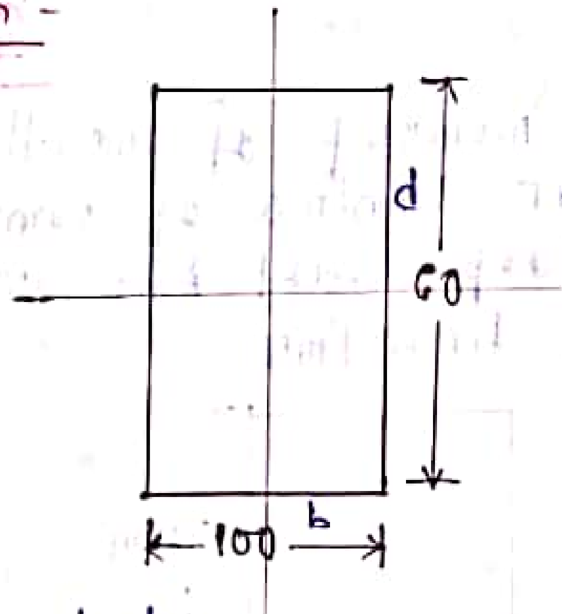
Moment of inertia of rectangular section about an axis passing through & parallel to X-X axis

$$MI_{cg} = \frac{db^3}{12}$$

$$= \frac{800 \times 40^3}{12} = 4266666.667$$

$$= \cancel{42.6766}\ 42.67 \times 10^5$$

Problem:-



Given data.

$b = 100mm$.

$d = 60mm$

Moment of inertia of rectangular about an axis passing through the centre of gravity & parallel (xx) axis

$$MI_{cg} = \frac{bd^3}{12}$$

$$= \frac{100 \times 60^3}{12} = 1800000$$

$$= 18 \times 10^5$$

Moment of inertia of rectangular section about an axis passing through & parallel to yy-axis.

$$MI_{cg} = \frac{db^3}{12}$$

$$= \frac{60 \times 100^3}{12}$$

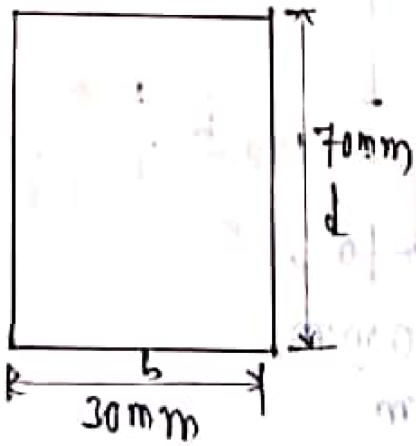
$$= 5000000$$

$$= 50 \times 10^5$$

PROBLEM 1-

Dt. 12.03.19
Tuesday

Find the moment of inertia of a rectangular lamina of 30mm wide & 70mm deep about its cg & least radius of direction



Given data,

$b = 30mm$

$d = 70mm$

$$MI_{cg_{xx}} = \frac{bd^3}{12}$$

$= \frac{30 \times 70^3}{12}$

$= 857500$

$= 8.57 \times 10^5 \; mm^4$

$MI_{cg}YY = \frac{db^3}{12}$

$= \frac{70 \times 30^3}{12}$

$= 157500$

$= 1.57 \times 10^5 \; mm^4$

RADIUS OF DIRECTION :—

$MI = Ak^2$

Where,

K = Radius of direction

$\Rightarrow K^2 = \frac{MI}{A}$

$\Rightarrow K = \sqrt{\frac{MI}{A}}$

Least radius direction,

$K = \sqrt{\frac{MI_{YY}}{A}}$

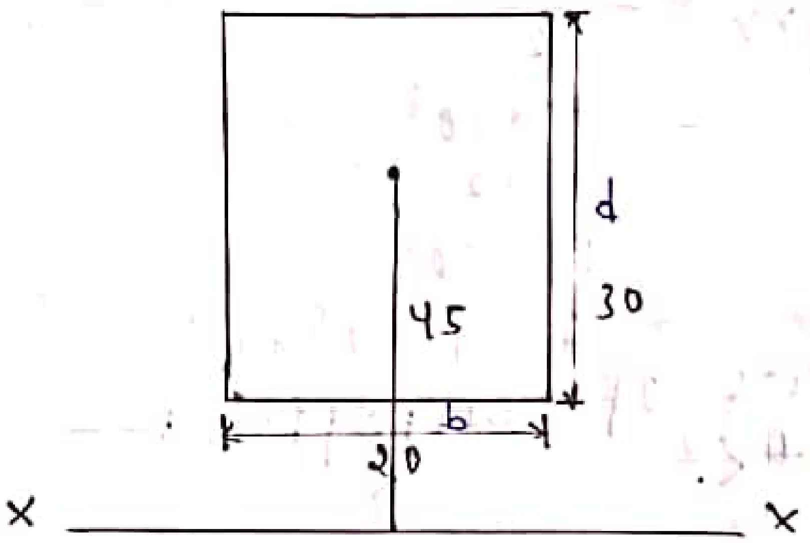
$A = 70 \times 30$
$= 2100$

$= \sqrt{\frac{1.57 \times 10^5}{2100}}$

$= \cancel{74.761}\; 8.646$

PROBLEM - 2

Find the moment of ~~area~~ inertia of a rectangle 20mm wide & 30mm deep about a given axis which is has a distance of 45mm from its centroid.



Given data,

$b = 20mm$

$d = 30mm$

Distance between centroid of gravity & given axis .XX - axis is 'l'.

$$MI_{cg} = \frac{bd^3}{12}$$

$$= \frac{20 \times 30^3}{12}$$

$$= 45,000 \ mm^4$$

$$A = b \times d$$

$$= 20 \times 30$$

$$= 600$$

$MI_{xx} = MI_{cg} + AL^2$

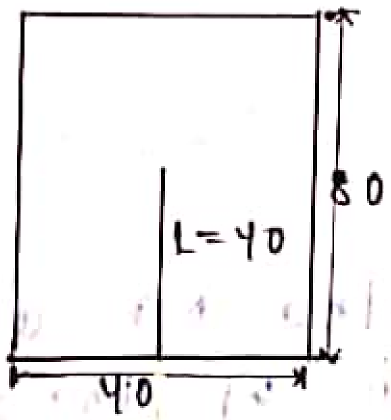
$\Rightarrow MI_{xx} = 45,000 + 600 \times 45^2$

$= 1260000 \ mm^4$

or 1.26×10

PROBLEM - 3

calculate the MI of a rectangle about centroid axis & also findout MI about its base AB.



Given data,

$b = 40mm$

$d = 80mm$

$$MI_{cg\,xx} = \frac{bd^3}{12}$$

$$= \frac{40 \times 80^3}{12}$$

$$= 1706666.667$$

$$MI_{cg\,YY} = \frac{d\,b^3}{12}$$

$$= \frac{80 \times 40^3}{12}$$

$$= 426666.6667$$

$$A = b \times d$$

$$= 40 \times 80$$

$$= 3200$$

$$MI_{AB} = MI_{cg} + AL^2$$

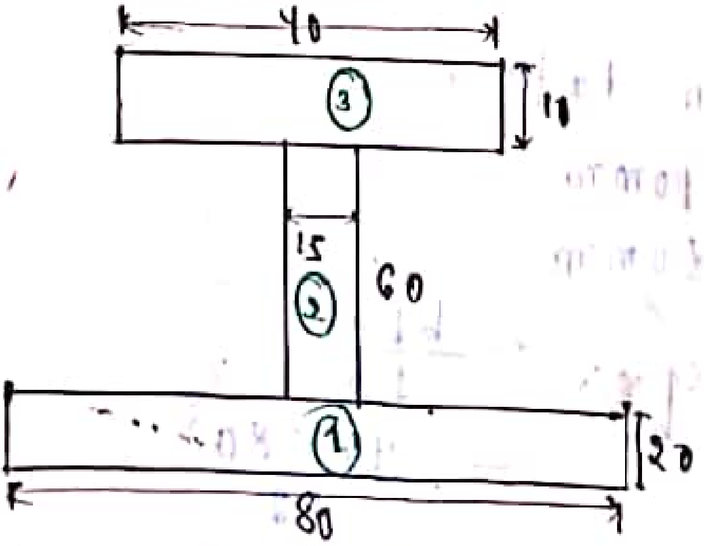
$$= 1706666.667 + 3200 \times 40^2$$

$$= 6826666.667\ mm^4$$

Dt. 15.03.19
Friday

PROBLEM:-

calculate the MI about the centroid axis of the following.



$\frac{3.5}{70/2}$

The given 'I' section is symmetrical about Y-Y axis.
Divide the 'I' section is rectangle

BF = Rect - 1

$a_1 = 1600$

$y_1 = \frac{20}{2} = 10$

Web

$a_2 = 15 \times 60$
$= 900$

$\cancel{y_2 = \frac{60}{2} = 30 - \frac{80}{2} + 20 = 600}$

$y_2 = \frac{60}{2} + 20 = 50$

TP.

$a_3 = 400$

$y_3 = 20 + 60 + \frac{10}{2}$
$= 85$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{1600 \times 10 + 900 \times 50 + 400 \times 85}{1600 + 900 + 400}$$

$= 32.75$

$L_1 = \bar{y} - y_1$
$= 32.75 - 10$
$= 22.75$

$L_2 = y_2 - \bar{y}$
$= 50 - 32.75$
$= 17.25$

$L_3 = y_3 - \bar{y}$
$= 85 - 32.75$
$= 52.25$

BF

$$MICg\,BF = \frac{bd^3}{12}$$

$$= \frac{80 \times 20^3}{12} = 53333.33333$$

$$MICg\,BF_{xx} = MICg\,BF + A_1 L_1^2$$
$$= 53333.33 + 1600 \times (22.75)^2$$
$$= 881433.33$$

$$MICg\,wed = \frac{b \cdot d^3}{12}$$

$$= \frac{15 \times 60^3}{12} = 270000 \text{ mm}^2$$

$$MICg\,wed_{xx} = MICg\,wed + A_2 L_2^2$$
$$= 270000 + 900 \times (17.25)^2$$
$$= 537806.25$$

$$MI_{cg\,top} = \frac{bd^3}{12}$$

$$= \frac{40 \times 10^3}{12} = 3333.33$$

$$MI_{cg\,top\,xx} = MI_{cg\,top} + A_3 L_3^2$$

$$= 3333.33 + 400 \times (52.25)^2$$

$$= 1095358.33$$

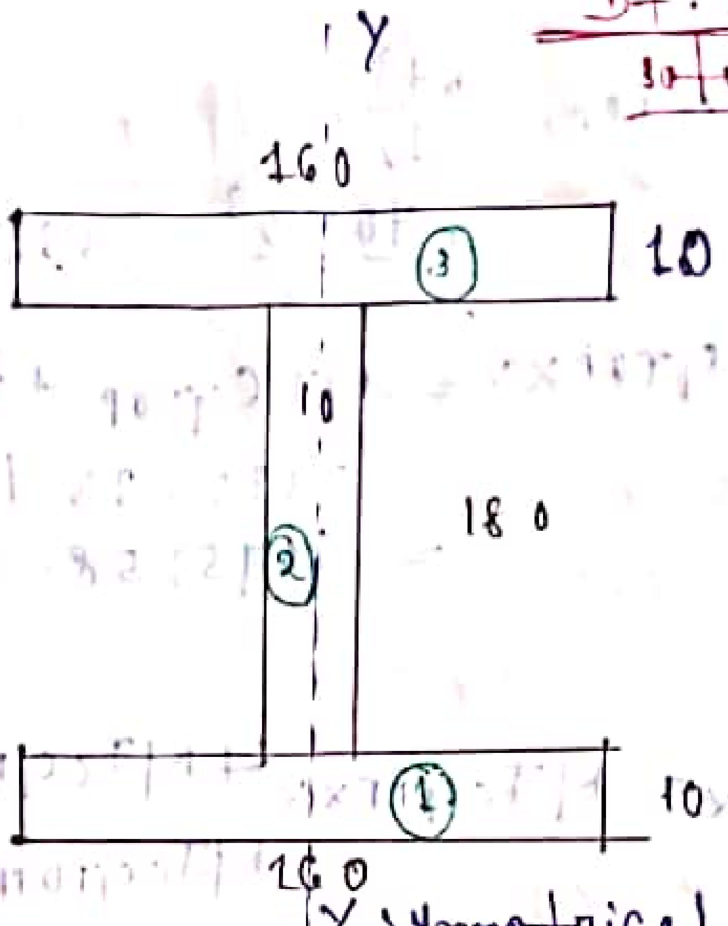
$$MI_{xx} = MI_{cg\,bf\,xx} + MI_{cg\,web\,f\,xx} + MI_{cg\,top\,f\,xx}$$

$$MI_{xx} = 881433.33 + 537806.25 + 1095358.33$$

$$= 2514597.91$$

Dt. 16.03.17
Saturday

1



Find the MI of an 'I' section of the following dimensions about the centroid axis. Find the radius of gyration also. Top & bottom flange 160×10mm, web 180×10mm

The 'I' section symmetrical about 'X X' axis. YY axis

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

BF / Rect 1

$a_1 = 160 \times 10$
$= 1600 \text{ mm}^2$

$y_1 = \frac{10}{2} = 5$

Web / Rect 2

$$a_2 = 180 \times 10$$
$$= 1800$$

$$y_2 = 10 + \frac{180}{2} = 100$$

~~Bottom~~ R·TF / Rect 3 :-

$$a_3 = 160 \times 10$$
$$= 1600 mm^2$$

$$y_3 = 10 + 180 + \frac{10}{2} = 195 mm$$

$$\bar{Y} = \frac{1600 \times 5 + 1800 \times 100 + 1600 \times 195}{1600 + 1800 + 1600}$$

$$= 100$$

$$\bar{Y} = 100$$

$$L_1 = \bar{Y} - y_1$$
$$= 100 - 5 = 95$$

$$L_2 = \bar{Y} - Y_2$$
$$= 100 - 100 = 0$$

$$L_3 = y_3 - \bar{Y}$$
$$= 195 - 100$$
$$= 95$$

$$MI_{cg\,xx\,tf} = MI_{cg\,BF} + a_1 L_1^2$$

$$= \frac{bd^3}{12} + 1600 \times (95)^2$$

$$= \frac{160 \times 10^3}{12} + 14440000$$

$$= 5.13333.33333 + 14440000$$

$$= 14453333.33$$

$$= 14.45 \times 10^6$$

$$MI_{cg\,web\,xx} = MI_{cg\,web} + a_2 L_2^2$$

$$= \frac{bd^3}{12} + 1600 \times 0$$

$$= \frac{10 \times 180^2}{12}$$

$$= 4860000$$

$$= 4.86 \times 10^6$$

$$MI_{cg\,xx\,top} = MI_{cg\,TF} + a_3 L_3^2$$

$$= \frac{bd^3}{12} + 1600 \times (95)^2$$

$$= \frac{160 \times 10^2}{12} + 14440000$$

$$= 14.45 \times 10^6$$

$$MI_{xx} = MI_{xx\,BF} + MI_{xx\,web} + MI_{xx\,TF}$$

$$= 14.45 \times 10^6 + 4.86 \times 10^6 + 14.45 \times 10^6$$

$$= 33760000$$

$$= 33.76 \times 10^6$$

Moment of inertia about 'yy' axis

Dt. 09.03.19
SATURDAY

MOMENT OF INERTIA :-

(i) The moment of a force is the product of the force & perpendicular distance between a refference point & the line of action of the force. The moment of force = $F \times x$.

(ii) The second moment of the force is the product of moment of their force & the same perpendicular distance.
$MI = Fx^2$

UNIT OF MOMENT OF INERTIA :-
The formula for area of moment of inertia is $MI = ax^2$
$= mm^2\ mm^2 = mm^4$

Where, a = Area
x^2 = perpendicular distance.

PARALLEL AXIS THEOREM:-

It states that if the moment of inertia of a plane area about an axis through its centre of gravity is denoted by MI_{cg}, then moment of inertia of the area about any other axis X-X axis, which is parallel to the first said axis & at a distance from the centre of gravity is written as:

$$MI_{xx} = MI_{cg} + AL^2$$

Where,

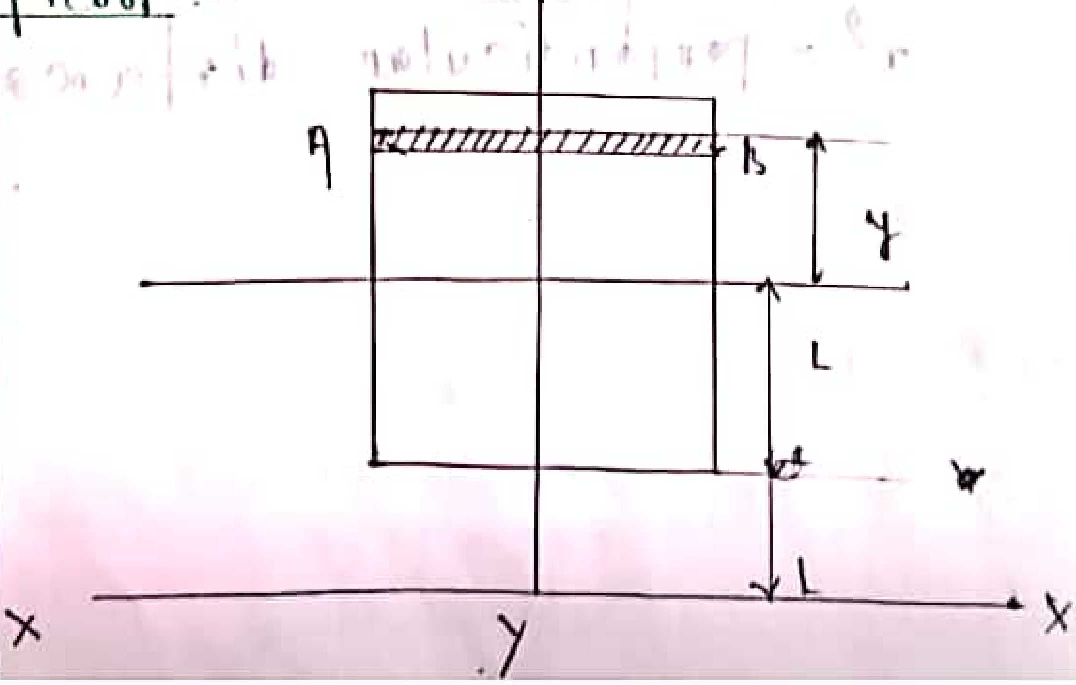
MI_{xx} = Moment of inertia of the plane area about x-x axis.

MI_{cg} = Moment of inertia of the plane about centre of gravity

A = Area of the plane

L = Distance b/w cg & x-x axis

proof:-



Let us consider a rectangular of plane area (A) as shown in figure.

Let us assume a small strip AB at a distance of 'y' from the C.G.

- The plane area is symmetrical about 'yy' axis.
- The C.G point is at a distance of 'L' from the 'xx' axis.

δa = Area of small strip AB assume.

Moment of inertia of the strip AB, about an axis passing through the C.G.

$MI = \delta a y^2$

Moment of inertia of the whole plane area about C.G.

$MI_{CG} = \Sigma \delta a y^2$ —— (1)

Moment of inertia of strip 'AB' about 'xx' axis,

$MI_{xx} = \delta a \times (L + y)^2$.

Moment of inertia of the whole area about 'yy' axis

$MI_{xx} = \Sigma \delta a \, (L + y)^2$

$\Rightarrow MI_{xx} = \Sigma \delta a \, (L^2 + y^2 + 2Ly)$ —— (2)

$= \Sigma \delta a L^2 + \Sigma \delta a y^2 + \Sigma \delta a \, 2Ly$

- The first term of the above eqn is $\Sigma \delta a L^2$

$= AL^2 \;\; (\because \Sigma \delta a = A)$ —— (3)

The second term to can be written by the comparing the eqn (1).

$M.I_{cg} = \Sigma \delta a y^2$ ——— (4)

In the above term $\Sigma \delta a y$ is the algebric sum of the moment of the all strip area about an axis through the c.g of the plane area is written as .

Hence the 2nd term is 0. Putting the value of eqn (3) & (4) in eqn (2)

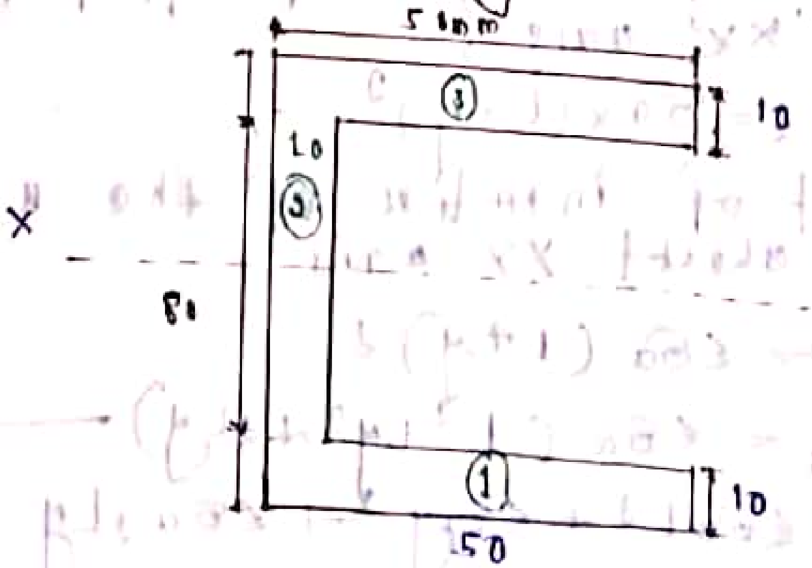
$\Rightarrow M.I_{xx} = AL^2 + M.I_{cg}$

$\Rightarrow \boxed{M.I_{xx} = M.I_{cg} + AL^2}$

Proved.

D- 18.03.19
Monday

Find the M.I of a channel section as shown in figure.



'c' section is symmetrical about xx axis.

$\bar{y} = 0$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

sec - 1

$a_1 = 50 \times 10 = 500$

$x_1 = \frac{50}{2} = 25$

Section - 2

$a_2 = 80 \times 10 = 800$

$x_2 = \frac{10}{2} = 5$

sec - 3

$a_3 = 50 \times 10 = 500$

$x_3 = \frac{50}{2} = 25$

$$\bar{x} = \frac{500 \times 25 + 800 \times 5 + 500 \times 25}{500 + 800 + 500}$$

$= 16.11$

$\bar{x} = 16.11, \; x_1 = 25, \; x_2 = 5, \; x_3 = 25$

$L_1 = x_1 - \bar{x} = 25 - 16.11 = 8.89$

$L_2 = \bar{x} - x_2 = 16.11 - 5 = 11.11$

$L_3 = x_3 - \bar{x} = 25 - 16.11 = 8.89$

$MI_{cg\,of\,yy} = MI_{cg\,of} + a_1L_1^2$

$= \frac{db^3}{12} + 500 \times 8.89$

$= \frac{10 \times 50^3}{12} + 500 \times 8.89^2$

$= 143682.7167$

$$M.I_{cg\,web\,yy} = M.I_{cg\,web} + A_2 h_2^2$$

$$= \frac{80 \times 10^3}{12} + 800 \times 11.11$$

$$= 105412.3467$$

$$M.I_{cgTF\,yy} = M.I_{cgTF} + A_3 h_3^2$$

$$= \frac{10 \times 50^3}{12} + 500 \times 8.89^2$$

$$= 143682.7167$$

$$M.I_{cnyy} = M.I_{cg\,BF\,yy} + M.I_{cg\,web} + M.I_{cgTF}$$

$$= 143682.7167 + 105412.3467 + 143682.7167$$

$$= 3.92 \times 10^5$$

$$M.I_{yy} = Ak^2$$

$$\Rightarrow k^2 = \frac{M.I_{yy}}{A}$$

$$\Rightarrow k = \sqrt{\frac{M.I_{yy}}{A}}$$

$$A = a_1 + a_2 + a_3$$

$$= 500 + 800 + 500$$

$$\Rightarrow k = \sqrt{\frac{3.92 \times 10^5}{500 + 800 + 500}}$$

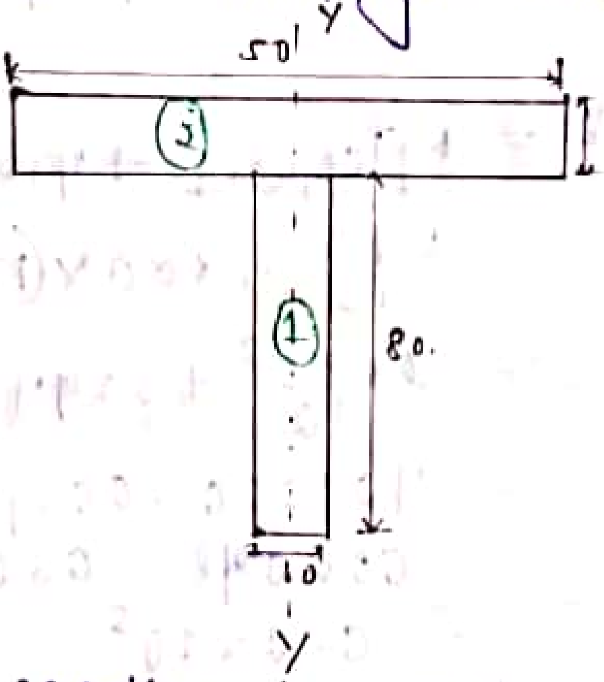
$$= 14.75729575$$

$$= 14.75 \times 10^8$$

17.02.19, Tuesday.

PROBLEM:-

A bar of 'T' section has flange 50mm wide & 10mm thick the web is 80mm deep & 10mm thick as shown in figure. find the MI of the section about the centrifugal axis XX & YY.



split 'T' section is rectangle

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

Rect - 1

$a_1 = 10 \times 80 = 800 mm^2$

$y_1 = \frac{80}{2} = 40 mm$

Rect - 2

$a_2 = 50 \times 10 = 500$.

$y_2 = 80 + \frac{10}{2} = 85$

$$\bar{y} = \frac{800 \times 40 + 500 \times 85}{800 + 500}$$

$= 57.30$

$MI_{xx\,web} = MI_{cg\,web} + a_1 L_1^2$

$L_1 = \bar{y} - y_1$

$= 57.30 - 40 = 17.3$

$L_2 = y_2 - \bar{y}$

$= 85 - 57.30 = 27.7$

$L_2 =$

$MI_{xx\,web} = MI_{cg\,web} + a_1 L_1^2$

$= \frac{bd^3}{12} + 800 \times (17.3)^2$

$= \frac{10 \times 80^3}{12} + 239432$

$= 426666.6667 + 239432$

$= 666098.6667$

$= 6.6 \times 10^5$

$MI_{xx\,flange} = MI_{cg\,flange} + a_2 L_2^2$

$= \frac{bd^3}{12} + 500 \times (27.7)^2$

$= \frac{50 \times 10^3}{12} + 383645$

$= 4166.66 + 383645$

$= 387811.66$

$= 3.87 \times 10^5 \ mm^4$

$MI_{xx} = MI_{xx\,web} + MI_{xx\,flange}$

$= 6.67 \times 10^5 + 3.87 \times 10^5$

$= 1045000$

MI I_{yy} MI of the 'T' section is yy axis, where $L_1 = 0$; $L_2 = 0$

$$MI_{yy\,web} = MI_{cg\,yy\,web} + A_1 L_1^2$$

$$= \frac{db^3}{12} = \frac{80 \times 10^3}{12}$$

$$= 6.6 \times 10^3$$

$$MI_{yy\,flange} = MI_{cg\,flange} + A_2 L_2^2$$

$$= \frac{db^3}{12} = \frac{10 \times 50^3}{12}$$

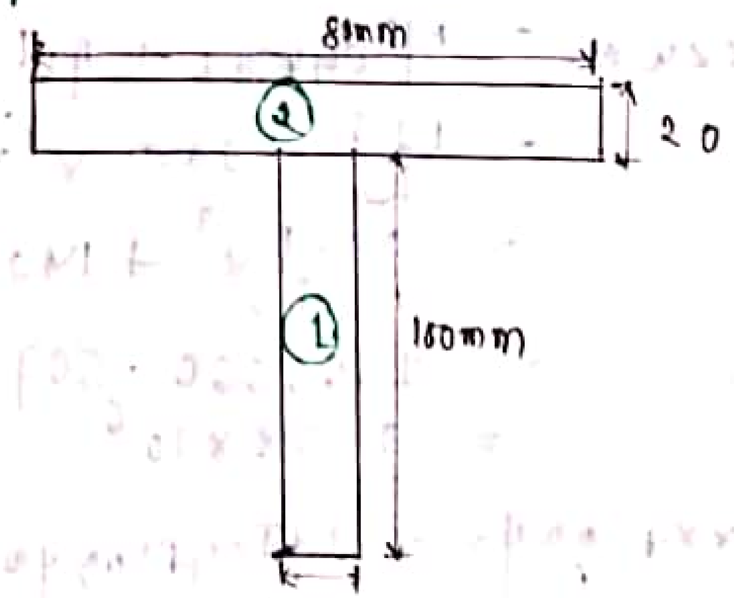
$$= 1.04 \times 10^5$$

$$MI_{yy} = MI_{cg\,web} + MI_{cg\,flange}$$

$$= 6.6 \times 10^3 + 1.04 \times 10^5$$

$$= 110600$$

PROBLEM:-



split 'T' section is rectangle

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

Rec-1 - 1

$a_1 = 20 \times 100 = 2000$

$y_1 = \frac{100}{2} = 50$

Rec-1 - 2

$a_2 = 80 \times 20 = 1600$

$y_2 = 100 + \frac{20}{2} = 110$

$$\bar{Y} = \frac{2000 \times 50 + 1600 \times 110}{2000 + 1600}$$

$= 76.66$

$MI_{xx\,web} = MI_{cy\,web} + a_1 L_1^2$

$L_1 = \bar{Y} - Y_1$

$= 76.66 - 50$

$= 26.60$

$L_2 = Y_2 - \bar{Y}$

$= 110 - 76.66$

$= 33.34$

$MI_{xx\,web} = MI_{cy\,web} + a_1 L_1^2$

$= \frac{bd^3}{12} + 2000 \times (26.66)^2$

$= \frac{20 \times 100^3}{12} + 1421511.2$

$= 1666666.667 + 1421511.2$

$= 3.08 \times 10^6$

$MI_{xx\text{-}flange} = MI_{cy\,flange} + a_2 L_2^2$

$= \frac{bd^3}{12} + 1600 \times (33.34)^2$

$= \frac{80 \times 20^3}{12} + 1778488.96$

$= 53333.3333 + 1778488.96$

$= 1.83 \times 10^6$

$M.I_{xx} = M.I_{xx\,web} + M.I_{xx\,flange}$

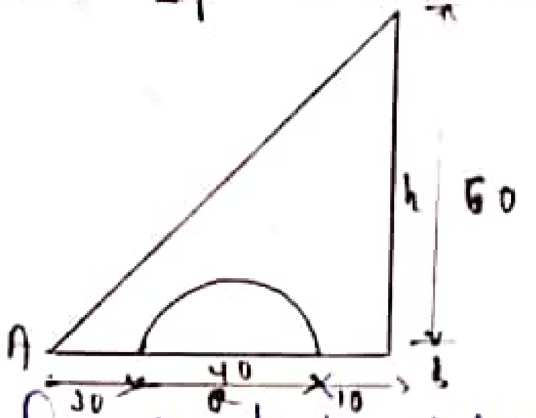
$= 3.08 \times 10^6 + 1.83 \times 10^6$

$= 4910000$

D 25.03.19
Monday

PROBLEM :-

Find the MI of a shaded area.



The MI of a triangle is,

$$MI \text{ of } \Delta = \frac{ah^3}{12}$$

$$= \frac{80 \times 60^3}{12}$$

$$= 1.44 \times 10^6 \text{ mm}^4$$

MI of the semicircle AB = MI of half circle $= \frac{1}{2} \times \frac{\pi}{64} d^4$

$$= \frac{1}{2} \times \frac{\pi}{64} \times 8.40^4$$

$=$

Now MI of shadded area is

MI of Δ = MI of half circle.

$$= 1.44 \times 10^6 - 6.28 \times 10^4$$

$$= 1.3 \times 10^6 \ mm^4$$

CHAPTER- 5

SIMPLE MACHINES

A machine is a device, which receives energy in sum available form & uses in it doing a useful work.

Ex:- steam engine.

Input - steam energy.
Output - mechanical energy

IC engine (Internal combustion engine)

Input - fuel energy or chemical energy.
Output - Mechanical energy

Leath machine -
Input - Electrical energy.
Output - Mechanical energy.

SIMPLE MACHINES:-

PRIMARY SIMPLE MACHINE:-

1. Leavers
2. Inclined plane
3. Wedge
4. Wheel & axil & axle
5. Screw

6 pulley:

secondary simple Machine:

1- Differential pulley.
2- pulley of 3 system.
3- ~~corr~~ worm & worm wheel.
4- Rack and pi
5- Differential screw jack
6- Differential wheel & axle.

FUNDAMENTAL TERMS OF SIMPLE MACHINE:

Mechanical advantages:

$$\boxed{M.A = \frac{W}{P}}$$

- The mechanical advantages is the ratios of weight lifted (W) to the effort applied P.

Velocity Ratio (VR):

$$\boxed{VR = Y/x}$$

$$\boxed{\eta = \frac{MA}{VR} \times 100}$$

PROBLEM:-

The velocity ratio of a simple machine is 10, the effort apply is 150N, determine the η if load lifted is 1200N.

Ans- Given data,

$VR = 10$

$P = 150N$

$\eta = ?$

$W = 1200N$

$$\eta = \frac{MA}{VR} \times 100$$

$$= W/P = \frac{1200}{150} = 8$$

$$\eta = \frac{MA}{VR} \times 100$$

$$= \frac{8}{10} \times 100$$

$$= 80\%$$

PROBLEM - 2

In a simple lifting machine & effort of 500N raised in load 12.5KN. What is the mechanical advantages if the machine has efficiency is 65%.

Ans:- Given data,

$P = 500N$

$W = 12.5KN = 12.5 \times 1000N$

$= 12500 N$

$MA = ?$

$\eta = \frac{65}{100} = 0.65$

$VR = ?$

$MA = \frac{W}{P} = \frac{12500}{500} = 25$

$VR = \eta = \frac{MA}{VR} \times 100$

$VR = \frac{MA}{\eta} = \frac{25}{0.65}$

$= 38.46$

PROBLEM. 3

In a simple lifting machine an effort of 250N raised a load of 6KN. What is the mechanical if the machine has efficiency is 60% & also find the velocity ratio.

Given data.

$P = 250N$

$W = 6KN = 6 \times 1000 N$
$= 6000$

$m.A = ?$

$\eta = \frac{60}{100} = 0.6$

$VR = ?$

$m.A = \frac{W}{P} = \frac{6000}{250} = 24$

$\eta = \frac{mA}{VR} \times 100$ $\qquad 60 = \frac{24}{VR}$

$V.R = \frac{mA}{\eta} = \frac{24}{0.6} = 40$.

Dt · 26 · 03 · 1?
Tuesday

PROBLEM 1 :-

A simple machines an effort of 100N by moving through a distance of 1 m. leaves a load of 1000 N. through a distance of 0.1m find VR, MA & η of the simple machine each the simple machine ideas.

Ans: Given data,

effort,
$P = 100N$

effort distance,
$Y = 1m$

$W = 1000N$

$x = 0.1m$

velocity Ratio,

$$VR = \frac{Y}{x} = \frac{1}{0.1} = 10.$$

Mechanical advantage,

$$MA = \frac{W}{P} = \frac{1000}{100} = 10$$

efficiency,

$$\eta = \frac{MA}{VR} \times 100$$

$$= \frac{10}{10} \times 100 = 100\%.$$

PROBLEM 2:-

In trolly jack and effort of 250N is lifting a load of 2 turns in lifting the load through a distance of 15cm the operator performes 40 ~~pumping~~ stroke of the handle is of width is 50cm long. calculate the MA, VR & η of the jack

Given data,

let effort,

$P = 250N$

Load $W = 2$ tunns

$= 2 \times 1000 \times 9.81$ N

$= 19620$ N

Load the moves through a distance

$x = 15$ cm

pumpink stroke ps = 40

stroke length of the handle = 50cm

Distance move by effort,

$y = ps \times sLH$

$= 40 \times 50 = 2000$ cm

Mechanical Advantage,

$$MA = \frac{W}{P} = \frac{19620}{250}$$

$$= 78.48$$

velocity ratio;

$$VR = \frac{y}{x} = \frac{2000}{15}$$

$$= \cancel{1.333 \times 10^2}$$

$$= 133.33$$

$$\cancel{\eta \neq \frac{MA}{VR} \times 100 \; 78.48}$$

Efficiency ;

$$\eta = \frac{MA}{VR} \times 100$$

$$= \frac{78.48}{133.33} \times 100$$

$$= 58.861\%$$

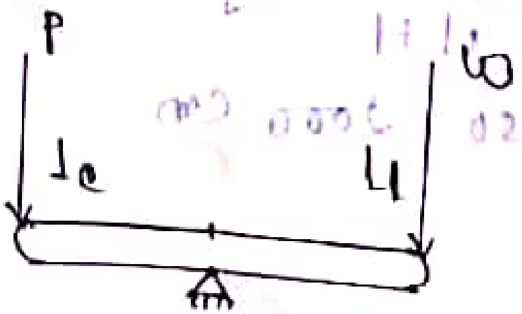
LEVERS:-

CLASSIFICATION OF LEVER:-

There are 3 types of lever

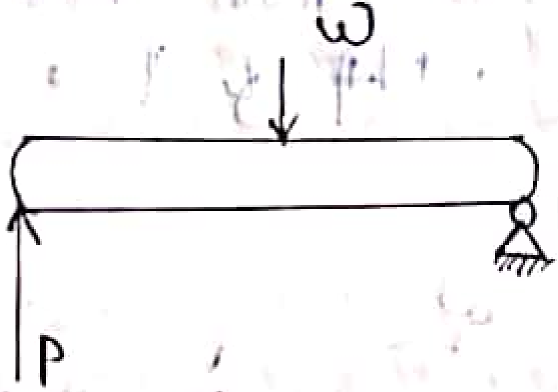
LEVER OF 1st ORDER:-

In this type of lever the effort and load act on the opposite side of the ~~fuld~~ fulcrum.



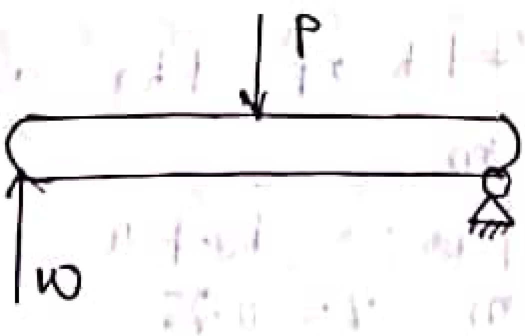
LEVER OF 2nd ORDER:-

In this type of lever the load acting between effort lever & fulcrum.



LEVER OF 3rd ORDER:-

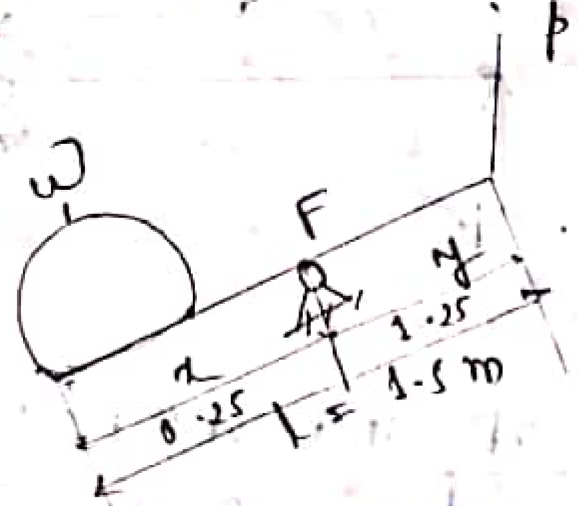
In this type of lever



Dt. 29.03.19
Friday

PROBLEM - 1

A load of 2000 N is lifted with crobar of length 1.5 m the fulcrum is placed at a distance of 0.25 m from the load, find the VR, MA & η & effort any.



Load to be lifted by the crobar
$W = 2000$ N

The length of the crobar,
$l = 1.5$ m

The distance betn the load & fulcrum $x = 0.25$

The distance betn the effort & of the fulcrum $y = 1.25$

$$VR = \frac{y}{x} = \frac{1.25}{0.25} = 5$$

Taking ~~the~~ moment about the fulcrumm = clockwise moment = Anticlock wise momen

$W \times x = P \times y$

$\Rightarrow 2000 \times 0.25 = P \times 1.25$

$\Rightarrow P = \frac{2000 \times 0.25}{1.25}$

$= 400$

$MA = \frac{W}{P}$

$= \frac{2000}{400} = 5$

$\eta = \frac{MA}{VR} \times 100$

$= \frac{5}{5} \times 100$

$= 100\%$

REVERSIBLE MACHINE :-

A machine which is capable of doing sum work in the reverse direction, after removal of effort is called reversible machine.

IMP CONDITIONS FOR REVERSIBILITY :-

Let us consider,

W = load lifted by the machine

P = effort required to lift the load.

y = distance through which the effort is move.

x = distance through which the load is move.

Reversible machine will work in reverse direction after removal of the effort.

- That means the output will act as input & the effort is 0.

- The to do the work the reverse direction the output has to overcome the friction in the machine the input of the machine = $P \times y$

the output of the machine = $W \times x$

Friction of the machine = input - output

$= (P \times y) - (W \times x)$

The machine is not running in reverse direction after removal of the effort.

that means the output is less than the friction in the machine

$$W \times x < (P \times y) - (W \times x)$$

$$\Rightarrow (W \times x) + (W \times x) > (P \times y)$$

$$\Rightarrow 2(W \times x) > (P \times y)$$

$$\Rightarrow \frac{W \times x}{P \times y} > \frac{1}{2}$$

$$\Rightarrow \frac{W/P}{y/x} > \frac{1}{2}$$

$$\Rightarrow \frac{MA}{VR} > \frac{1}{2}$$

$$\Rightarrow \eta > 50\%.$$

Dt. 30.03.19
saturday

SELF LOCKING OR IRREVERSIBILITY!—

A machine which is not capable of doing sum work in the reverse direction after removal of the effort is called self locking.

CONDITION FOR IRREVERSIBLE:-
Let us consider,

W = Load lifted by the machine

P = effort required to lift the Load.

y = distance through which the effort is move.

x = distance through which the load in move.

PROBLEM 1:-

In a simple machine an effort of 290 N is apply through a distance 220 cm to lift a load of 1200 N through a distance 40cm from the data. check them whether the machine is reversible or irreversible?

Ans:- Given data,

Effort applied $P = 240\,N$

$y = 220\,cm = \frac{220}{100} = 2.2\,m$

$W = 1200\,N$

$x = 40\,cm =$

$\eta = \frac{MA}{VR}$

$MA = W/P = \frac{1200}{290} = 4.13$

$VR = y/x = \frac{2.2}{0.4} = 5.5$

$\eta = \frac{MA}{VR} = \frac{4.13}{5.5} \times 100$

$= 75\,\%$

Hence the efficiency is greater than 50% show the machine is reversible.

OR

$P = 240\,N$

$y = 220\,cm = 2.2\,m$

$w = 1200\,N$

$x = 40\,cm = 0.4\,m$

$output = w \times x = 1200 \times 0.4 = 480$

$input = P \times y = 290 \times 2.2 = 638$

$friction = input - output$

$= 638 - 480$

$= 158$

$480 > 158$

output > friction

PROBLEM-2

A crowber of length 2m is lifting a weight of 800N so crowber is suport is a distance of 0.6m from the load, determine the effort appliy, vetiVR, MA, η.

Ans:- Load to be lifted by the crowbar

$W = 800N$

The length of the crowbar,

$l = 2m$

The distance betn the load & fulcrum $x = 0.6m$

The distance betn the effort & of the fulcrum $y = 1.4m$

$$VR = \frac{y}{x} = \frac{1.4}{0.6} = 2.33$$

Taking moment about the fulcrum :-

clockwise moment = Anticlockwise moment

$$W \times x = P \times y$$

$$\Rightarrow 800 \times 0.6 = P \times 1.4$$

$$\Rightarrow P = \frac{800 \times 0.6}{1.4} = 342.85$$

$\Rightarrow$

$$MA = \frac{W}{P} = \frac{800}{342.85} = 2.33$$

$$\eta = \frac{MA}{VR} \times 100$$

$$= \frac{2.33}{2.33} \times 100 = 100\%$$

PROBLEM3:-

In a simple machine g effort 300N which applied through a distance of 200 cm to lift a load of 1300N through a distance 35cm from the data check from the machine is reversible g irreversible.

Ans:- Given data.

Effort applied $P = 300\,N$

distance through which load

$y = 200\,cm = 2\,m$

$W = 1300\,N$

$x = 35\,cm = 0.35$

$\eta = \frac{MA}{VR}$

$MA = W/P = \frac{1300}{300} = 4.33$

$VR = y/x = \frac{2}{0.35} = 5.71$

$\eta = \frac{MA}{VR} \times 100$

$= \frac{4.33}{5.71} \times 100$

$= 75\%$

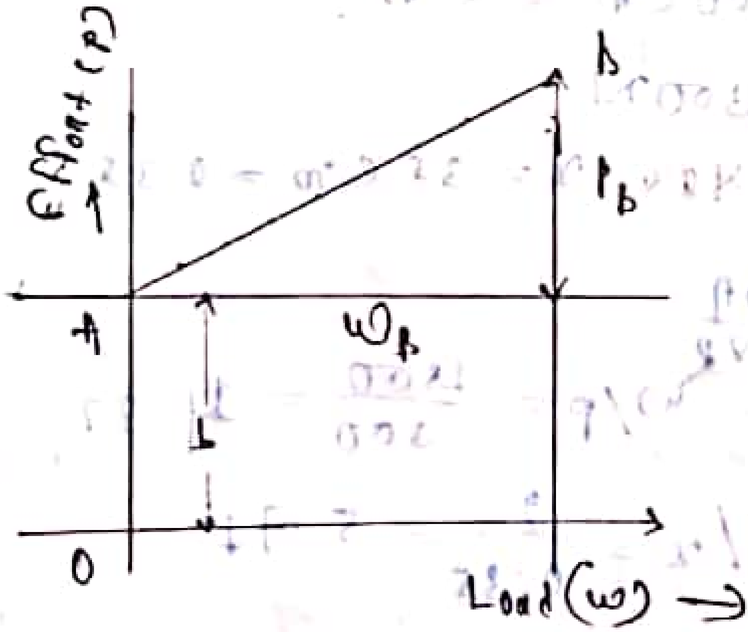
Hence the efficiency is greater than 50% show the machine is reversible

02.04.19
Tuesday

Law of Machines :-

The relationship betn effort apply & the load lifted is called law of machine. Let us considered any machine and record efforts required to lift difference loads.

Let a graf betn effort & load.



$$\boxed{\text{slope } (a) = \frac{P_B}{W_B}}$$

The relationship betn effort and load is a straight line.

According to co-ordinate geometry the straight line is written as eqn $\boxed{P = aW + b}$

Where,

P = effort apply

W = load to be lifted

a = slop of the straight line

Intersect of straight line on y-axis i.e. OA

$$\text{slope } (a) = \frac{P_B}{W_B}$$

$$\Rightarrow a = \frac{\text{effort at point B}}{\text{Load at point B}}$$

PROBLEM-1

In a lifting machine & effort of 98N lifts a load of 2450 N & effort of 127.4 N lifts a load of ~~3900~~ 3920. find the law of machine. (i) calculate the effort required to the load of 5S0N

(ii) Find the load that can be lifted 190N

(iii) What is the max^m efficiency of the machine 75.

Given data,

$P_1 = 98N$

$W_1 = 2450N$

$P_2 = 127.4N$

$W_2 = 3920N$

According to law of machine,

$P_1 = aW_1 + b$

$P_2 = aW_2 + b$

$P_1 = aW_1 + b$

$98 = a2450 + b$ —— (1)

$P_2 = aW_2 + b$

$127.4 = a3920 + b$ —— (2)

Subtract eqn (1) by eqn (2)

$29.4 = -1470 \Rightarrow a = \frac{29.4}{1470} = 0.02.$

putting the value of a in eqn (1)

$\Rightarrow 98 = 0.02 \times 2450 + b$

$\Rightarrow 98 = 49 + b$

$\Rightarrow -b = 49 - 98$

$\Rightarrow \cancel{-}b = \cancel{-}49$

$\Rightarrow b = 49$

$P = aW + b$

$\Rightarrow P = 0.02W + 49$

Dt. 05.04.19
Friday

(i) $P = 0.02 \times 5880 + 49$

$= 166.6\,N$

(ii) $P = 196\,N$

$\Rightarrow 196 = 0.02W + 49$

$\Rightarrow \cancel{196 - 49} =$

$\Rightarrow 0.02W = 196 - 49 = 147$

$\Rightarrow 0.02W = 147$

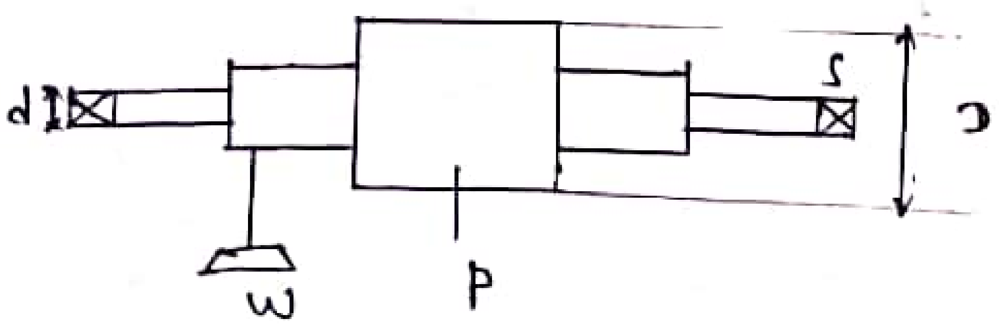
$\Rightarrow W = \dfrac{147}{0.02} = 7350$

$$\text{Maximum efficiency} = \frac{1}{a \times VR} \times 100$$

$$= \frac{1}{0.02 \times 75} \times 100$$

$$= 66.6\,\%$$

SIMPLE WHEEL AND AXLE:-



D = Diameter of the wheel

d = Diameter of the axle

P = effort apply

W = load to be lifted.

The displacement of the effort wheel a in one rotation.

$y = \pi D$

The displacement of the load axle be in on revolution =

$x = \pi d$

$$VR = y/x = \frac{\pi D}{\pi d} = \frac{D}{d}$$

$$MA = \frac{W}{P}$$

$$\eta = \frac{MA}{VR} = \frac{\frac{W}{P}}{\frac{VR}{1}} \Rightarrow \eta = \frac{W}{P} \times \frac{1}{VR}$$

$$\Rightarrow \eta = \frac{W}{P} \times \frac{1}{D/d}$$

$$\Rightarrow \frac{W}{P} \times \frac{d}{D} = \frac{Wd}{PD}$$

$$\Rightarrow \boxed{\eta = \frac{Wd}{PD}}$$

PROBLEM - 1

In a simple wheel & axle the radius of the effort wheel is 240mm & that of axle is 40mm. determine the efficiency if a load of 20940 N can be lifted by a and effort of 588 N.

Given data,

$R = 240mm$

$D = 2R = 2 \times 240 = 480 mm$

$r = 40mm$

$d = 2 \times r = 2 \times 40 = 80mm$

$\eta = ?$

$W = 2940 N$

$P = 588 N$

$$\eta = \frac{W \times d}{P \times D} = \frac{2940 \times 80}{588 \times 480} \times 100$$

$$= 0.83 \times 100$$

$$= 83\%$$

$$VR = \frac{D}{d} = \frac{480}{80} = 6$$

Dt. 06.04
saturday

DIFFERENTIAL WHEEL AND AXLE :-

D = Diameter of the wheel
d_1 = Diameter of the axle 1
d_2 = diameter of the axle 2
P = effort apply
W = Load lifted.

$$VR = \frac{2D}{(d_1 - d_2)}$$

$$MA = W/P$$

$$\eta = \frac{MA}{VR} = \frac{W/P}{\frac{2D}{(d_1 - d_2)}}$$

$$\eta = \frac{W(d_1 - d_2)}{2DP}$$

PROBLEM- 1

In a differential wheel & axle the diameter of the effort wheel is 400 mm. The ~~radial~~ radius of the axles are 150 mm & 100 mm respectively the diameter of rop is 10 mm. Find the load which can be lifted by an effort of 19G N assuming that the η of the machines is 75%

Given that,

$D = 400\,mm + 10 = 410$

$r_1 = 150\,mm$ $\quad d_1 = 2r_1 = 300\,mm \times 10 = 310$

$r_2 = 100\,mm$ $\quad d_2 = 2r_2 = 2 \times 100 = 200\,mm + 10$
$= 210$

$l = 10$

$P = 196\,N$

$\eta = 75\% = \frac{75}{100} = 0.75$

$$\eta = \frac{W(d_1 - d_2)}{2DP}$$

$$\Rightarrow 0.75 = W \frac{310 - 210}{2 \times 410 \times 196}$$

$$= W = \frac{0.75 \times 2 \times 410 \times 196}{100}$$

$$= 1205.4\,N$$

PROBLEM - 2

In a differential wheel & axle the diameter of the wheel is 200mm & the diameter of the axle are 50mm & 40mm to lift a load of 800N & effort of 40N is apply, find the efficiency of the machine and the effort loss in friction.

Given data,

$D = 200\ mm$

$d_1 = 50mm$

$d_2 = 40mm$

$W = 800N$

$P = 40N$

$\eta = ?$

$$\eta = \frac{W(d_1 - d_2)}{2DP} = \frac{800(50-40)}{2 \times 200 \times 40} \times 100$$

$$= 50\%$$

$$VR = \frac{2D}{(d_1 - d_2)} = \frac{2 \times 200}{50 - 40} = 40$$

$$MA = W/P = \frac{800}{40} = 20$$

effort loss in friction $= P - MA = 40 - 20 = 20$

Dt·08·04·19

SINGLE PURCHASE CRAB OR WINCH:-

D = Rotating diameter of the handle.

$\frac{D}{2} = R$: Radius of the handle or length of the handle.

d = diameter of the load drum.

T_1 = number of teeth on pinion gear.

T_2 = number of teeth on screw gear.

$$y = \pi D$$

$$x = \pi d \times \frac{T_1}{T_2}$$

$$VR = y/x = \frac{\pi D}{\pi d \times \frac{T_1}{T_2}} = \frac{DT_2}{dT_1}$$

$$\boxed{VR = \frac{DT_2}{dT_1}}$$

$$MA = \frac{W}{P}$$

$$\eta = \frac{MA}{VR} = \frac{W/P}{\frac{DT_2}{dT_1}} = \frac{WdT_1}{PDT_2}$$

PROBLEM 1:-

A single purchase crab has the following data length of the handle = $R = 120mm$, gear ratio $G = 6$, diameter of the load drum $d = 50mm$, load lifted to the $W = 900N$, effort apply $P = 100N$, calculate the η of the machines, Find the VR.

Ans: Given data,

$R = 120mm$

$D = 2 \times 120 = 240mm$

$G = \frac{T_2}{T_1} = 6$

$d = 50mm$

$W = 900N$

$P = 100N$

$$VR = \frac{DT_2}{dT_1}$$

$$\Rightarrow VR = \frac{D}{d} \times \frac{T_2}{T_1}$$

$$\Rightarrow VR = \frac{240}{50} \times 6 = 28.8$$

$$\eta = \frac{wd}{PD} \times \frac{T_1}{T_2}$$

$$= \frac{900 \times 50}{100 \times 240} \times 6$$

$$= 1125$$

DOUBLE PURCHASE CRAB OR [illegible] :-

D = diameter of the handle

R = Radius of the handle.

T_1 = number of teeth on pinion gear 1.

T_2 = number of teeth on screw gear 1.

T_3 = number of teeth on pinion gear 2.

T_4 = number of teeth on screw gear 2.

P = effort apply

W = load lifted

$$VR = \frac{DT_2 T_4}{dT_1 T_3}$$

$$M.A = W/P$$

$$\eta = \frac{W d T_1 T_3}{P D T_2 T_4}$$

PROBLEM:-

A double purchase crab has a following dimension, length of the handle R = 120 cm, radius of load drum r = 25 cm, load lifted to the W = 6000 N, number of teeth of pinion 30 & 40, no of teeth on screw gear 60 & 80, η of the machine 60%, calculate the effort apply.

Ans- Given data,

$R = 120$, $2R = 2 \times 120 = 240$

$r = 25$, $d = 2 \times 25 = 50$

$W = 6000 N$

$T_1 = 30$

$T_3 = 40$

$T_2 = 60$

$T_4 = 80$

$\eta = \frac{60}{100} = 0.6$

We know double purchase crab winch is,

$$\eta = \frac{WdT_1T_3}{PDT_2T_4}$$

$$P = \frac{WdT_1T_3}{\eta DT_2T_4}$$

$$= \frac{600 \times 50 \times 30 \times 40}{0.6 \times 240 \times 60 \times 80}$$

$$= 52.08 N$$

WORM & WORM WHEEL :-

D = Rotating diameter of the handle
d = diameter of the load pully.
W = Load lifted
P = effort apply.
T = no of teeth of ~~own~~ worm wheel

$$VR = \frac{DT}{d}$$

$$MA = \frac{W}{P}$$

$$\eta = \frac{MA}{VR} = \frac{W/P}{\frac{DT}{d}} = \frac{Wd}{PDT} \Rightarrow \eta = \frac{Wd}{PDT}$$

If the worm has n number of thread that,

$$VR = \frac{DT}{nd}$$

$$\eta = \frac{nWd}{PDT} \quad (n = \text{no of thread})$$

PROBLEM:-

In a worm & worm wheel, the worm wheel has 40 teeth, the diameter of worm pully is 180mm the diameter of the load drum is 120mm, with an η of 55% of the worm & worm wheel lifts a load of 3800N. Find the effort for the above condition.

Ans:- Given data,

$T = 40$

$D = 180mm$

$d = 120mm$

$\eta = 55\% = \frac{55}{100} = 0.55$

$W = 3800N$

$P = ?$

form the formula of worm & worm wheel is,

$$\eta = \frac{Wd}{PDT}$$

$$P = \frac{Wd}{\eta DT} = \frac{3800 \times 120}{0.55 \times 180 \times 40} = 115N$$

PROBLEM:-

In a double threaded worm & worm wheel the no of on the worm wheel is 80, the diameter of the ~~worm effort~~ wheel is 300mm & the diameter of the load drum is 160mm, 'η' of the machine is 45%, determine the VR & effort required to ~~load~~ lift a load ~~120mm~~ 120N.

Given data,

$T = 80$

$D = 300mm$

$d = 160mm$

$\eta = 45\% = \frac{45}{100} = 0.45$

$W = 120N$

$$VR = \frac{DT}{2d} = \frac{300 \times 80}{2 \times 160} = 75$$

$$\eta = \frac{Wd}{PDT}$$

$$\Rightarrow P = \frac{Wd}{\eta DT} = \frac{2 \times 120 \times 160}{0.45 \times 300 \times 80}$$

$$= 3.55\ N$$

SIMPLE SCREWJACK :-

L = Length of effort worm.

p = pitch of the screw.

W = Load to be lifted

P = effort apply.

$y = 2\pi l$

$x = p$

$$VR = \frac{y}{x} = \frac{2\pi l}{p}$$

$$MA = \frac{W}{P}$$

$$\eta = \frac{MA}{VR} = \frac{\frac{W}{P}}{\frac{2\pi l}{p}} = \frac{Wp}{2\pi l P}$$

PROBLEM:-

The pitch of screwjack is 7.5mm, the length of leaver rod at the end of which effort apply is 450mm, find the effort required to rise the load of 500N, if the 'η' is 45%.

Given data,

$p = 7.5\,mm$

$L = 450$

$P = ?$

$W = 500$

$\eta = 45\% = \frac{45}{100} = 0.45$

$VR = \frac{y}{x} = \frac{2\pi l}{p} = \frac{2\times\pi\times 450}{7.5} = \cancel{377.14}$ 376.99

$\eta = \frac{Wp}{2\pi l P}$

$P = \frac{Wp}{2\pi l\eta} = \frac{500\times 7.5}{2\times\pi\times 450\times 0.45} = 2.94N$

Dt. 9.04.19
Tuesday

CHAPTER - 6
DYNAMICS

NEWTON'S LAW OF MOTION :-

1st Law :-

A body continuous in its state of rest or uniform motion in a straight line, untill it is acted upon by any external force.

2nd Law :-

The rate of change of momentum is proportional to the resultant force & takeplace in the direction of a straight line in which the force acts.

3rd Law :-

To every actions there is an equal in opposite reaction..

NEWTON'S LAW OF MOTION :- (Lift)

W = weight carret of the lift.

m = mass of the lift.

a = a uniform accoleration of the lift

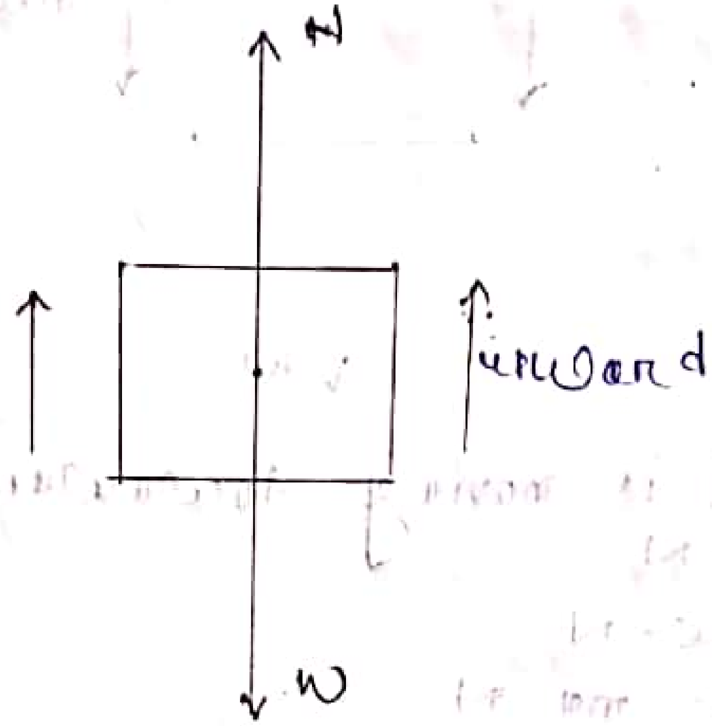
N = Normal reaction or tension in the cable.

Case - 1

When a lift moving upward:-

W = downward force of the lift.

N = upward force of the lift.



The lift is moving upward hence

$N > W$

Net force $F = N - W$

$\Rightarrow ma = N - \cancel{W}\, mg \quad (\because F = ma,\ W = mg)$

$\Rightarrow N = ma + mg$

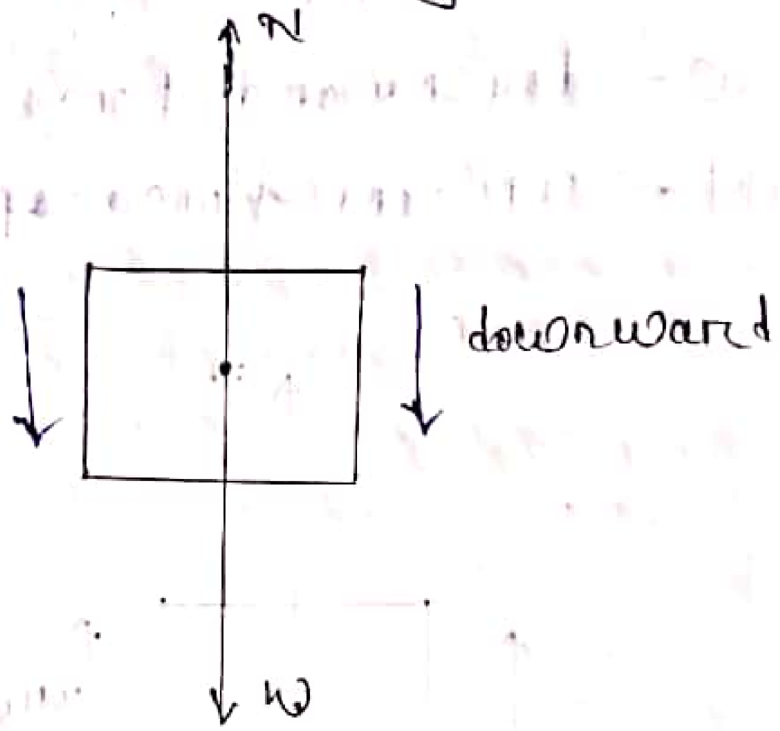
$\Rightarrow \boxed{N = m(a+g)}$

$\Rightarrow N = \frac{W}{g}(a+g)$

$\Rightarrow \boxed{N = W\left(\frac{a}{g} + 1\right)}$

Case - 2

When a lift moving downward



Lift is moving downward hence
$W > N$

$F = W - N$

$\Rightarrow ma = mg - N$

$\Rightarrow N = mg - ma$

$\Rightarrow \boxed{N = m(g-a)}$

$\Rightarrow N = \frac{W}{g}(g-a)$

$\Rightarrow N = W\left(\frac{g}{g} - \frac{a}{g}\right)$

$\Rightarrow \boxed{N = W\left(1 - \frac{a}{g}\right)}$

PROBLEM - 1

An elevator (lift) lifting a weight of 4450 N, starts to move upward with a uniform accn of $0.6\ m/sec^2$ find the tension in the cable during upward motion..

Given data,

$W = \cancel{4400}\ 4450\ N$

$a = 0.6\ m/sec^2 \qquad g = 9.81\ m/sec^2$

$F = N - W$

$\Rightarrow ma = N - mg$

$\Rightarrow N = ma + mg$

$\Rightarrow N = m(a+g)$

$\Rightarrow N = \frac{W}{g}(a+g)$

$\Rightarrow N = W\left(\frac{a}{g}+1\right)$

$= 4450\left(\frac{0.6}{9.81}+1\right)$

$= 4722\ N$

D| .12.04
Friday

PROBLEM - 2

An elevator weighing 6000 N is acending with an acceleration of 1 m/sec^2 during acend its operator whose weight is 600N is standing on the floor. What is the tension in the cable.

Given data,

$W_e = 6000 N$ $\quad g = 9.81 m/sec^2$

$a = 1 m/sec^2$

$W_o = 600 N$

$N = ?$

$N = w\left(\frac{a}{g}+1\right)$

$= \cancel{600\left(\frac{10}{9.81}\right)+1}$

$W = W_e + W_o$

$= 6600$

$N = w\left(\frac{a}{g}+1\right)$

$= 6600\left(\frac{10}{9.81}+1\right)$

$= 13327 N$

INERTIA:-

Inertia is a force which ~~offers~~ occurs resistance to the change of ~~scale~~ state of rest or uniform motion of the body.

Momentum:-

(i) If a body of mass 'm' is moving with a velocity 'v', then the product mv is called the momentum of the body.

(ii) Momentum is the unit of kg x m/s.

RATE OF CHANGE OF MOMENTUM:-

Let a body moving in a straight line

m = mass of the body

u = initial velocity of the body

v = final velocity of the body

a = acceleration of the body

t = time taken by the body to change in velocity u - v.

F = Force required to change velocity u to v.

→ Initial momentum of the body
= mu.

→ Final momentum after 't' second
= mv

→ change in momentum = $mv - mu$

→ Rate of change of momentum =
$$\frac{mv - mu}{t - 0}$$

$$\Rightarrow \frac{m(v-u)}{t}$$

According Newton's second law of motion :- the rate of change of momentum is proportional to the force.

$\therefore$ $F \propto$ rate of change of momentum

$$F \propto \frac{m(v-u)}{t}$$

$$F \propto ma \quad \left(\because a = \frac{v-u}{t}\right)$$

$$\Rightarrow F = Kma$$

$$\Rightarrow F = ma$$

LAW OF CONSERVATION OF MOMENTUM

The total momentum of a system of bodies remains constant, even after the mutually action between them.

Mathematically,

$$m_1u_1 + m_2u_2 + \dots\dots = m_1v_1 + m_2v_2 \dots\dots$$

$$\frac{w_1}{g}u_1 + \frac{w_2}{g}u_2 + \dots\dots = \frac{w_1}{g}v_1 + \frac{w_2}{g}v_2 \dots\dots$$

PROBLEM

Find the magnitude of the force required to move a body of mass 100 kg with an acceleration of $2 m/sec^2$.

$m = 100 kg$

$a = 2 m/sec^2$

$F = ma$

$= 100 \times 2$

$= 200 N$

Dt. 15. 04. 17
Monday

PROBLEM:-

A sphere of mass 50kg moving at 10m/sec & collides with another sphere on mass 30kg moving at 5m/sec in the same direction. Find the common velocity after impact.

Ans:- Given data,

$m_1 = 50kg$

$u_1 = 10m/sec$

$m_2 = 30kg$

$u_2 = 5m/sec$

According to as per law of conser momentum,

$m_1u_1 + m_2u_2 = m_1v + m_2v$

$\Rightarrow m_1u_1 + m_2u_2 = v(m_1 + m_2)$

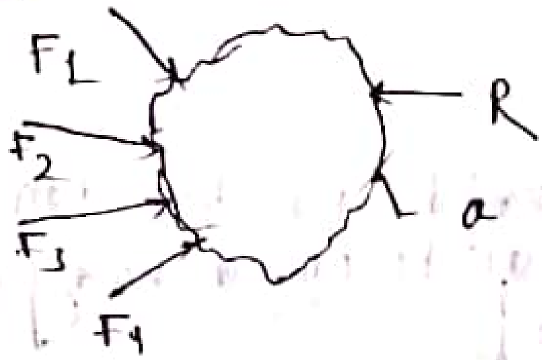
$\Rightarrow v = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2} = \dfrac{50 \times 10 + 30 \times 5}{50 + 30}$

$= 8.125 \ m/sec$

D'ALEMBERT'S ~~D'Alembert~~ PRINCIPLES :-

Let a body be subjected to a system of forces which causes the body to move with an acceleration a in the direction of resultant force.

Let 'R' be the resultant of the force



If a force of magnitude = R & is applied on the body along the line of action of the resultant in a directions opposite to the

$R = ma$

$\Rightarrow R - ma = 0$

$\Rightarrow R + (-ma) = 0$

We may consider '-ma' is an imaginary force along with the real force acting on a body this force is called inertia force.

→ The above equation is known as D'ALEMBERT'S PRINCIPLE.

→ D'ALEMBERT'S PRINCIPLE states that the system of force acting in a body in motion, which is in dynamic equilibrium with the inertia force of the body.

Work:-

Work is said to be done, when a force acts on a body & causes displacement

Workdone (W)

→ $W = F \times d$

→ Workdone is the unit of Nm (Joule)

→ 1 Joule = N m

$W = F\cos\theta \times d$ (inclined)

POWER:-

Power is the time rate of change of workdone.

→ $\text{power} = \frac{\text{Work done}}{\text{Time}} = \frac{F \times \text{displacement}(d)}{dt}$

→ $\text{power} = F \times V$

→ 1 kW = 1000 watt

→ power is the unit of $\frac{Nm}{sec}$ (watt)

→ 1 hp = 746 watt

ENERGY:-

E.g - Mechanical energy, solar energy, Heat energy, Electrical energy, chemical energy, wind energy, tidal energy.

"The energy of a body its capacity to do the work".

RECOIL OF GUN:-

→ When a bullet is a fired a gun, the opposite reaction of the bullet exists and it is called a recoil of gun.

→ It is application of Newton's 3rd law.

M = mass of the gun

m = mass of the bullet.

V = velocity of the gun.

v = velocity of the bullet.

∴ momentum of the bullet after firing . mv

Momentum of the gun after firing MV

$MV = mv$

POTENTIAL ENERGY:-

The energy possessed by a body by virtue of its position it is called potential energy.

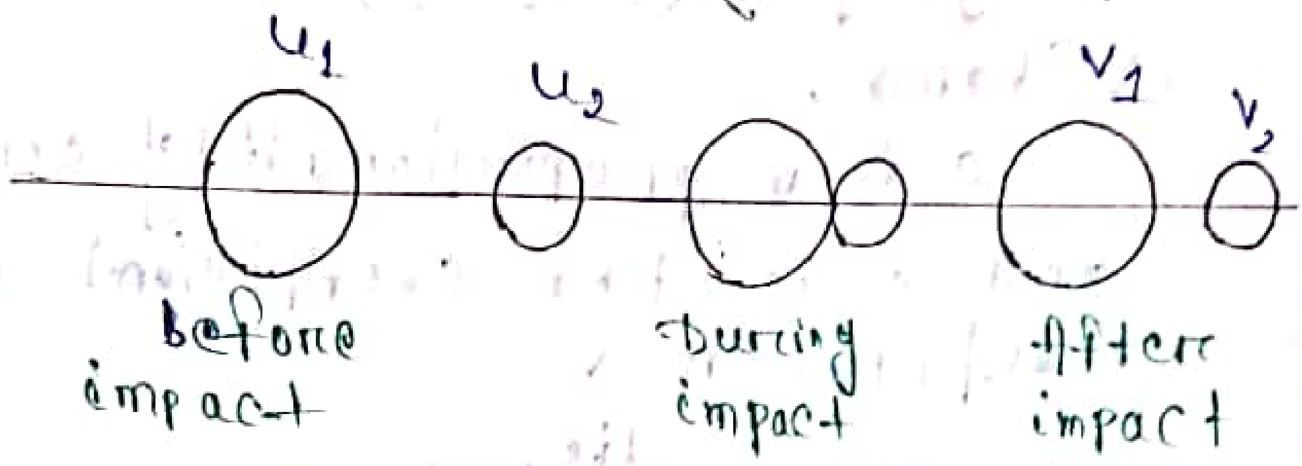
→ $P.E = mgh$

KINETIC ENERGY:-

The energy possessed by a body by virtue of its motion is called kinetic energy.

→ $K.E = \frac{1}{2} mv^2$

Coefficient of Restitutions:-



Let u_1 = initial velocity of the 1st body.

v_1 = final velocity of the 1st body

u_2 = initial velocity of the 2nd body

v_2 = final velocity of the 2nd body

A little consideration will show that the impact will take place only of ~~you to~~ u_2, after impact the separation of the 2 only of v_2

Therefore the velocity of separation will be equal to $(v_2 - v_1)$

→ Now as per Newton's law of collision of elastic bodies equal to $v_2 - v_1 = C \cdot K(u_1 - u_2)$

Where,

c is a proportionality constant and 'c' is the coefficient of restitution.

→ Its values lie between 0 & 1

→ If c = 0 & the body is inelastic.

→ When c = 1 perfectly elastic

PROBLEM:—

A body is move through a distance of 10m along a horizontal surface the force applied is 250N acting at 40° to the direction of motion. Find the work done.

Given data,

$d = 10m$

$F_x = 250N$

$\theta = 40^\circ$

$W = F d$

$F = F_x \cos\theta$

$= 250 \cos 40$