Unit and Dimension

Physical Quantity

A physical quantity is a property of a material or system that can be quantified by measurement.

- It is of 2 types.

1) Fundamental quantity:-
physical quantities are a set of suitably choosen independent observable which are defined operationally.
ex. mass, Length, time

2) Derived:-
The physical quantities which are defined in terms of physical quantity.
ex Velocity, Acceleration, Force etc.

S.I unit

physical quantity	S.I unit	symbol
1) ~~Length~~ Mass	kilogram	kg.
2) Length	metre	m
3) Time	second	s
4) Electric current	Ampere	A
5) Temperature	kelvin	K
~~6) Cand~~		
6) Luminous Intensity	Candela	cd
7) Amount of Substance	mole	mol.

Metric prefix

prefix	power of 10	Symbol
deci	10^{-1}	d
centi	10^{-2}	c
mili	10^{-3}	m
micro	10^{-6}	μ (mu)
nano	10^{-9}	n
pico	10^{-12}	P
femto	10^{-15}	f
atto	10^{-18}	a

power of 10	prefix	Symbol
10^{+1}	Deca	D
10^{2}	Hecto	H
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	Giga	G
10^{12}	Tera	T
10^{15}	peta	P
10^{18}	Exa	E

* Momentum, P = mass x velocity

$$= [M^1 L^0 T^0] \times [M^0 L^1 T^{-1}]$$
$$= [M^1 L^1 T^{-1}]$$

* Force = ma

$$= [M^1 L^0 T^0] \times [M^0 L^1 T^{-2}]$$
$$= [M^1 L^1 T^{-2}]$$

* Work = force x distance

$$= [M^1 L^1 T^{-2}] \times [M^0 L^1 T^0]$$
$$= [M^1 L^2 T^{-2}]$$

* Power = W/t

$$= \frac{[M^1 L^2 T^{-2}]}{[M^0 L^0 T^1]}$$

$$= [M^1 L^2 T^{-3}]$$

* Kinetic energy $= \frac{1}{2} mv^2$

$$= [M^1 L^0 T^0] \times [M^0 L^1 T^{-1}]^2$$

$$= [M^1 L^0 T^0] \times [M^0 L^2 T^{-2}]$$

$$= [M^1 L^2 T^{-2}]$$

* Potential energy = mgh / mah

$$= [M^1] \, [M^0 L^1 T^{-2}] \, [L^1]$$

$$= [M^1 L^2 T^{-2}]$$

* Pressure = force / area

$$= \frac{[M^1 L^1 T^{-2}]}{[M^0 L^2 T^0]}$$

$$= [M^1 L^{-1} T^{-2}]$$

* Angle $= [M^0 L^0 T^0]$
* Impulse, $I = Ft$

$$= [M^1 L^1 T^{-2}] \, [T^1]$$

$$= [M^1 L^1 T^{-1}]$$

* Work and energy are having same dimensional formula.
* Dimesional formula of momentum and Impulse are same.
* Angle is a dimensionless quantity.

Properties of unit

A unit must possess the following properties.

1) It should be invariable (Can't be changed)
2) It should be easily available for compression with various measurements.
3) It should be convenient in size.

Fundamental Quantities

For the study of physics we need 7 fundamental quantities:-

1) mass
2) Length
3) Time
4) Electric Current
5) Temperature
6) Luminosity
7) Amount of substance.

System of Unit

1) C.G.S system (Gaussian system)

- It is a system of measurement in which the fundamental units of the measurement of length, mass and time are taken as 1cm, 1gm and 1sec repectively.

* This system contain many derived units which are small in size.

2) M.K.S System (metric system).

- It is the system of measurement in which the fundamental units of the length, mass and time are meter, kilogram and second respectively.
- The magnitude of units in this system are of moderate size.
- So, it is very commonly used for scientific measurement.

3) S.I ~~unit~~ system. (Systemed International)

C.G.S system and M.K.S system have 3-fundamental units. These units are sufficient for studying mechanics. once we move out of mechanics, we can't measure in terms of these 3 units.

- A new system of 7 fundamental units for S.I

i) Unit of mass → Kilogram (kg)
ii) Unit of length → metre (m)
iii) Unit of time → Second (s)
iv) " " electric current → Ampere (A).
v) " " temperature → Kelvin (K)
vi) Luminosity → Candela (cd)
vii) Amount of substance → mole (mol)

Physical Quantity

A quantative description of any physical phenomena / process always involves certain measurable quantities like force, velocity, time, density, charge, temperature and host of others are called observable or physical quantities.

on the other hand those quantities like - volume and speed, whose defining operations are based on the use of some other physical quantities are called derived quantities.

Unit:

To make the measurement of a physical quantities we have to create a standard for that measurement so that different measurement of some physical quantity can be expressed relative to each other. That standard is called "a unit" of that physical quantity.

F.P.S System (Foot, pound, second) (British system)

In this system, length, mass and time are expressed in terms of foot, pound and second.

* Unit of angle → radian (rd)

* Unit of Solid angle - Steradian (sr)
- Steradian is the solid angle subtended at the centre of a sphere by a surface area of the sphere whose magnitude is equal to the square of radius of the sphere.

* Unit of Area → m^2

Velocity → m/s

acceleration → m/s^2

force → N

momentum → kg m/s.

Practical units for microscopic and macroscopic

1) Astronomical unit (AU)
- It is the mean distance of the earth from the sun. It is equal to 1.496×10^{11} m.

2) Light year:-
- It is the distance travelled by light in vaccum in one year.

* velocity of light in vaccum $= 3.10^8$ m/s

$$\text{Time} = 1\text{year} = 365\tfrac{1}{4}\text{ days}$$
$$= 365\tfrac{1}{4} \times 24 \times 60 \times 60\text{ s}$$

* $$1\text{ Light year} = 3\times10^8 \times 365\tfrac{1}{4} \times 24 \times 60 \times 60\text{ m}$$
$$= 9.467 \times 10^{15}\text{ m (approx.)}$$

* 1 micron $= 10^{-6}$ m

~~cm~~ $= 10^{-4}$ cm

* $1\text{Å} = 10^{-10}$ m

1 Fermi $= 10^{-15}$ m

Q. A body moves with a velocity of 36km/hr what is the value of the velocity in m/s?

Ans $\frac{36000}{3600} = 10m/s$

Q. Convert an acceleration of $20m/s^2$ into km/hr^2

Soln $1m = \frac{1}{1000}$, $1s = \frac{1}{3600}$

$20m/s^2 = 20 \times \frac{1}{1000} \times \frac{1}{\left(\frac{1}{60\times60}\right)^2}$

$= \frac{20}{1000} \times 3600 \times 3600$

$= 72 \times 3600$

$= 259200$

$= 2.592 \times 10^5 \ km/hr^2$

HW

Q1) Calculate the realtion between

i) milligram & Nanogram

Sol: $1mg = 10^{-3}g$

$\Rightarrow 1g = \frac{1}{10^{-3}} = 10^3 mg$

$1ng = 10^{-9}g$

$\Rightarrow 1g = \frac{1}{10^{-9}} = 10^9 ng$

$\Rightarrow 10^3 mg = 10^9 ng$

$\Rightarrow 1mg = \frac{10^9}{10^3} = 10^6 ng$

Q. petameter and micrometer

$$1p = 10^{15} m$$

$$\Rightarrow 1m = \frac{1}{10^{15}} = 10^{-15} pm.$$

$$1\mu = 10^{-6} m.$$

$$\Rightarrow 1m = \frac{1}{10^{-6}} = 10^{6} \mu$$

$$\Rightarrow 1pm = \frac{10^{-6} \mu m}{10^{-15} pm}$$

$$= 10^{6+15} \mu m$$

$$= 10^{21} \mu m.$$

Dimension:

- The word dimension has a special meaning in physics. It defines the qualitative nature of a physical quantity.
- Mass, Length and time are considered to be the base dimensions are denoted by bracketed capitals letters [M], [L], [T], respectively.
- [M], [L], [T] indicates the nature of the dimesions but not the magnitude. The dimensions of a derived physical quantity may be defined as the powers to which its base unit must be raised to represent it completely.

Dimensional Formula

A dimensional formula is an expression which shows how and which of the fundamental units entre into the units of a physical quantity. An equation written "Area = $[M^0L^2T^0]$" in this manner is called dimensional equation.

D. formula of volume = $[M^0L^3T^0]$

Velocity = $[M^0L^1T^{-1}]$

acceleration = $[M^0L^1T^{-2}]$

momentum = $[M^1L^1T^{-1}]$

Force = $[M^1L^1T^{-2}]$

* Charge (q)

$I = \frac{q}{t}$

$\Rightarrow q = It \quad [A^1T^1]$

$Q = ms\theta$

$\Rightarrow s = \frac{Q}{m\theta}$

$= \frac{[M^1L^2T^{-2}]}{[M^1][K^1]}$

$\Rightarrow s = [M^0L^2T^{-2}K^{-1}]$

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Uses of Dimensional analysis

It has 3 uses:-

- To convert the values of a physical quantity from one system to another.
- To check the correctness of a given relationship.
- To derive a relation between various physical quantities.

* Using dimensional analysis and applying the principle of homogenity we can get the numerical value of a physical quantity in any system if its value is in other system is known.

Principle of homogenity

- The dimensional formula of every term on the two sides of a correct relation must be same.

* Dimension of each term of LHS must be same as dimension of each term of RHS.

ex. $[M^a L^b T^c] = [M^1 L^1 T^{-2}]$ by applying

$a = 1, b = 1, c = -2$

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Q. Convert a work of 1 joule to erg.

System-1 (M.K.S)

$M_1 = 1kg$

$L_1 = 1m$

$T_1 = 1s$

$n_1 = 1$

Work $[M^1L^2T^{-2}]$

System-2 (C.G.S)

$M_2 = 1gm$

$L_2 = 1cm$

$T_2 = 1s$

$n_2 = ?$

Physical quantity as represented in system-1

$\Rightarrow n_1 [M_1^a L_1^b T_1^c]$

In system $\Rightarrow n_2 [M_2^a L_2^b T_2^c]$

Since the quantity is same in both the system $\rightarrow n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$

$$\Rightarrow n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$a = 1, b = 2, c = -2$

$$\Rightarrow n_2 = 1 \left[\frac{1kg}{1g}\right]^1 \left[\frac{1m}{1cm}\right]^2 \left[\frac{1s}{1s}\right]^{-2}$$

$$\Rightarrow n_2 = 1 \left[\frac{1000g}{1g}\right]^1 \left[\frac{100cm}{1cm}\right]^2 \left[\frac{1s}{1s}\right]^{-2}$$

$= 1 \times 1000g \times 10000\,cm$

$= 1 \times 10^3 g \times 10^4 cm$

$\Rightarrow n_2 = 10^7$

So, 1 joule $= 10^7$ erg.

Q. Convert a pressure of 76 cm of mercury column into MKS units.

System-1

C.G.S

$M_1 = 1g$

$L_1 = 1cm$

$T_1 = 1s$

$n_1 = 76 \times 13.6 \times 980$

System-2

M.K.S

$M_2 = 1kg$

$L_2 = 1m$

$T_2 = 1s$

$n_2 = ?$

pressure $= [M^1 L^{-1} T^{-2}]$ $(a=1, b=-1, c=-2)$

physical quantity as represented in system-1

$= n_1 [M_1^a L_1^b T_1^c]$

System-2 $= n_2 [M_2^a L_2^b T_2^c]$

Since the quantity is same in both the system

$\Rightarrow n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$.

$n_2 \Rightarrow n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$\Rightarrow n_2 = 76 \times 13.6 \times 980 \cdot \left[\frac{1g}{1kg}\right]^1 \left[\frac{1cm}{1m}\right]^{-1} \left[\frac{1s}{1s}\right]^{-2}$

$= 76 \times 13.6 \times 980 \left[\frac{1}{1000}\right]^1 \left[\frac{1}{100}\right]^{-1} \left[\frac{1}{1}\right]^{-2}$

$= 76 \times 13.6 \times 980 \times (10^{-3})^1 (10^{-2})^{-1}$

$= 76 \times 13.6 \times 980 \cdot \times 10^{-3} \times 10^{-2}$

$= 1.0129 \times 10^5 \; N/m^2$

Type-2

To check the correctness of the relation i.e. $s = ut + \frac{1}{0} at^2$

Sol: Dimensional formula of $s = [M^0L^0T^0]$

$$ut = [M^0L^1T^{-1}][T^1]$$
$$= [M^0L^1T^0]$$

Now, $at^2 = [M^0L^1T^{-2}].[T^2]$
$$= [M^0L^1T^0]$$

LHS = RHS (proved)

Since, same dimensional formula representation all the three terms involved in the equation, the relation is correct.

Q. $v^2 - u^2 = 2as$

Dimensional formula of

$$as = [M^0L^1T^{-2}][M^0L^1T^0]$$
$$= [M^0L^2T^{-2}]$$

$$v^2 = [M^0L^1T^{-1}]^2$$
$$= [M^0L^2T^{-2}]$$

$$u^2 = [M^0L^1T^{-1}]^2$$
$$= [M^0L^2T^{-2}]$$

Q_3 $v = u + at$

Dimensional formula of $v = [M^0 L^1 T^{-1}]$

$\Rightarrow u = [M^0 L^1 T^{-1}]$

$\Rightarrow at = [M^0 L^1 T^{-2}] \cdot \times [M^0 L^0 T^1]$

$= [M^0 L^1 T^{-1}]$

Q_4 $t = 2\pi \sqrt{\frac{g}{l}}$

Dimensional formula of $t = [M^0 L^0 T^1]$

$$\frac{g}{l} = \frac{[M^0 L^1 T^{-2}]}{[M^0 L^1 T^0]}$$

$= [M^0 L^0 T^{-2}]$

Type-3

To derive a relation between various physical quantity. $F = \frac{mv^2}{r}$

Sol: Let us obtain an expression for centripetal force required to move to body of mass 'm' with velocity 'v' in a circle.

Let the centripetal force (F) depend upon m, v, and r.

So, $F \propto M^a$

$F \propto V^b$

$F \propto r^c$

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$\Rightarrow F \propto M^a v^b r^c$

$\Rightarrow F = k m^a v^b r^c$

Dimensional Formula of Force

$\Rightarrow [M^1 L^1 T^{-2}] = [M^1]^a [L^1 T^{-1}]^b [L^1]^c$

$= [M^a L^{b+c} T^{-b}]$

Comparing both side, we get $a = 1$

$b + c = 1$

$-b = -2 \Rightarrow b = 2$

So $b + c = 1$

$\Rightarrow 2 + c = 1$

$\Rightarrow c = -1$

$F = k m^a v^b r^c$

$= k m^1 v^2 r^{-1}$

$= k \frac{mv^2}{r}$ $\boxed{k=1}$

So, $F = \frac{Mv^2}{r}$

Q2 Force of viscosity (F) acting on a spherical body moving through a fluid depends upon its velocity radius and co-efficient of viscosity (η). Using method of dimension obtain an expression for Force.

Sol: $F \propto v^a$

$F \propto r^b$

$F \propto \eta^c$

So, $F \propto v^a r^b \eta^c$

dimensional formula of force $= [M^1 L^1 T^{-2}]$

$= [L^1 T^{-1}]^a [L^1]^b [M^1 L^{-1} T^{-1}]^c$

$= [M^c] [L^{a+b-c}] [T^{-a-c}]$

$= [M^c L^{a+b+c} T^{a-c}]$

Comparing both sides, we get.

$c = 1$, $a+b-c = 1$, $a-c = -2$

$a - 1 = -2$

$-a = -2+1 = 1$.

Now $a+b-c = 1$

$\Rightarrow 1 - 1 + b = 1$

$\Rightarrow 0 + b = 1$

$\Rightarrow b = 1$

$F = K v^a r^b \eta^c$

$\underline{Q_3}$: convert a force of 50dyne into newton by the method of dimensional analysis.

System-1 (C.G.S)

$M_1 = 1gm$

$L_1 = 1cm$

$T_1 = 1sec$

$n_1 = 50$

System-2 (M.K.S)

$M_2 = 1Kg$

$L_2 = 1m$

$T_2 = 1s$

$n_2 = ?$

physical quantity as represented in system-1

$\Rightarrow n_1 [M_1^a L_1^b T_1^c]$

In system-2 $= n_2 [M_2^a L_2^b T_2^c]$

Since

ee

Since, the quantity is same in both the System - $n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$

$$\Rightarrow n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

D. formula of force $= [M^1 L^1 T^{-2}]$

$(a=1, b=1, c=-2)$

$$\Rightarrow n_2 = n_1 \left[\frac{1gm}{1kg}\right]^1 \left[\frac{1cm}{1m}\right]^1 \left[\frac{1s}{1s}\right]^{-2}$$

$$= n_1 \left[\frac{1g}{1000g}\right]^1 \left[\frac{1cm}{100cm}\right]^1 \left[\frac{1s}{1s}\right]^{-2}$$

$= 50 \times (10^{-3}) . (10^{-2})$

$= 50 \times 10^{-3} \times 10^{-2}$

$= 50 \times 10^{-5}$ N / 5×10^{-4} N

Hence 50 dyne = 0.0005N

Scalar and vector

Scalar quantity

These quantity are those quantities which require only the magnitude for their complete specifications.

ex. Mass, length, time, volume etc.

Vector quantity:-

These quantities are the quantities which require magnitude as well as direction for their complete specification.

ex. Displacement, velocity, acceleration, force, electric intensity, magnetic intensity etc.

Vector:- A directed line segment is called a vector.

* vector quantities can-not be added algebraically.

Representation of a vector:

A vector can be represented by observing the following steps:-

- Draw a line parallel to the direction of vector.
- Cut a length of the line so that it represents the magnitude of the vector on a certain convenient scale.
- put an arrow head in the direction of the vector. Those line with the arrow head represents the given vector.

- A vector is written with an arrow head over its symbol like $\vec{x}$.

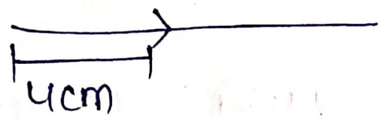
Q: Consider a body moving with a velocity 40m/s due east. We want to represent it vectorically?

Sol: Let us select a scale

10m/s = 1cm

=) 4cm = 40m/s

Draw a line, 4cm long in west-east direction put an arrowhead pointing east. This vector represents a velocity of 40m/s due east.



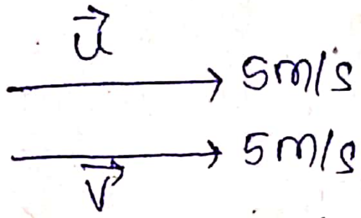
Some terms connected with vectors

1) Null vector:- It is a vector having '0' magnitude and arbitrary direction.

Properties:-

- It has '0' magnitude.
- It has arbitrary direction.
- It is represented by a point.
- When a null vector is added or substracted from a given vector the resultant vector is same as the given vector.
- Dot product of a ~~magnitude~~ null vector with any vector is always zero.
- Cross product ~~is~~ ~~also~~ of a null vector with any other vector is also a null vector.

2) **Equal vectors:-** Two vectors are said to be equal vectors if they possess same magnitude and same direction.

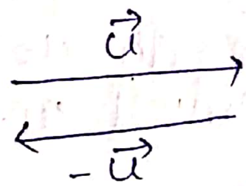


($\vec{u}$ & $\vec{v}$ are equal vectors)

$\Rightarrow \vec{u} = \vec{v}$

3) **Negative vectors:-**

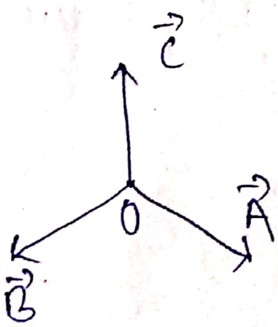
A vector is said to be a negative vector of another vector, if it is represented by a line having same length as that of second vector and is directed in opposite direction.



(Negative vector)

4) **Co-initial vectors:-**

A number of vectors having a common initial point are called Co-initial vectors.



5) Co-linear vector :-

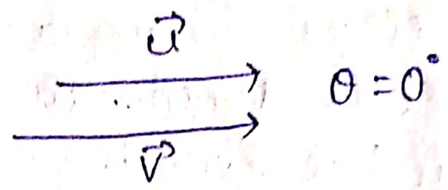
- Vectors having a common line of action are called co-linear vector.
- These are 2 types ~~of~~ :-

  i) A-parallel vectors

  ii) Anti-parallel vectors.

i) Parallel vectors :-

- Two vectors which may have different magnitudes acting along same direction are called parallel vectors.

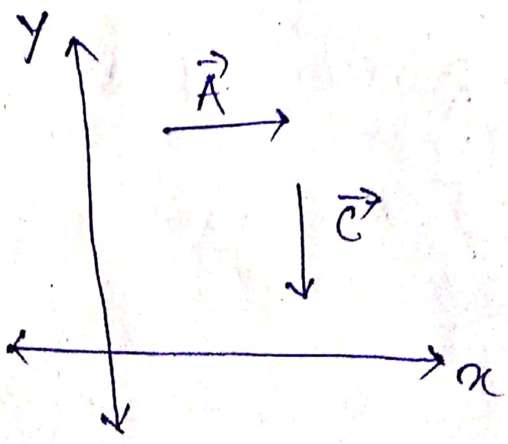


ii) Anti-parallel vectors :-

- Two vectors which are directed in opposite direction are called anti-parallel vector.
- Angle between them is 180°.

6) Coplanar vector :-

Vectors situated in one plane are called co-planar vectors. Their direction may be random.



Localised vector:-

Vector whose initial point is fixed is said to be a localised vector.

- It is also called as fixed vector.

ex. position vector of every points starts from origin
∴ i.e called as localised vector

Non-localised vector:-

Vector whose initial point is not fixed is called non-localised vector. It is also known as free vector.

ex. force, momentum, Impulse etc.

Triangle's law of vector

It is a law for the addition of 2-vectors

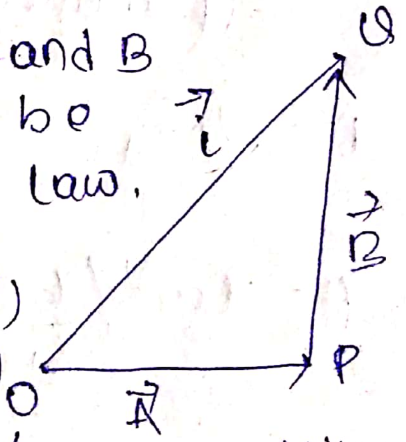
Statement:-

If two vectors are represented in magnitude and direction by the two sides of a triangle, taken in the same ~~time~~ order, then their resultant is represented in magnitude and direction by the 3rd side of the triangle taken in opposite order.

→ Resultant of 2 vectors A and B acting at a point can be determined by triangle law.

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{—(i)}$$

(magnitude)

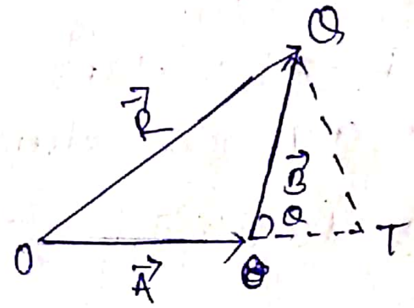


$$\beta = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right) \quad \text{—(2)}$$

(direction)

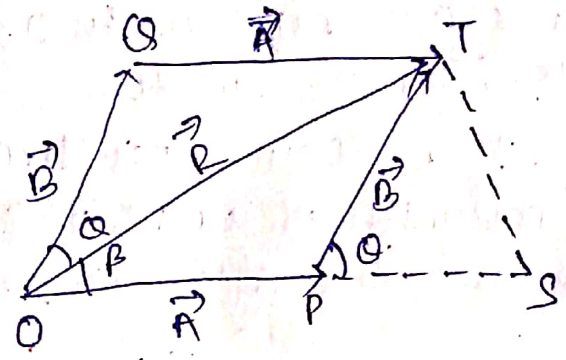
(Vector addition by triangle law)

eq (1) and (2) gives the magnitude and direction of the resultant 'R' respectively.



Parallelogram Law of vector

It states that if two vectors acting simultaneously, at a point are represented in magnitude and direction by the two sides of a llgm drawn from a point then their resultant is given in magnitude and direction by the diagonal of the llgm passing through that point



(parallelogram Law of vector)

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad -(1)$$

$$\beta = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right) \quad -(2)$$

Eq (i) and (2) give the magnitude and direction of the resultant.

If $\theta = 0°$

$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos 0}$

$= \sqrt{(A+B)^2}$

$= A+B$

So, $R = A+B$

$\beta = \tan^{-1}(0)$

$= 0°$

If $\theta = \cancel{0}180°$ ($\cos 180° = -1$)

$\Rightarrow R = A - B$

$\beta = \tan^{-1}\left(\frac{B}{A}\right)$

If $\theta = 90°$

$R = \sqrt{A^2 + B^2}$

Q, Two forces, one of 5N and another of 20N at an angle of 120° between them. Find the resultant force in magnitude and direction.

Sol: $F_1 = 5N$ $F_2 = 20N$ $\cancel{At}$ $\theta = 120°$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{(5)^2 + (20)^2 + 2(5)(20)\cos 120}$

$= \sqrt{25 + 400 + \cancel{200}^{100}\left(-\frac{1}{\cancel{2}}\right)}$

$= \sqrt{\cancel{625}_{3}\left(-\frac{1}{2}\right)}$

$= \sqrt{425 - 100}$

$= \sqrt{325} = 18.03N$

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$$\beta = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

$$= \frac{20 \sin 120^\circ}{5 + 20\cos 120^\circ}$$

$$= \frac{20 \times \sqrt{3}/2}{5 + 20(-1/2)} = \frac{10\sqrt{3}}{5-10} = \frac{10\sqrt{3}}{-5} = -2\sqrt{3}$$

Q. Two forces equal in magnitude have magnitude of their resultant equal to either. Find the angle between them.

Sol:

$F_1 = F_2 = F = R$

$\Rightarrow R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$\Rightarrow F = \sqrt{F^2 + F^2 + 2F^2 \cos\theta}$

$\Rightarrow F = \sqrt{2F^2 + 2F^2\cos\theta}$

$\Rightarrow F = \sqrt{2F^2(1+\cos\theta)}$

$\Rightarrow \cancel{F^2} = 2\cancel{F^2}(1+\cos\theta)$

$\Rightarrow 1 = 2(1+\cos\theta)$

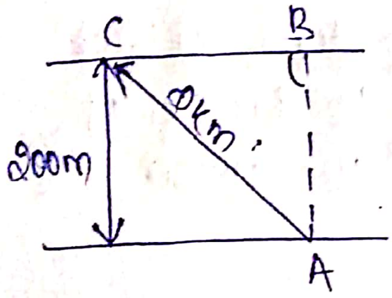
$\Rightarrow \frac{1}{2} = 1 + \cos\theta$

$\Rightarrow \cos\theta = 1/2 - 1$

$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = \theta = 120^\circ$

Q3 A stream 200m broad is running downward at the rate of 1Km/hr. A swimmer who can swim at the rate of 2Km/hr wishes to ~~each~~ reach point just opposite. Along what time should he strike and how long will he take in crossing?

Sol: As the man tries to swim in water, he is acted upon by two velocity 2km/hr due to his own and 1km/hr due to river.



$AB^2 = AC^2 - BC^2$

$\Rightarrow V^2 = 2^2 - 1^2$

$= 4-3$

$\Rightarrow V^2 = 3$

$\Rightarrow V = 3$

$\cos\theta = \frac{AB}{AC} = \frac{\sqrt{3}}{2} = \cos 30° = \theta = 30°$

If α is the angle between the direction of resultant direction of motion of stream then

$\Rightarrow d = 90 + \theta$

$\Rightarrow d = 90 + 30 = d = 120$

$V = S/t \Rightarrow t = S/V$

Time 't' to cross the river

$t = \frac{\text{distance } AB}{\text{Velocity } AB}$

$= \frac{0.2 km/hr}{\sqrt{3} km/hr} = \frac{0.2}{1.732} = \frac{2}{17.32}$ hr

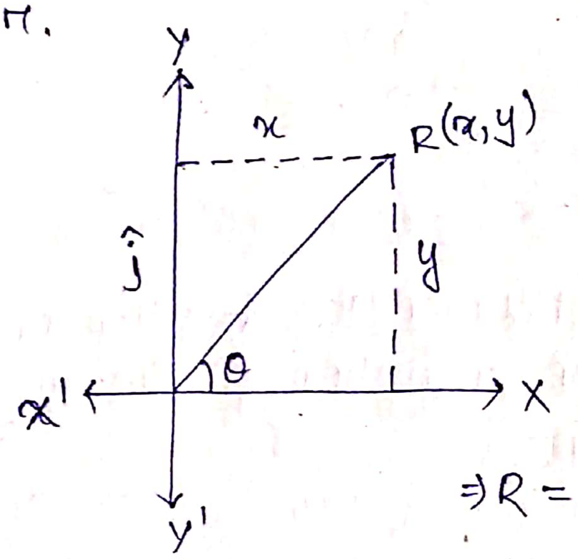
$= \frac{2}{17.32} \times 60$ min

$= 6.94$ mm

Resolution of vector in a plane

Composition of vector is a process of obtaining a single vector which produces the same effect as produced by a no. of vectors acting simultaneously. A process just opposite to this is called resolution of vectors.

- Resolution of vector is the process of obtaining the component vectors which when combined according to laws of vector addition produces the given vector.



In ΔOAR

$\cos\theta = x/R$

$\Rightarrow x = R\cos\theta$

In ΔOAR, $\sin\theta = y/R$

$\Rightarrow y = R\sin\theta$

$\Rightarrow R = x\hat{i} + y\hat{j}$

$\Rightarrow y = R\sin\theta$

$\Rightarrow \cancel{\vec{R} = x} \Rightarrow \vec{R} = x\hat{i} + y\hat{j}$

$= (R\cos\theta)\hat{i} + (R\sin\theta)\hat{j}$

Q. Determine the direction and magnitude of the resultant of following velocities impressed on a particle.

i) 8m/s due south

ii) 12m/s due east

iii) $3\sqrt{2}$ m/s due N.E

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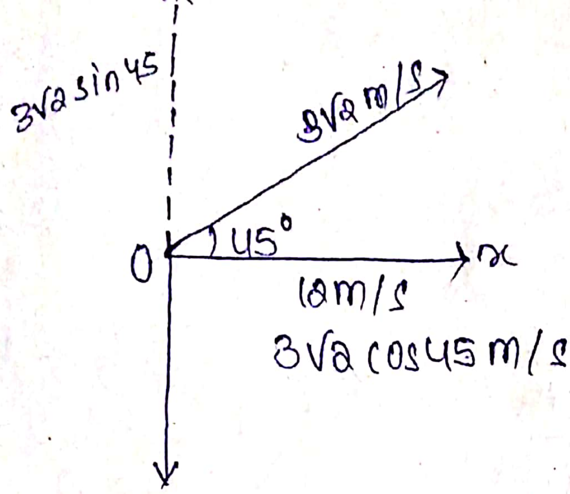


$3\sqrt{2}\cos 45^\circ = 3\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 3 m/s$ along OX

$3\sqrt{2}\sin 45^\circ = 3\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 3 m/s$ along OY.

Net velocity along OX $= 12 + 3 = 15 m/s$
Net velocity along OY $= 8 - 3 = 5 m/s$

$$\boxed{\begin{array}{l} \vec{R} = x\hat{i} + y\hat{j} \\ R = \sqrt{x^2 + y^2} \end{array}}$$

Resultant, $R = \sqrt{(15)^2 + (5)^2}$

$= \sqrt{225 + 25}$

$= \sqrt{250} = 15.81 m/s$

$\tan\beta = \frac{5}{15}$

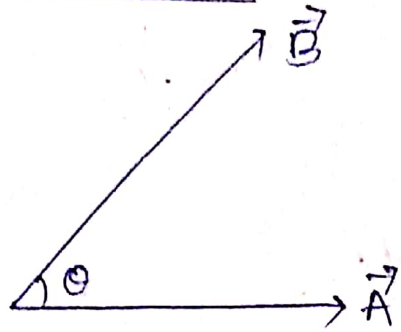
$\Rightarrow \beta = \tan^{-1}(5/15)$ along southeast.

<u>Product of two vectors</u>

1) Scalar product / Dot product $(\cdot)$

2) vector / cross product $(\times)$

1) Dot product :-



$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$

Properties

i) Commutative property ex $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

ii) Distributive $\phi \to \vec{A}\cdot(\vec{B}+\vec{C}+\vec{D}+\cdots)$

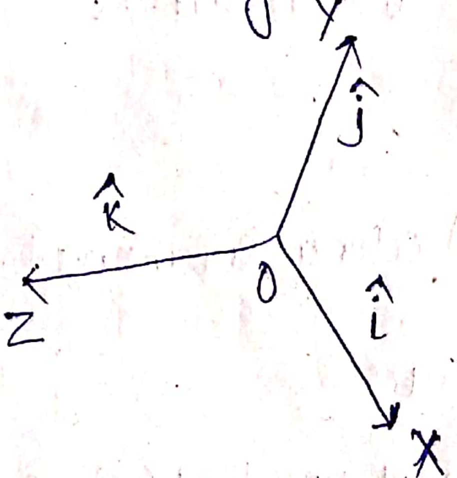
$= \vec{A}\vec{B} + \vec{A}\vec{C} + \cdots$

iii) perpendicular → The dot product of two non-zero vectors which are perpendicular to each other is always zero (0).

ex. $\theta = 90°$, $\vec{A}\vec{B} = AB\cos 90$

$= 0$

- This statement is known as "condition of perpendicularity".



$\hat{i}$, $\hat{j}$ and $\hat{k}$ are mutually perpendicular

$\hat{i}\cdot\hat{j} = 0$	$\hat{j}\cdot\hat{k} = 0$	$\hat{k}\cdot\hat{i} = 0$

iv) Co-linear Vectors:-

→ parallel → $\theta = 0°$

$\Rightarrow \vec{A}.\vec{B} = AB\cos 0°$

$= AB$

→ Anti-parallel → $\theta = 180°$

$\vec{A}.\vec{B} = AB\cos 180 = -AB$

- The dot product of co-linear vectors is equal to the do product of their magnitude. It is positive if they parallel and negative if they are antiparallel. This statement is called as "condition of co-linearity.

v) Equal vectors:-

$\theta = 0$

$\Rightarrow \vec{A}.\vec{A} = AA\cos 0°$

$= A^2$

$$\boxed{\begin{matrix}\hat{i}.\hat{i} = 1 \\ \hat{j}.\hat{j} = 1 \\ \hat{k}.\hat{k} = 1\end{matrix}}$$

vi) Dot product in terms of rectangular component

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_x\hat{i}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$
$+ A_z\hat{k}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_x(\hat{i}.\hat{i}) + A_xB_y(\hat{i}.\hat{j}) + A_xB_z(\hat{i}.\hat{k}) + A_yB_x(\hat{j}.\hat{i})$
$+ A_yB_y(\hat{j}.\hat{j}) + A_yB_z(\hat{j}+\hat{k}) + A_zB_x(\hat{k}.\hat{i}) + A_zB_y(\hat{k}.\hat{j})$
$+ A_zB_z(\hat{k}.\hat{k})$

$\Rightarrow A_xB_x + A_yB_y + A_zB_z$

$$\boxed{\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

d) Vector/cross product

$$\vec{A} \times \vec{B} = \vec{C}$$
$$= \boxed{AB \sin\theta \, \hat{n}}$$

Properties:-

i) Not-commutative

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

ii) Distributive:-

$$\vec{A}(\vec{B} + \vec{C} + \dots +)$$
$$\Rightarrow \vec{A}\vec{B} + \vec{A}\vec{C} + \dots$$

iii) Co-linear vector

- parallel, $\theta = 0°$

$$\vec{A} \times \vec{B} = AB \sin 0° \, \hat{n}$$
$$= (0)\hat{n} \text{ (Null vector)}$$

* Cross product of two co-linear vectors is always a null vector. This statement is known as "condition of co-linearity".

iv) Equal vectors:-

$\theta = 0°$

$$\vec{A} \times \vec{A} = AA \sin 0° \, \hat{n}$$
$$= (0)\hat{n} \text{ (Null vector)}$$

* Cross product of two equal vectors is a null vector.

$$|\hat{i} \times \hat{i}| = |\hat{j} \times \hat{j}| = |\hat{k} \times \hat{k}| = 0$$

v) perpendicular vectors.

$\theta = 90^\circ$

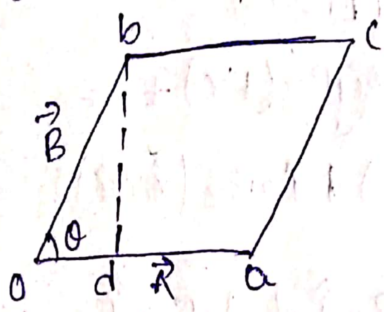
$\vec{A} \times \vec{B} = AB\sin 90^\circ \hat{n}$

$= AB\hat{n}$

$\boxed{\begin{array}{l} \hat{i} \times \hat{j} = \hat{k} \\ \hat{j} \times \hat{k} = \hat{i} \\ k \times \hat{i} = \hat{j} \end{array}}$

$\boxed{\begin{array}{l} \hat{j} \times \hat{i} = -\hat{k} \\ \hat{k} \times \hat{j} = -\hat{i} \\ \hat{i} \times \hat{k} = -\hat{j} \end{array}}$ (anti-clock wise)

Ar. of llgm



Ar. of llgm

$= Oa \times bd$ (b×h)

$= A \times B\sin\theta$

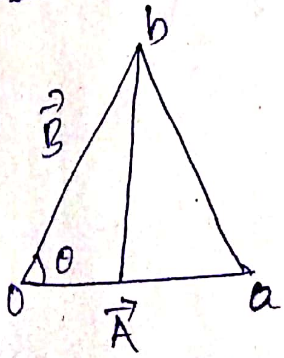
$= |\vec{A} \times \vec{B}|$

In ΔOBD, $\sin\theta = \frac{bd}{ob}$

$\Rightarrow bd = ob\sin\theta$

$= B\sin\theta$

Area of Δ



Ar. of $\Delta = \frac{1}{2} \times oa \times bc$

$= \frac{1}{2} \times A \times B\sin\theta$

$= \frac{1}{2} |\vec{A} \times \vec{B}|$

In Δobc

$\sin\theta = bc/ob$

$\Rightarrow bc = b\sin\theta$

$\Rightarrow B\sin\theta$

Cross-product in terms of rectangular components.

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_x\hat{i}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_z\hat{k}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x(\hat{i}\cdot\hat{i}) + A_xB_y(\hat{i},\hat{j}) + A_xB_z(\hat{i}\cdot\hat{k}) + A_yB_x(\hat{j}\cdot\hat{i}) + A_yB_y(\hat{j}\cdot\hat{j}) + A_yB_z(\hat{j}\cdot\hat{k}) + A_zB_x(\hat{k}\cdot\hat{i}) + A_zB_y(\hat{k}\cdot\hat{j}) + A_zB_z(\hat{k}\times\hat{k})$$

$$= A_xB_y\hat{k} - A_xB_z\hat{j} - A_yB_x\hat{k} + A_yB_z\hat{i} + A_zB_x\hat{j} - A_zB_y\hat{i}$$

$$= \hat{k}(A_xB_y - A_yB_x) + \hat{j}(A_zB_x - A_xB_z) + \hat{i}(A_yB_z - A_zB_y)$$

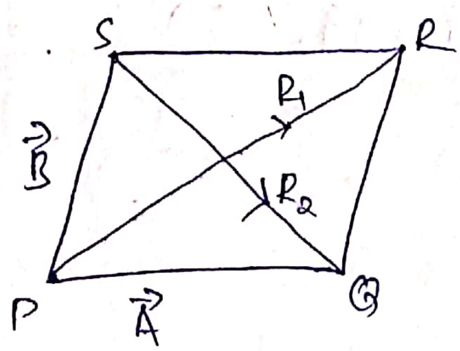
$$= \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

$$|\vec{A}\times\vec{B}| = \sqrt{(A_yB_z - A_zB_y)^2 + (A_zB_x - A_xB_z)^2 + (A_xB_y - A_yB_x)^2}$$

$$\vec{A}\times\vec{B} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

Q. The diagonals of a ||gm are represented by $\vec{R_1} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{R_2} = 5\hat{i} + 6\hat{j} - 3\hat{k}$. Find the area of the ||gm.



Consider a ||gm PQRS. Let its diagonal PR and SQ be represented by $\vec{R_1}$ and $\vec{R_2}$ respectively

$$\vec{R_1} = \vec{A} + \vec{B} \quad \text{—— (1)}$$

$$\vec{R_2} = \vec{A} - \vec{B} \quad \text{—— (2)}$$

Solving eq (1) & (2)

$$\vec{R_1} + \vec{R_2} = 2\vec{A}$$

$$\Rightarrow \vec{A} = \frac{\vec{R_1} + \vec{R_2}}{2}$$

$$\vec{R_1} - \vec{R_2} = 2\vec{B}$$

$$\Rightarrow \vec{B} = \frac{\vec{R_1} - \vec{R_2}}{2}$$

$$\cancel{\vec{A} = \left(\frac{3\hat{i} - 2\hat{j} + 7\hat{k}}{2}\right)}$$

$$\vec{A} = \frac{(3\hat{i} - 2\hat{j} + 7\hat{k}) + (5\hat{i} + 6\hat{j} - 3\hat{k})}{2}$$

$$= \frac{8\hat{i}+4\hat{j}+4\hat{k}}{2} \quad \text{(all term by 2)}$$

$$= \boxed{4\hat{i}+2\hat{j}+2\hat{k}}$$

$$\vec{B} = \frac{(3\hat{i}-2\hat{j}+7\hat{k})-(5\hat{i}+6\hat{j}-3\hat{k})}{2}$$

$$= \frac{-2\hat{i}-8\hat{j}+10\hat{k}}{2}$$

$$= \cancel{\hat{i}+4} \boxed{-\hat{i}-4\hat{j}+5\hat{k}}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 2 \\ -1 & -4 & 5 \end{vmatrix}$$

$$= \hat{i}(10+8) - \hat{j}(20+2) + \hat{k}(-16+2)$$

$$= 18\hat{i} - 22\hat{j} - 14\hat{k}$$

$$|\vec{A}\times\vec{B}| = \sqrt{(18)^2+(22)^2+(14)^2}$$

$$= \sqrt{1004} = 31.68$$

Chapter 3 Kinematics

Particle:- All the bodies can be considered to be a collection large number of small particles packed together.

Rest and motion:-

A body is said to be at rest if it does not change its position with respect to its surrounding.

→ A body is said to in motion if it changes its position with respect to the surrounding.

→ The position of a particle can be specified by its projections on the third axis of a rectangular co-ordinate system having its origin 'o' at a fixed point. Distance of these ~~per~~ projections from 'o' are called its coo-ordinates.
If the co-ordinates do not change with time then the particle is said to be at rest. If there change in time then the particle is said to be in motion.

Displacement:-

When the position of a body changes with time, then the body is said to have been displaced.

→ Displacement of a body is a vector connecting the initial and final position of the body and is directed away from initial towards the final position.

→ The displacement has unit or length

M.K.S → m

C.G.S → cm

Dimension $[M^0 L^1 T^0]$

Speed:

- Speed of body is defined as the distance covered by the body in one second.

$$\rightarrow \text{speed} = \frac{\text{distance}}{\text{time}}$$

→ Speed is a scalar quantity.

M.K.S → m/s

C.G.S → cm/s

Dimension → $[M^0 L^1 T^{-1}]$

Velocity

- Velocity of a body is defined as the rate of change of displacement.

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

- Velocity is a vector quantity.

Acceleration

- It is defined as change in velocity in one second.

$$\text{Acceleration} = \frac{\text{Velocity}}{\text{time}}$$

M.K.S = m/s^2

C.G.S = cm/s^2

Dimension = $[M^0 L^1 T^{-2}]$

Force:-

- It is defined as the product of mass and acceleration

Force = mass X acceleration

- MKS - Newton (N)

CGS - dyne (D)

Dimension = $[M^1 L^1 T^{-2}]$

Equations of motion under gravity

- If a body moves freely under the action of gravity then corresponding equations of motion can be obtained by replacing 'a' with g in kinematics equation of motion.

i) Velocity - time relation

$\Rightarrow v = u + gt$

ii) Displacement time relation

$\Rightarrow s = ut + \frac{1}{2}gt^2$

iii) Velocity displacement relation

$\Rightarrow v^2 - u^2 = 2gs$

iv) Displacement in n^{th} second

$\Rightarrow S_{nth} = u + \frac{g}{2}(2n-1)$

Q. A body starts from rest and aquires velocity of 12m/s in 5 sec. Calculate the acceleration and distance travelled.

Sol: $u = 0 \quad v = 12 m/s \quad t = 5 sec$

$v = u + at$

$\Rightarrow a = \frac{v-u}{t}$

$\Rightarrow a = \frac{12-0}{5} = 2.4 m/s^2$

~~Sit~~

$s = ut + \frac{1}{2}at^2$

$= 0 \times 5 + \frac{1}{2} \times 2.4 \times (5)^2$

$= 1.2 \times 25$

$= 30m$

Q.2 A car moving with a velocity of 30m/s is stopped by the application of brakes which impact a retardation of $6 m/s^2$ to the car. Find

i) how long does it take for the car to come to a stop.

ii) How far does the car travel during the time brakes are applied.

Sol: $u = 30 m/s$ $\quad v = 0 m/s$ $\quad a = -6$

$a = -6 m/s^2$

$\Rightarrow v = u + at$

$\Rightarrow t = \frac{v-u}{a}$

$= \frac{0-30}{-6}$

$= \frac{\cancel{-30}}{\cancel{-6}}$

$t = 5 sec$

$s = ut + \frac{1}{2}at^2$

$= 30 \times 5 + \frac{1}{\cancel{2}}(\cancel{-6})^{3} \times 5^2$

$= 150 + (-75)$

$= (150 - 75)$

$= 75 m$

Rotational kinematics

- It is a branch of physics which deals with study of rotational motion without going into the cause of motion.

Angular displacement:-

- Angular displacement of a particle undergoing rotational motion is defined as the angle turned by its radius vector (θ).

Angular velocity (ω):-

- Angular velocity of a particle undergoing rotational motion is defined as the rate of change of angular displacement with time.

$$\boxed{\omega = \frac{\theta}{t}}$$

unit s^{-1}.

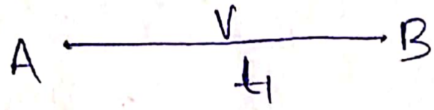
Angular acceleration (α)

- Angular acceleration of a body is defined as the rate of change of its angular velocity ~~of~~ with time.

$$\boxed{\alpha = \frac{\omega}{t}}$$

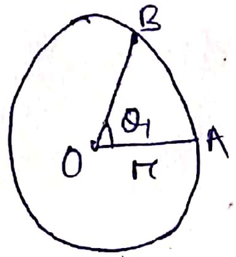
Relation between linear ($\vec{v}$) and angular ($\vec{\omega}$) velocity

Let, v be the magnitude of linear velocity of the body. Since, it takes a time t_1 to go from A to B.



$$\boxed{AB = vt_1} \quad \text{--- (1)}$$

Angular



$$\boxed{AB = r\theta_1} \quad \text{--- (2)}$$

From eq (1) & (2)

$$vt_1 = r\theta_1$$

$$\boxed{\Rightarrow v = \frac{r\theta_1}{t_1}} \quad \text{--- (3)}$$

But $\omega = \dfrac{\theta}{t}$

$$\Rightarrow \boxed{v = r\omega}$$

Relation between linear acceleration and angular acceleration

$$\boxed{\alpha = \frac{\omega}{t}} \quad \text{— (i)}$$

$v = r\omega$

$$\Rightarrow \boxed{\omega = \frac{v}{r}} \quad \text{— (2)}$$

putting the value of ω in eq (i)

$\cancel{\alpha = \frac{\omega}{t}}$

$\alpha = \frac{v}{rt}$

$\Rightarrow \alpha = \frac{a}{r} \quad \left[\frac{v}{t} = a\right]$

$\Rightarrow \boxed{a = r\alpha}$

Linear acceleration = radius x angular acceleration

$\Rightarrow a = r\alpha$

Linear velocity = radius x angular velocity

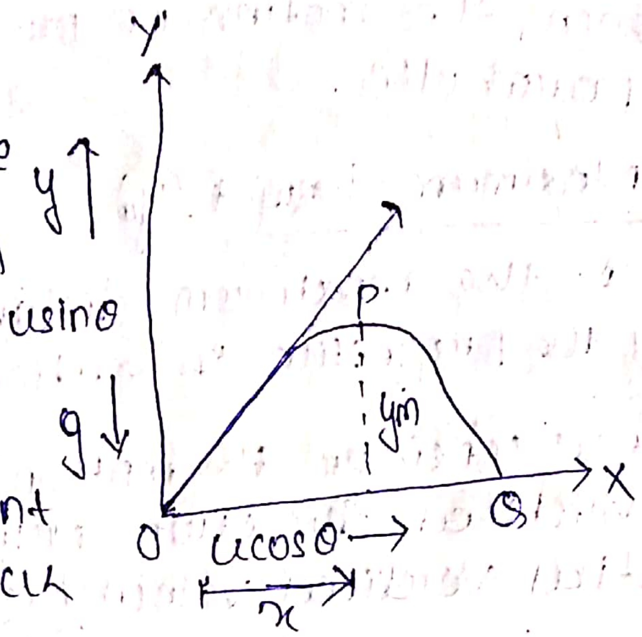
$\Rightarrow v = r\omega$

Projectile:-

- A body projected into space and is no longer being propelled by fuel.
- Some energy is given to the projectile at the initial state. As it moves through spaces then there is no supply of energy to it and it moves freely under the action of gravity.

ex. A bullet fired from a rifle.
A cricket ball thrown into space.

Projectile fired at an angle θ with the Horizontal



- Consider a particle fired with velocity 'u' at an angle 'θ' with the horizontal. The projectile reaches to the highest point 'P' and falls back at 'Q' lying on the same level of projection.

Resolve 'u' into two components

(i) ucosθ along horizontal and it is uniform.

(ii) usinθ along vertically and upward direction and it is non-uniform.

Following imp. result can be derived in this case.

i> Equation of trajectory:-

Horizontal equation of motion (uniform)

$$x = vt$$
$$= \boxed{u\cos\theta . t} \quad \text{—(i)}$$

Vertical eq. of motion (non-uniform)

$$\boxed{y = u\sin\theta . t - \frac{1}{2} g t^2} \quad \text{— (2)}$$

From eq.(i) ~~&(2)~~ we get $= \boxed{t = \frac{x}{u\cos\theta}}$ —(3)

putting the value of t in eq.(2)

$$\Rightarrow y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \boxed{y = x\tan\theta - \frac{1}{2} \frac{-gx}{u^2\cos^2\theta}} \quad \text{— (4)}$$

Eq. (4) is the eq. of parabola
Hence, the motion of projectile is parabolic.

ii) Maximum height (y_0)

- It is the maximum distance travelled by the projectile in vertical direction.

Let us consider the motion of projectile in vertical direction only. A point 'O' initial vertical velocity is
Acceleration = $-g$. vertical distance travelled = y_0
Applying kinematic relations

$$v^2 - u^2 = 2as$$

$$\Rightarrow 0 - (u\sin\theta)^2 = 2(-g)y_0$$

$$\Rightarrow -u^2\sin^2\theta = -2gy_0$$

$$\Rightarrow \boxed{y_0 = \frac{u^2\sin^2\theta}{2g}} \quad -(5)$$

iii) Time of ascent (t)

It is the time taken by the projectile to rise to the highest point.
Hence, the initial vertical velocity = $u\sin\theta$
Final, vertical velocity = 0, accln = $-g$ $T = t$
Applying kinematics relation.

$$\Rightarrow v = u + at$$

$$\Rightarrow 0 = u\sin\theta + (-g)t$$

$$\Rightarrow 0 = u\sin\theta - gt$$

$$\Rightarrow gt = u\sin\theta \Rightarrow \boxed{t = \frac{u\sin\theta}{g}} \quad -(6)$$

4> <u>Total time of Flight (T)</u>

It is the time taken by the projectile to come back to the same level from which it was projected.

$$T = 2t$$

$$\Rightarrow \boxed{T = \frac{2u\sin\theta}{g}} \quad -(7)$$

- Total time of flight is twice the time taken to reach the highest point.

5> <u>Horizontal range (x)</u>

- It is the distance travelled by the projectile in the horizontal direction.

$x =$ horizontal velocity $\times$ total time of flight

$$\Rightarrow x = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$= \frac{u^2 2\sin\theta\cos\theta}{g}$$

$$\boxed{x = \frac{u^2\sin 2\theta}{g}} \quad -(8)$$

6> <u>Condition for maximum horizontal range</u>

$$\sin 2\theta = 1 = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

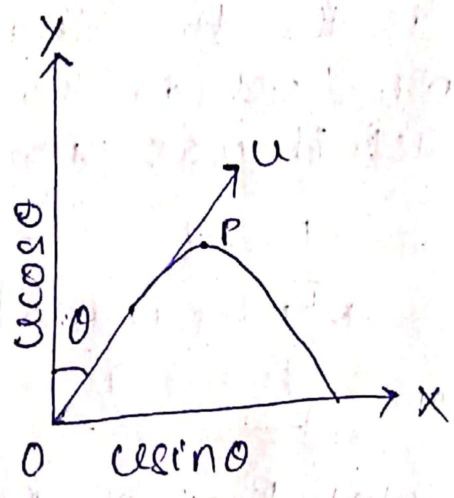
$$\Rightarrow \theta = 45^\circ$$

$$\boxed{x_{max} = \frac{u^2}{g}} \quad -(9)$$

eq (9) is the maximum horizontal range.

Projectile fired at angle θ with the vertical

- When a body is fired with a velocity 'u' making an angle 'θ' with the vertical. Then components of u interchanged



- Horizontal components is $u\sin\theta$ and vertical component is $u\cos\theta$.

i) Equation of trajectory (y)

$$y = x\cot\theta - \frac{1}{2}\left(\frac{gx^2}{2u^2\sin^2\theta}\right)$$

ii) Time of ascent (t)

$$t = \frac{u\cos\theta}{g}$$

iii) Total time of flight (T)

$$T = \frac{2u\cos\theta}{g}$$

iv) max. height (y_0)

$$y_0 = \frac{u^2\cos^2\theta}{2g}$$

v) Horizontal range

$$x = \frac{u^2\sin 2\theta}{g}$$

Friction

Work:- Work is said to be done if a force acting on a body displaces the body through a certain distance and the force has some component along the displacement

$$\boxed{W = FS}$$

OR

- Work is defined as the dot product of force and displacement

$$\boxed{W = \vec{F} \cdot \vec{S}}$$

- Unit of work :- Joule/erg

Dimension → $[M^1 L^2 T^{-2}]$

Friction: Whenever a body tends to slide over another surface, an opposing force called force of friction comes into play. This force acts tangentially to the interface of two bodies.

It is of 3 types:-

i) Sliding friction

ii) Rolling friction

iii) Fluid friction.

- It has been seen that if we cut off the engine of a moving car, it comes to rest after covering some distance. According to Newton's first law, motion could only be destroyed by an external force. This means external force is there which opposes the motion.

Sliding Friction:-

- The motion of a body over another surface is said to be sliding if the points of contact of first body with other always remain constant.
- The force of friction which comes into play between ~~two~~ two surfaces when one tends to slide over the other is called sliding friction

Static Friction:-

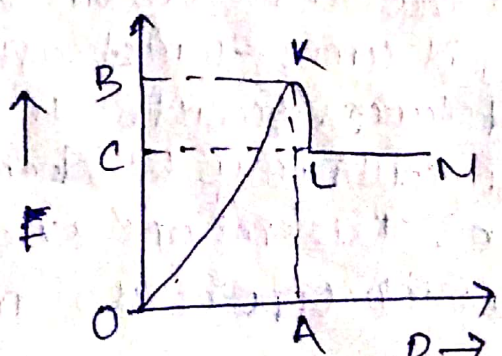
- It is the force of ~~sliding~~ friction between two surfaces so long as there is no relative motion between them.

Dynamic Friction:-

- Dynamic friction is the force of friction which comes into play between two surfaces when there is some relative motion between them.

Limiting friction:-

- It is the maximum value of force of friction between two surfaces so long as there is a no relative motion between them.



Limiting Friction

→ Static Friction is always equal to the applied Force. It is seen that the value of Static friction increases to certain maximum value OB. Beyond which if the applied force is increased then the body starts moving. This maximum value of force of friction is called Limiting friction.

- On increasing P beyond OA the body starts sliding then we can see the force of friction decreases slightly from OB to OC and then remains constant throughout. Then increase in P will produce an acceleration in the body. This force of friction is called Dynamic friction.

Laws of Limiting Friction

1) The direction of force of friction is always opposite to the direction of motion
2) The force of limiting friction depends upon the nature and state of polish of the surfaces in contact an α tangentially to the interface between two surfaces.
3) The magnitude of limiting friction α is directly proportional to the magnitude of the normal reaction are between the two surfaces in contact

$F \propto R$.

4) The magnitude of limiting friction between two surfaces is independent of the area and shape of the surfaces in contact so long as the normal remains the same.

Co-efficient of friction

- Co-efficient of friction of a pair of surface in contact is defined as the ratio between the force of limiting friction 'F' to the Normal reaction 'R'.
- It is denoted by μ.

$$\boxed{\mu = \frac{F}{R}}$$

- since F depends upon the nature and state of polish of the surface in contact so that 'μ' also depends upon these factors.

Q. Find the horizontal for required to move a body of mass 200 kg on a rough horizontal surface having co-efficient of friction 0.35.

Sol:- $\mu = 0.35$ $M = 200kg$

$\boxed{R = Mg}$ $(g = 9.8N)$

$\Rightarrow \mu = \frac{F}{R}$

$\Rightarrow F = \mu R$

$= \mu Mg$

$= 0.35 \times 200 \times 9.8N$

$= 686N$

Q. A Force of 10N is just sufficient to pull a block 20N over a flat surface. What is the angle of friction?

Sol: $F = 10N$ $R = 20N$

$$\mu = \frac{10}{20} = \frac{1}{2}$$

$$\Rightarrow \mu = \tan\theta$$

$$\Rightarrow \theta = \tan^{-1}(\mu)$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

Method to reduce friction

- Friction can be reduced if we try to remove the cause of friction.

Methods:-

1) By rubbing and polishing:-

~~The irregularities~~
The irregularities of the surface are smoothened by ~~rea~~ rubbing so that it reduces the friction between them.

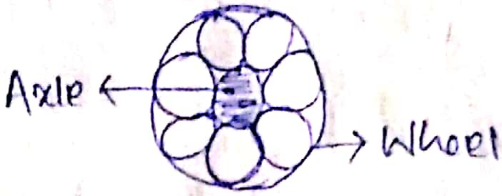
ii) By lubricants:-

A lubricant is an oil which when spread over the surfaces and fills the irregularities and forms a thin layer between.

→ The sliding now occurs between upper surface and the layer of lubricant. This friction is very small than between two surface.

iii) By converting sliding into rolling friction

- If we slide a heavy object on the floor then it needs a big force. Again if we put it on wheels we can move it easily. This is due to the reason that rolling friction is much lesser than sliding friction.



(Ball bearing system)

- Sliding friction can be converted to rolling friction by a system known as Ball bearing system.
- A no. of steel balls are inserted between the wheel and axle which reduces the force of friction by large amount.

iv) By streamlining:-

- When a body is moving through a fluid then fluid friction depends upon the shape of the body.
- It is minimum for a shape known as streamline shape.
- This shape is a pin-pointed one so that all high speed ~~bod~~ bodies like aeroplanes, rockets have pin-pointed shapes.

Gravitation

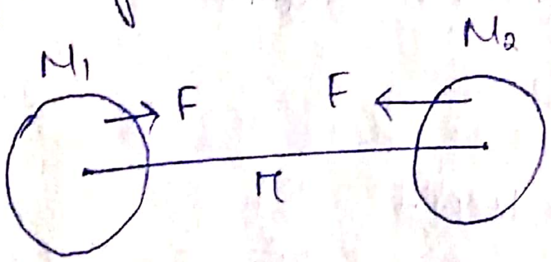
- When a body is ~~rotated~~ release in free space then it falls towards the earth. It appears that earth attracts everything towards it. This observation led Newton to discover the process called gravitation.

In astronomy various terms like stars, planets, and satellite are used.

Newton's law of gravitation

Statement:

- Every particle of matter in this universe attracts every other particle with a force which varies as the product of the masses of 2-particles and inversely as the square of the distance between them.
- The force of attraction between any two bodies in the universe is known as the force of gravitation.
- This force is mutual and acts along the line joining the centres of two bodies.



(Gravitation attraction between two bodies)

Consider two bodies of masses M_1 and M_2,
r = the distance between their centres.
Let F be the magnitude of force of attraction between them.

According to law gravitation,

$$F \propto M_1 M_2 \quad \text{---} (1)$$

$$F \propto 1/r^2 \quad \text{---} (2)$$

Combining eq. (1) & (2)

$$F \propto \frac{M_1 M_2}{r}$$

$$\Rightarrow \boxed{F = G \frac{M_1 m_2}{r^2}} \quad \text{---} (3)$$

where G = Constant of proportionality.
- This constant has the same value ~~ever~~ everywhere and is known as universal Gravitation constant or Gravitational constant.

Universal Gravitational Constant

$$F = G \frac{m_1 m_2}{r^2}$$

When $m_1 = m_2 = 1$ unit

$r = 1$ unit

$$\boxed{F = G}$$

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- The Gravitational Constant is defined as the magnitude of force of attraction between two bodies each of unit mass and separated by a unit distance from each other.

$$G = \frac{Fr^2}{m_1 m_2} \quad \left(F = G\frac{m_1 m_2}{r^2}\right)$$

$$= \frac{dyne \cdot cm^2}{gm^2}$$

$$= dyne\ cm^2\ gm^{-2}\ (C.G.S)$$

<u>M.K.S</u>

$$G = \frac{Newton \cdot m^2}{kg^2} = Nm^2 kg^{-2}$$

<u>Dimension</u>

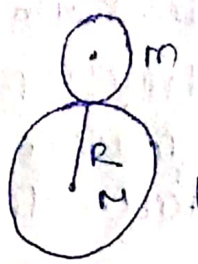
$$G = \frac{[M^1 L^1 T^{-2}][L^2]}{[M]^2}$$

$$= [M^{-1} L^3 T^{-2}]$$

$$\boxed{\begin{aligned} G &= 6.67 \times 10^{-11} Nm^2 kg^{-2} \\ &= 6.67 \times 10^{-8} dyne\ cm^2 gm^{-2} \end{aligned}}$$

<u>Acceleration due to gravity</u>

- The acceleration produced by gravity is called acceleration due to gravity. It is denoted by 'g'.
- The force between earth and a body near it is called gravity.
- The force with which a body is attracted towards earth is called weight. So, weight of a body and the gravity are similar things.



(Attraction due to earth)

Gravity, $F = mg$ — (i)

Since, the body is placed on the surface of earth of mass M and radius R.
According to newton's law of gravitation.

$$F = \frac{GMm}{R^2} \quad \text{— (2)}$$

Comparing eq (1) & (2) we get,

$$Mg = \frac{GMm}{R^2}$$

$$\boxed{g = \frac{GM}{R^2}} \quad \text{— (3)}$$

eq (3) is the relation between g & G.

unit of 'g' in MKS → m/s^2

C.G.S → cm/s^2

Dimension = $[M^0 L^1 T^{-2}]$

- Since 'g' depends upon the mass of planet and radius of planet so that it is not a universal constant.

<u>Mass</u>:

- The amount of matter present inside a body is called mass.

Weight :-

- The force by which a body is attracted towards earth is called weight.

Q1. A mass of 2kg experienced a weight of 18N on a planet. What is the value of 'g' on the planet.

Sol:

Mass = 2kg weight = 18N

$W = mg$

$\Rightarrow g = \frac{W}{m} = \frac{18}{2} = 9 m/s^2$

Q2. Find the force of gravitational attraction between two neutrons whose centres are 10^{-2}m apart.

Sol:

$r = 10^{-12} m$

$G = 6.67 \times 10^{-11} Nm^2kg^{-2}$

$M_1 = M_2 = 1.67 \times 10^{-27} kg$

$F = G\frac{m_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times (1.67 \times 10^{-27})^2}{(10^{-12})^2}$

$= 1.8 \times 10^{-40} N$

Q3 Two bodies of masses 2kg and 5kg are placed separated by a distance of 0.4m Assuming that the only force acting on them are due to gravitational interaction. Find their initial accⁿ?

Sol:

$G = 6.67 \times 10^{-11} Nm^2kg$

$M_1 = 2kg \quad M_2 = 5kg \quad r = 0.4m$

$F = G\frac{m_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 2 \times 5}{(0.4)^2} = \frac{6.67 \times 10^{-11} \times 10}{0.16}$

$F = 41.7 \times 10^{-10} N$

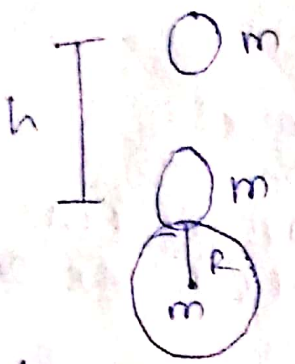
$F = ma$

$\Rightarrow a = \frac{F}{m}$

$a_1 = \frac{F}{m_1} = \frac{41.7 \times 10^{-10}}{2} = 20.85 \times 10^{-10} \, m/s^2$

$a_2 = \frac{F}{m_2} = \frac{41.7 \times 10^{-10}}{5} = 8.34 \times 10^{-10} \, m/s^2$

Variation of g with Altitude



(Effect of altitude on g)

- Consider a body of mass m placed on the surface of earth.

  M = mass of earth

  R = Radius of earth

  g = acceleration due to gravity on the free surface of earth.

Then $\boxed{g = \frac{GM}{R^2}}$ — (i)

- When the body is taken to a height 'h' above the surface of earth.
- Let the value of acceleration due to gravity at this height be g'.

So, $\boxed{g' = \frac{GM}{(R+h)^2}}$ — (2)

Divide eq (2) by (1)

$$\Rightarrow \frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

$$= \frac{R^2}{(R+h)^2}$$

$$\Rightarrow \frac{g'}{g} = \frac{R^2}{\left[R\left(1+\frac{h}{R}\right)\right]^2}$$

$$= \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow \frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

Since h is very small than R, so $\frac{h}{R}$ is very small

Binomial theorem $(1+x)^{-n} = (1-nx)$

$$\text{so } \frac{g'}{g} = \left(1-\frac{2h}{R}\right)$$

$$\Rightarrow g' = g\left(1-\frac{2h}{R}\right)$$

$$\Rightarrow \boxed{g' = g - \frac{2h}{R} g}$$

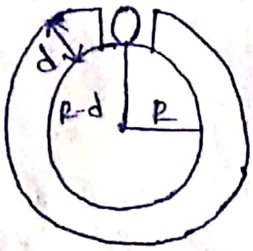
$g - g'$ = change in g

Since, the value of 'g' at a given height on the earth is constant and R is also constant then

$$\boxed{g - g' \propto h}$$

- It is ~~Clean~~ Clear from the derivation that if 'h' increases than g' must decreases because g is constant.
- So, the value of accⁿ due to gravity decreases with increase in height above the surface of earth.

<u>Variation of g with depth</u>



(Effect of depth on g)

- Consider the earth to be a homogenous sphere of radius 'R' and 'M'. Let ρ = mean density of the surface of earth where the value of accⁿ due to gravity is 'g'. Let the body be taken to a depth 'd' below the free surface of earth. Let, g' = the value of accⁿ due to gravity at depth 'd'.

$$g = \frac{GM}{R^2}$$

$$\Rightarrow g' = \frac{GM}{(R-d)^2}$$

$$\boxed{g' = g\left(1 - \frac{d}{R}\right)}$$

$\boxed{g - g' \propto d}$ as R is constant

- It is clear ~~the~~ from the eq that if d increases then g' must decreases because g is constant so, the value of acceleration due to gravity decreases when the depth increases.

- The weight of body at the centre of earth is equal to 0.
- The value of acceleration due to gravity at a height is the same as the value of acceleration due to gravity at a depth $d = 2h$.

Kepler's law of Planetary motion

1st law :-

Law of Eliptical orbit :-

A planetary moves around the sun in an elliptical orbit with sun situated at one of its

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2nd law Law of Areal Velocities:-

- A planet moves around the sun in such a way that its areal velocity is constant.

3rd law Law of Time period

- It is also known as the Harmonic law.
- A planet moves around the Sun in such a way that the square of its time period is proportional to the cube of ~~semi-conductor~~ semi-major axis of its elliptical orbit.

$$\Rightarrow T^2 \propto R^3.$$

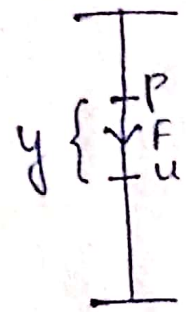
→ If, T_1 and T_2 are the time periods of two planets having R_1 and R_2 as the semi-major axis of the orbits of two planets respectively then

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Oscillation & Waves

SHM (Simple Harmonic Motion)

- A particle is said to be in SHM if its acceleration is proportional to the displacement and is always directed towards the mean position.



$\Rightarrow a \propto y$

$= \boxed{a = -ky}$

The -ve sign indicates that the acceleration is always directed towards the mean position and it will oppose the increasing displacement.

(Displacement in SHM)

OR

SHM is the ~~motion~~ an motion in which the restoring force is proportional to displacement from the mean position and opposes its increase.

$\Rightarrow F \propto y$

$\Rightarrow F = -ky$

Ex. of SHM:-

- Vibration of pendulum
- Vibration of stretched string
- vertical vibration of a loaded spring
- Vibration of a liquid contains in the two limbs of a U-tube.

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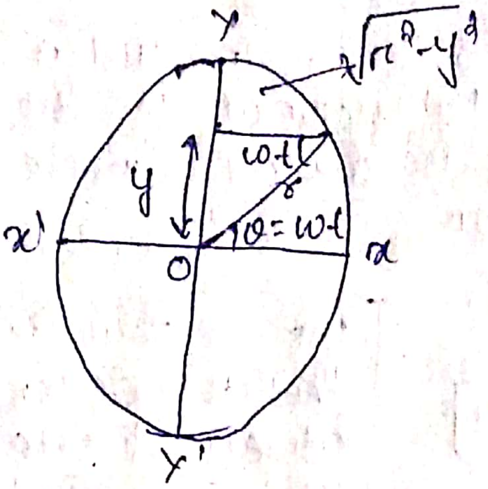
Expression for displacement, velocity, acceleration of a body / particle in SHM.



(SHM as projection of uniform Circular motion)

<u>Displacement (y) or Amplitude (r)</u>

- Displacement of a particle vibrating in SHM at any instant is defined as its ~~diameter~~ distance from the mean position at that instant.
- Let P be the position of projection of A at time 't'. Time is measured from the instant when P was at 'O'.

in ΔOAP, $\sin\theta = \frac{OP}{OA}$

$\Rightarrow OP = OA\sin\theta$

$\Rightarrow y = r\sin\theta$ —— (1)

$\therefore y = r\sin\omega t$ —— (2)

$\boxed{\omega = \theta/t \quad \theta = \omega t}$

ω = angular velocity

- It is seen from eq (2) that 'y' changes with time. 'y' is maximum when 'sin ωt' is maximum.

Since, extreme values of $\sin\omega t = \pm 1$.

so, $\boxed{y = \pm r}$

r = amplitude of vibrations.

- Amplitude (r) of a particle vibrating in SHM is defined as its maximum displacement on either side of the mean position.

→ Unit of y or r is m/cm.

Velocity:-

Differentiating eq (2) we get

$$V = \frac{dy}{dt} = \frac{d}{dt}(r\sin\omega t)$$

$$= r\cos\omega t \cdot \omega$$

$$\boxed{V = r\omega\cos\omega t} \quad -(3)$$

In ΔOAP, $\cos\omega t = \frac{AP}{OA}$ $\sqrt{\frac{r^2 - y^2}{r}}$

$$V = V\sqrt{\frac{r^2 - y^2}{r}}$$

Again. $V = r\omega$

$$\Rightarrow V = \frac{r\omega\sqrt{r^2 - y^2}}{r}$$

$$\Rightarrow \boxed{V = \omega\sqrt{r^2 - y^2}} \quad -(4)$$

At point 'O', $y=0$

$$V = \omega\sqrt{r^2-0}$$

$$= \omega r$$

$\boxed{V=v}$ (max velocity)

At point y or y'

$\Rightarrow y = \pm r$

$$V = \omega\sqrt{r^2-y^2}$$

$$= 0$$

- A particle vibrating in SHM passes with maximum velocity through the mean position and is at rest at the extreme position.

Acceleration :-

- Acceleration 'a' of the particle can be obtained by differentiating eq (3).

$$V = v\cos\omega t$$

$$\Rightarrow a = \frac{dV}{dt} = \frac{d}{dt}(v\cos\omega t)$$

$$\Rightarrow a = v(-\sin\omega t)\,\omega$$

$$= -v\omega\sin\omega t$$

$$= -v \cdot \frac{v\sin\omega t}{r}$$

$$\Rightarrow \boxed{a = \frac{-v^2}{r}\sin\omega t} \quad -(5)$$

In $\triangle OMP$

$\sin\omega t = \frac{OP}{OA} = \frac{y}{r}$

$\Rightarrow a = -\frac{v^2}{r} \cdot \frac{y}{r}$

$\Rightarrow a = -\frac{v^2}{r^2} y \quad (v = r\omega)$

$\Rightarrow a = -\frac{r^2\omega^2}{r^2} y$

$\Rightarrow \boxed{a = -\omega^2 y}$ — (6)

At point O,

$y = 0$

So, $a = -\omega^2 y \Rightarrow a = -\omega^2 0$

$\Rightarrow a = 0$

At point Y or Y'

$\Rightarrow Y = \pm r$

$\Rightarrow a = \pm\omega^2 y$

$\Rightarrow a = \pm\omega^2 r$

- A particle vibrating in SHM has zero acceleration while passing through mean position and it has maximum acceleration while at extreme position.

Time period (T)

- It is the time taken by the particle to complete one vibration.

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$$\boxed{T = \frac{2\pi}{\omega}}$$

From eq (6)

$$a = -\omega^2 y$$

$$\Rightarrow \omega^2 = \frac{a}{y}$$

$$\Rightarrow \omega = \sqrt{\frac{a}{y}}$$

$\left[\begin{array}{l} a = \text{acc}^n \\ y = \text{displacement} \end{array} \right.$

$$\Rightarrow T = \frac{2\pi}{\sqrt{\frac{a}{y}}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{y}{a}}} \quad -(7)$$

– Using eq (7), the displacement and accleration of the particle at a particular time, its time period can be calculated.

Q1 If a particle executes simple harmonic motion of time period 8 sec and amplitude 0.4m then. find the maximum velocity and accleration?

Sol: Given, $T = 8s$

$$T = \frac{2\pi}{\omega}$$

$$\Rightarrow \omega = \frac{2\pi}{T}$$

$$= \frac{2\pi}{4} \Rightarrow \omega = \frac{\pi}{4}$$

$$r = 0.4m$$

maximum velocity, $V = r\omega$

$$\Rightarrow V = 0.4 \times \frac{\pi}{4} = \frac{\pi}{10} = \frac{3.14}{10}$$

$$= 0.314 m/s$$

maximum acceleration

$$a = \omega^2 r$$

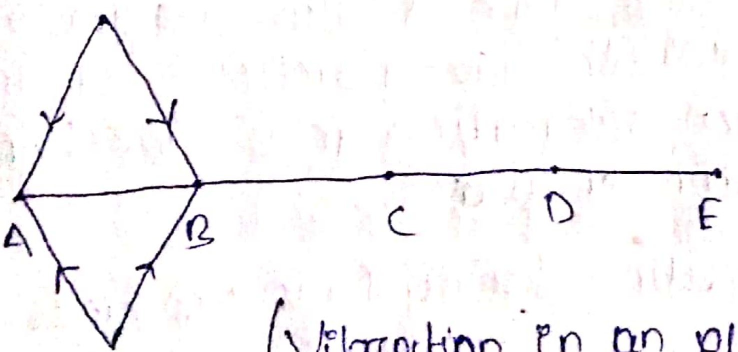
$$\Rightarrow a = \frac{\pi^2}{4} \times 0.4$$

$$\Rightarrow a = 0.2468 \; m/s^2$$

Wave motion :-

- Wave motion is the disturbance that travells through the medium and is due to repeated periodic motion of the particle of the medium, the motion being handed over from particle to particle.

(Ripple formation)



(Vibration in an elastic medium)

- The vibration of 'A' is communicated to other molecules like B, C, D, E, so it appears that a distrubance is ~~called~~ travelling from left to right.
- Travelling of these distrubance is called wave motion.

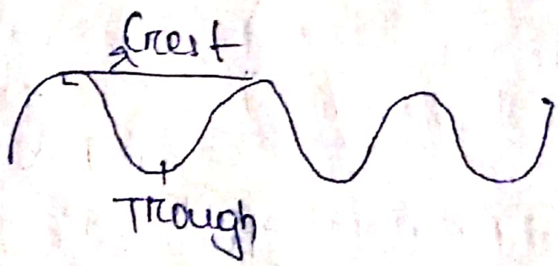
Types of waves

Waves are of 2 types:

i) Transverse:-

It is the type of wave motion in which the particles of the medium are vibrating in a direction at right angles to the direction of propagation of waves.

ex. A stone thrown in a pond of water results in the formation of transverse wave motion.



ii) Longitudinal:

It is the type of wave ~~motion~~ motion in which the particles of the medium are vibrating in a direction of propagation of wave.

ex. A vibrating tuning fork produces longitudinal waves.

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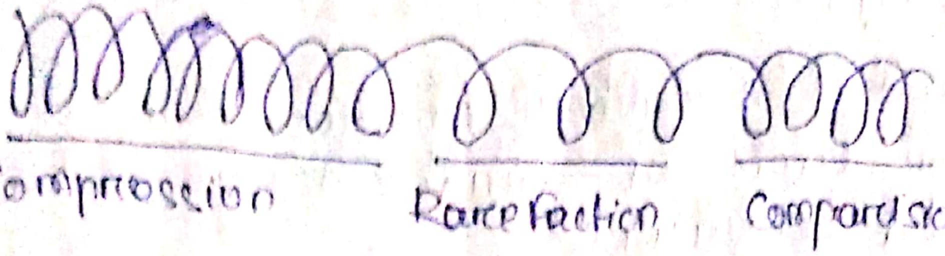


Q. Comparision between Transverse and Longitudinal wave.

Transverse	Longitudinal
- Vibration of the particles of medium are normal to the direction of propagation of wave.	- Vibration of the particle of medium are parallel to the direction of propagation of wave.
- Crest and through are form during its propagation.	- Compression and rarefaction are form during its propagation.
- It causes temporary change in size of the medium.	- It causes temporary change in size of the medium.
- It travelles through solid ~~bed~~ & shallow liquid but not in gas.	- It travels through solid liquid & gas
- velocity = $v = \sqrt{T/m}$	- velocity $V = \frac{F}{y}$

Ultrasonic:-

- Sound of frequency greater than the upper limit of audible range is called ultrasonic.
- Ultrasonic wave have a range of 2×10^4 to 10^9 cs^{-1} (cycle)

Properties of ultrasonic

- Ultrasonic waves are longitudinal in nature.
- propagation of ultrasonic through a medium results in the formation compression and rarefraction.
- These are the waves of very high frequency having a range of 2×10^4 to 10^9 Hz
- They travell with the speed of sound.
- Ultrasonic are highly energetic waves.
- Ultrasonic do not spread that much as audible sound due to their smaller wave length.
- Ultrasonic constitutes narrow beam.
- passage ultrasonic through liquid results in variation of density.

Application of Ultrasonic

- Ultrasonic are highly energetic waves, so these are used in industries and scientific research.

i) Echo sounding
ii) Thickness gauging
iii) Flow detection
iv) Ultrasonic welding
v) Ultrasonic Cleaning
vi) Fuel guging
vii) Drilling holes
viii) In biological effects.

Heat and Thermodynamics

Heat: It is a form of energy which possess the sensation of hotness.

Temperature:- It is the quantity which determines the direction of flow of heat from one body to other body when

Unit of heat

C.G.S = Calorie
MKS = Kilo calorie
SI = Joule
FPS = BTU (British Thermal Unit)

Dimension - $[M^1 L^2 T^{-2}]$

Celsius Scale - 0°C - 100°C
Kelvin Scale - 27K - 373K
Fahrenheit Scale - 32°F - 212F

Q. Convert 50°C into fahrenheit scale

$$\boxed{\frac{C}{100} = \frac{F-32}{180}}$$

$$\frac{50}{100} = \frac{F-32}{180}$$

$$\Rightarrow \frac{1}{2} = \frac{F-32}{180}$$

$$\Rightarrow 2F - 64 = 180$$

$$\Rightarrow 2F = 244$$

$$\Rightarrow F = 122$$

Specific heat

- Specific heat of a body is the amount of heat required to raise the temp. of unit mass of the body through 1°C.

- Let, some quantity of heat is supplied to a body.

M = mass of the body
θ = increases in temp. of the body
Q = amount of heat supplied.

$$Q \propto m$$
$$Q \propto \theta$$
$$\Rightarrow Q \propto m\theta$$
$$\Rightarrow \boxed{Q = ms\theta}$$

S = specific heat

* When m = 1 and θ = 1°C

then $\boxed{Q = S}$

$$Q = ms\theta \Rightarrow S = \frac{Q}{m\theta}$$

$$CGS \Rightarrow S = \frac{Cal}{gm\,^{\circ}C} = Cal\ gm^{-1}\ ^{\circ}C^{-1}$$

$$MKS = kcal\ kg^{-1}\ ^{\circ}C^{-1}$$

$$S.I = J kg^{-1} K^{-1}$$

$$\text{Dimension} = S = \frac{Q}{m\theta}$$

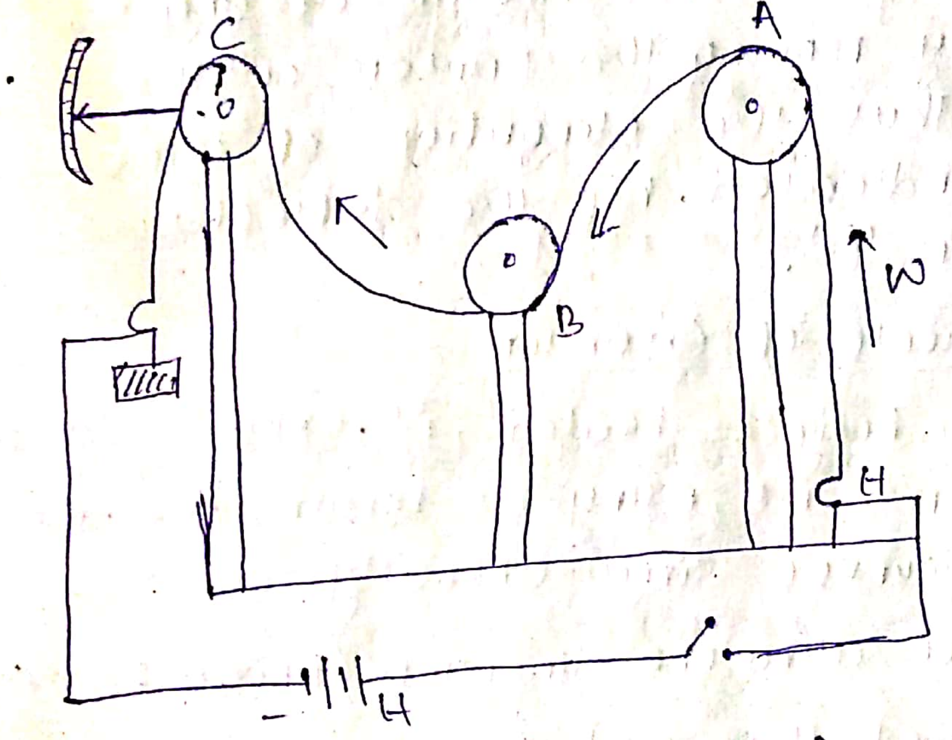
$$= \frac{[M^1 L^2 T^{-2}]}{[M][K^1]} = [M^0 L^2 T^{-2} K^{-1}]$$

specific heat of water = 1 cal gm^{-1} $°C^{-1}$
specific heat of Ice = 0.5 cal gm^{-1} $°C^{-1}$

Latent heat

- It is defined as the amount of heat absorbs by unit mass of the substance when it is converted from one state to other state.

Thermal Expansion



(Solid expand on heating)

- When an object is heated whether it is a Solid, liquid or gas, it expands.
- A simple experimental setup to know expansion solid is written below. A long wire 'W' has one end connected to a fixed hook H. and carries a weight 'Mg' at the other end. In between, the wire passes over a no. of pulleys A, B & C.

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Pulley 'C' carries a pointer 'P' which is capable of sliding over a scale 'S'. Electric current is allowed to flow through the wire by the help of a battery. Heat produced due to this current makes the wire to elongate in length. Due to elongation of wire, the weight Mg moves down then pulley C turns through an angle, there by moving the pointer on the scale. Thus the elongation of wire can be indicated by the deflection of pointer 'P'.

Co-efficients of Expansion

- When a body is heated, it expands in all dimension, i.e, along its length, breadth and thickness simultaneously.

1) Expansion along one dimension (Linear Expansion)

A long and thin rod can be considered to be one dimensional if its length is very large as compared to its diameter.

- Co-efficient of linear expansion of the material of a rod is defined as the change in length per unit length, at 0°C, per degree centigrade rise of temp.

$$\alpha = \frac{l_2 - l_1}{l_1 (t_2 - t_1)}$$

$$l_2 = l_1 [1 + \alpha (t_2 - t_1)]$$

ii) Expansion in two dimensional (Superficial expansion)

- A surface area having some length and breadth but having ~~negati~~ negligible thickness can be taken to be two dimensional.
- Co-efficient of superficial expansion is defined as the change in area of the surface per unit area at 0°C, per degree Centigrade rise of temperature.

$$S_t = S_0(1+\beta t)$$

$$\beta = \frac{S_t - S_0}{S_0 t}$$

iii) Expansion in three dimensional (Cubical expansion)

- A body having length, breadth, and thickness is said to be 3-D
- Coefficient of cubical expansion is defined as the change in volume per unit volume, at 0°C per degree Centigrade rise of temperature.

$$V_t - V_0 = \gamma V_0 t$$

$$\gamma = \frac{V_t - V_0}{V_0 t}$$

* α, β & γ are having the same unit.

MKS/CGS = $°C^{-1}$

SI = K

Relation between Expansion co-efficients

1) Relation between α and β.

- Consider a square sheet having each side $= l_0$ at $0°C$

Area S_0 at $0°C \Rightarrow S_0 = l_0^2$

On heating the sheet to $t°C$, each side expands to l_t.

Area of the sheet at $t°C$, $S_t = l_t^2 = l_0^2(1+\alpha t)^2$

$$\beta = \frac{S_t - S_0}{S_0 t}$$

$$= \frac{l_0^2[(1+\alpha t)^2 - 1]}{l_0^2 t}$$

$$\Rightarrow \beta = \frac{l_0^2[(1+\alpha t)^2 - 1]}{l_0^2 t}$$

$$= \frac{(1+\alpha t)^2 - 1}{t}$$

$$= \frac{1+\alpha^2 t^2 + 2\alpha t - 1}{t}$$

$$= \frac{\alpha^2 t^2 + 2\alpha t}{t}$$

$$= \frac{t(\alpha^2 t + 2\alpha)}{t}$$

$$= \alpha^2 t + 2\alpha$$

Since α is very small, the term $\alpha^2 t$ can be neglected being very small.

$$\Rightarrow \boxed{\beta = 2\alpha}$$

Relation between α and γ

Let V_0 and V_t be the volumes of a circuit at $0°C$ and $t°C$ respectively. If l_0 and l_t are the sides of these cube at $0°C$ and $t°C$

$$V_0 = l_0^3$$

$$V_t = l_t^3 = l_0^3 (1+\alpha t)^3$$

$$\gamma = \frac{V_t - V_0}{V_0 t} = \frac{l_0^3 (1+\alpha t)^3 - l_0^3}{l_0^3 t}$$

$$= \frac{l_0^3 \left[(1+\alpha t)^3 - 1\right]}{l_0^3 t}$$

$$= \frac{(1+\alpha t)^2 - 1}{t}$$

$$= \frac{1 + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3 - 1}{t}$$

$$= \frac{3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3}{t}$$

$$= \frac{\cancel{t}\,(3\alpha + 3\alpha^2 t + \alpha^3 t^2)}{\cancel{t}}$$

$$\gamma = 3\alpha + 3\alpha^2 t + \alpha^3 t^2$$

Since α is very small so, we neglect α^2 and α^3.

So, $\boxed{\gamma = 3\alpha}$

$\beta = 2\alpha$ $\qquad \alpha = \frac{\gamma}{3}$

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

$$\Rightarrow \boxed{\beta = \frac{2}{3}\gamma}$$

$$\Rightarrow \gamma = \frac{3\beta}{2}$$

Joule's Mechanical Equivalent of Heat (J)

Dr. James Joule concluded that there is equivalence between work and heat.

→ Whenever heat is converted into work or work into heat then the quantity of energy disappearing in one form is equivalent to the quantity of energy appearing in other.

- Joule's mechanical equivalent of heat is defined as the amount of work required to produce a unit quantity of heat.

$$\boxed{J = W/H}$$

Unit of Joule

$J = 4.2 \times 10^7 \text{ erg cal}^{-1}$ (CGS)

$J = 4.2 \text{ J cal}^{-1}$ (MKS)

First law of Thermodynamics

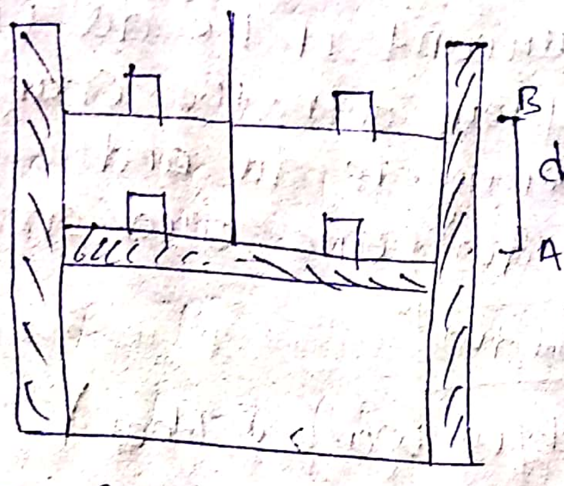
- If the quantity of heat supplied to a system is capable of doing work then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system & the external work done by it.

$$dQ = dU + dW$$

Q = heat

U = internal energy

W = Work



- Consider some gas enclosed in a barrel having insulating walls and conducting bottom. Let an amount of heat 'Q' be added to the system through the bottom.

U_1 = Initial energy of the system.

Total energy of the system in the beginning $= U_1 + Q$

- When we provide heat to the base then the gas tends to expand. Heat pushes to piston (P) from A to B. Then some work 'W' is done by the gas. The work is external work.

U_2 = Final internal energy of the system.

Total energy of the system at the end $= U_2 + W$

According to law of conservation of energy.

$\Rightarrow U_1 + Q = U_2 + W$

U_1, U_2, Q & W all are being taken in same unit.

Then $Q = U_2 - U_1 + W$

- When a small amount of he dQ is added to the system then the changes in internal energy ie dU and external work dW are also small.

Then $\boxed{dQ = dU + dW}$

(Relation b/w work & Heat)

Optics

Reflection:-

When a light comes from one medium and incident on a surface (mirror) and returns back to the same medium then this process is called as reflection.

Laws of reflection:-

1) The incident ray, the reflected ray and the normal to the reflecting surface, all lie in a ~~Sam~~ same plane.

2) The angle of incidence ($\angle i$) is equal to the angle of reflection ($\angle r$). $\angle i = \angle r$.

Refraction

- It is the process by virtue of which a ray of light going from one medium to the other medium undergoes a change in its velocity.

Laws of refraction:-

1) The incident ray, the reflected ray and the normal to the interface at the point of incidences all lie in one plane and that plane is perpendicular to the interface separating the two media.

2) The sine of the angle of incidence bears a constant ratio with the sine of the angle of refraction.

$$\frac{\sin i}{\sin r} = \text{constant}$$

- This law is also called as Snell's law

Refractive Index.

- Refractive index of a medium is a property of medium which determines its behaviour to propagation of light.

Defination in terms of angle of incidence and refraction.

- Refractive index of a medium with respect to another medium is defined as the ratio between sine of the angle of incidence to the sine of the angle of refraction.

$$\frac{\sin i}{\sin r} = {}^1\mu_2 \quad \left(\mu = \text{refractive index}\right)$$

Defination in terms of velocity of light

- Refractive index of medium 2 w.r.t medium 1 is defined as the ratio between velocity of light in medium 1 to the velocity of light in medium 2 $\left({}^1\mu_2 = \frac{V_1}{V_2}\right)$

- If the first medium is air or vaccum, the refractive index is written as μ and is known as "absolute refractive index".

$$\boxed{\mu = \frac{c}{v}} \quad C = \text{velocity of light}$$

Defination in terms of wave length

- Refractive index of second medium with respect to first medium is defined as the ratio between wave length of light in medium to the wave length of light in medium 2.

$$\boxed{{}^1\mu_2 = \frac{\lambda_1}{\lambda_2}}$$

In terms of absolute index.

Refractive index of 2nd medium with respect to 1st medium is defined as the ratio between absolute refractive of 2nd medium to the absolute refractive index of 1st medium.

$$\boxed{{}^1\mu_2 = \frac{\mu_2}{\mu_1}}$$

$${}^1\mu_2 \times {}^2\mu_1 = 1$$

Q. A ray of light travelling in water is incident at an angle of 30° on a water glass interface. Calculate the angle of refraction in glass of refractive index of water is 4/3 and refractive index of glass is 3/2.

Ans According to snell's law, $\frac{\sin i}{\sin r} = {}^1\mu_2$

$$= {}^w\mu_g$$

$$\Rightarrow {}^w\mu_g = \frac{\mu_g}{\mu_w}$$

$$\Rightarrow \frac{3/2}{4/3} = \frac{3}{2} \times \frac{3}{4}$$

$$= \frac{9}{8}$$

$$\Rightarrow \frac{\sin 30}{\sin r} = \frac{9}{8}$$

$$\Rightarrow \frac{1/2}{\sin r} = \frac{9}{8}$$

$$\Rightarrow \sin r = \frac{9}{8} \times \frac{2}{1}$$

$$= \frac{9}{4}$$

$$\Rightarrow \sin r = \frac{4}{9}$$

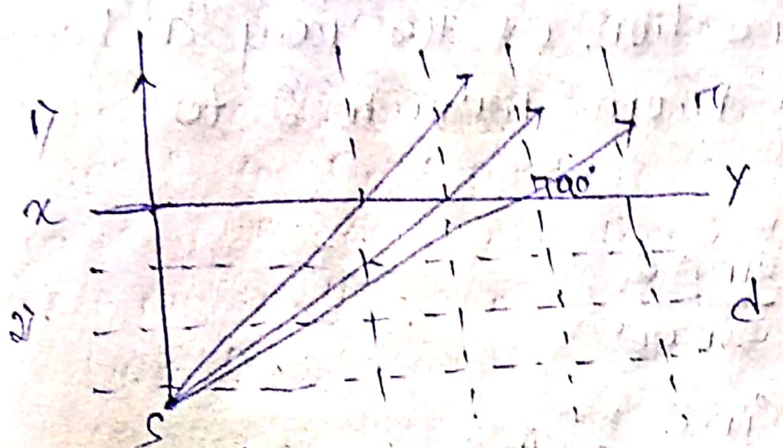
$$\Rightarrow r = \sin^{-1}\left(\frac{4}{9}\right)$$

Total internal Reflection

- It is the process by virtue of which a ray of light travelling from a denser medium to a rarer medium is send back in the same medium provided it is incident on the interface at an angle greater than critical angle.

Critical angle:-

It is the angle of incidence of a ray of light in denser medium such that its angle of ~~reflect~~ refraction in the rarer medium is 90°.



(Total internal reflection)

Let us take a source of light 's' in the denser medium (water). The rays starts from point 's' and it travells from water to air. A ray 'a' incident normally on the interface x-y goes un-deviated. The rays 'b' and 'c' are incident on the interface at gradually increasing angle of incidence. So, they deviate more and more way from the normal. A ray is ~~rade~~ incident at a particular angle of incidence 'c' such that the refracted ray is parallel to the surface. (r = 90°). These angle of incidence 'c' is called the ~~sin~~ critical angle. If the angle of incidence of the ray is increased further, it is reflected back into the same medium. This process is called "Total internal Reflection".

- The ray while suffering total internal reflection obey the laws of reflection.

- Consider refraction of the ray 'd' here the ray goes from medium 2 to medium 1.

$$^{2}\mu_1 = \frac{\sin c}{\sin 90^\circ}$$

$$= \frac{\sin c}{1} = \sin c$$

$$^{1}\mu_2 = \frac{1}{^{2}\mu_1} = \frac{1}{\sin c}$$

- If the first medium is air or vaccum the

$^{1}\mu_2 = \mu$ (absolute refractive index)

$$\mu = \frac{1}{\sin c}$$

So, the absolute refractive index of medium is equal to the reciprocal of the sine of the ciritical angle, for that medium.

<u>Refraction through a prism.</u>

- When a ray of light incident on one of the refriacting faces of a prism and proceeds through the prim. then it undergoes two changes:-

i) Deviation

ii) Dispersion

Deviation:-

A ray of light of monochromatic light while passing through a prism suffers a change in its path. This process is called deviation.

$$d = (\mu - 1) A$$

A = refracting angle of the path.
μ = refractive index
d = deviation

Dispersion:

- A ray of light containing more than 1 wavelength while passing through the prism splits up into a no. of rays. This process is called dispersion.

$$\frac{d_v - d_r}{d} = \frac{\mu_v - \mu_r}{\mu - 1} = \omega$$

ω (omega)

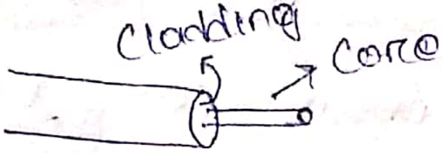
- Dispersive power of a prism is defined as the ratio between angular dispersion to ~~[illegible]~~ mean deviation produced by the prism.

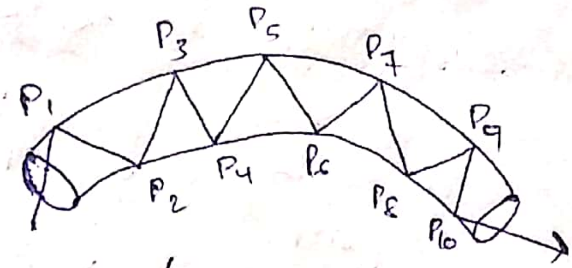
Fiber Optics

- The study of optical fibres is called fiber optics.

Optical fiber

- An optical fiber is a thin fiber of glass or plastic that can carry the light from one end to the other end.





(Optical fiber)

Proportios

- It has monomode ~~prop~~ propagation.
- It also has multimode propagation.

Application:-

- Here, the data is communicated at higher speed.
- The data transmission rate is 1Gbit s^{-1}
- No electromagnetic interference and lightning strikes.
- Here, there is no risk of short circuit or electrical spark.
- It is lighter in weight.
- Tempering of data is not easy.
- It is more resistive to atmosphere
- It has greater operational life.

Electrostatic & Magnetostatics

~~Statement~~ :

~~The electrostatic force of attraction or repulsion between two charged body is directly proportional to the~~

- Electrostatics is a branch of physics that explains electric charges at rest.

Coulomb's law in Electrostatics

Statement:

- The electrostatics force of attraction or repulsion between two charged body is directly proportional to the product of their charges and varies inversely as the square of the distance between two bodies

$$F \propto q_1 q_2$$

$$\propto \frac{1}{r^2}$$

$$\Rightarrow F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow \boxed{F = \beta \frac{q_1 q_2}{r^2}}$$

β = Constant of proportionality

- The value of β depends on the nature of the medium in which the two ~~charges~~ charges are situated. It also depends on the units in which the quantities F, q_1, q_2 and r^2 are measured.

Value of β

C.G.S
$$\beta = \frac{1}{K}$$

K = Dielectric constant of the medium

$$F = \frac{1}{K} \frac{q_1 q_2}{r^2}$$

S.I
$$\beta = \frac{1}{4\pi\epsilon_0\epsilon_r}$$

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1 q_2}{r^2}$$

$$\boxed{\epsilon = \epsilon_0\epsilon_r}$$

$$\Rightarrow F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$$

Unit Charge :-

C.G.S :- The units charge (.) is called e.s.u of charge or stat coulomb.

- Electrostatic unit of charge or Stat coulomb is that amount of charge which when placed in air at a distance of 1cm from a similar repels it with a force of 1 dyne.

S.I

one coulomb of charge is defined as that charge which when placed in air at a distance of one metre from an equal and similar charge repels it with a force of 9×10^9 N.

Relation between Coulomb and Stat coulomb

1 Coulomb = 3×10^9 stat coulomb.

ϵ = permittivity of the medium

ϵ_0 = permittivity of air or free space.

ϵ_r = relative permitivity.

ϵ_r (Relative permitivity)

- The relative permitivity of a medium defined as the ratio between the permittivity of medium and permittivity of air.

$$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0}}$$

* Relative permittivity of a medium is defined as the ratio of force between two charges when they are placed in air and the force between them when placed inside that medium.

$$\boxed{\epsilon_r = \frac{F_0}{F_m}}$$

* Relative permittivity of a medium is equal to the dielectric constant of that medium.

$$\boxed{\epsilon_r = K}$$

- ϵ_r is a unitless and dimensionless physical quantity.

* $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \, Nm^2C^{-2}$

$$\epsilon_0 = 8.85 \times 10^{-12} \, N^{-1} m^{-2} C^2$$

Electric potential :-

- It is defined as the quantity which determines the direction of flow of charge between two bodies.
- The difference in potential between two points is called the "potential difference".
- Electric potential at any point in an electric field is defined as the amount of work done in moving a unit positive charge between infinity to that point without any acceleration against the electric force.

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$ $\boxed{V = \text{potential}}$

Unit of potential

SI → Volt

C.G.S → Stat Volt

1 Volt = 1/300 Stat Volt [Electro static unit (e.s

e.m.u

1 Volt = 10^8 ab Volt

10^8 abVolt = $\frac{1}{300}$ Stat Volt

⇒ 1 Stat Volt = 300×10^8 ab Volt

= 3×10^{10} ab Volt.

Electric Field

When an electric charge is placed at a point, the properties of space around the charge get modified, the modified space around an electric charge is called electric field.

- The charge is known as "source of electric field"

- Electric field intensity due to a point charge

To measure the strength electric field at any point we place a test charge 'q_0' at that point. The test charge 'q_0' has its own electric field, we have to take precaution to keep the magnitude of test charge q_0 to be so small that it does not distrube the location of source charge or the charge distribution.

- The strength of an electric field is measured by ~~notho~~ ~~ty~~ tiny the force experienced by a unit positive charge. placed at that point.

$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

- The direction of head is given by the direction of motion of a unit positive charge if it were free to do so.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

* $F = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$ (Scalar form)

$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \vec{r}$ $\quad \hat{r} = \frac{\vec{r}}{|r|}$

Unit $\Rightarrow NC^{-1} \left(\frac{N}{C}\right)$

Q1. Two equal and similar charge 0.03m apart, in air, repel each other with a force of 4.5 kgf. (4.5 × 9.8N) Find the charge in coulomb.

Sol: $q_1 = q_2 = q$ (coulomb), $r = 0.03m$

$F = 4.5\ kgf$

$= 4.5 \times 9.8N$

Since $F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$

$\Rightarrow 4.5 \times 9.8 = 9 \times 10^9 \times \frac{q^2}{(0.03)^2}$

$\Rightarrow q^2 = \frac{4.5 \times 9.8 \times (0.03)^2}{9 \times 10^9} = \frac{441}{10^{14}}$

$\Rightarrow q = 21 \times 10^{-7} C$

Capacitance

- The capacity of a conductor is also defined as the charge required to raise it through a unit potential.

OR

- The capacity of a conductor is defined as the ratio between the charge on the conductor to its potential.

If 'V' is the potential of the conductor due to a charge 'Q', then $Q \propto V$ or $Q = CV$

$$\Rightarrow \boxed{Q = CV}$$

$$\boxed{C = \frac{Q}{V}}$$

Unit $\rightarrow \frac{\text{Coulomb}}{\text{Volt}}$ = Farad (F) (SI/MKS)

In esu $\rightarrow$ Stat farad
emu $\rightarrow$ ab farad

$1F = 9 \times 10^{11}$ stat farad

$1F = \frac{1}{10^9}$ ab farad

$\Rightarrow 9 \times 10^{11}$ stat farad $= \frac{1}{10^9}$ ab farad

$\Rightarrow 1$ stat farad $= \frac{1}{9 \times 10^{11} \times 10^9}$ ab farad

$= \frac{1}{9} \times 10^{-20}$ ab farad

$\cancel{[C^2 N^{-1} m^{-1}]}$ $[C^2 N^{-1} m^{-1}]$

$$C = \frac{Q}{V} = \frac{\text{Coulomb}}{\text{Volt}} = \frac{(\text{Coulomb})^2}{\text{Joule}}$$

$$= \frac{[A^1 T^1]^2}{[M^1 L^1 T^{-2} L^1]}$$

$$= \frac{[A^2 T^2]}{[M^1 L^1 T^{-2} L^1]}$$

$$= [A^2 M^{-1} L^{-2} T^4]$$

Grouping of capacitors

- If a no. of capacitors of capacities $C_1, C_2 .. C_n$ are grouped together, the combination behaves like a single capacitor of capacity 'C'. The value of 'C' depends upon the way in which grouping is made.

i) Capacitors in parallel:-

- The resultant capacity of a no. of capacitors connected in ll is equal to the sum of their individual capacities.

$$C = C_1 + C_2 + C_3 + \dots + C_n$$

ii) Capacitors in series.

- The reciprocal of the resultant capacity of a no. of capacitors connected in series is equal to the sum of the reciprocals of their individual capacities.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

Magnetostatics

Magnet:-

A piece of substance possess the property of attracting small pieces of iron towards it is called a magnet.

Natural magnet:-

- ~~The~~ If the property of magnetism occurs naturally then the magnet is known as natural magnet.

Artificial magnet:-

- When the magnet is made artificially, it is known as artificial magnet.

Properties of magnet.

i) Two poles of a magnet:-

A magnet has two poles North and South These poles situated a little distance inside the faces of the magnet.

- Face to face of the magnet is called geometric length and pole to pole length is called magnetic length of the magnet.
- The magnetic length is slightly less than geometric length.
- Magnetic length or ~~$2l\frac{7}{8}\times g$~~ or

$$2l = \frac{7}{8} \times \text{geometric length.}$$

2> **Attracting property of a magnet:-**

- A magnet is capable of attracting small pieces of iron towards it.

3> **Directional property of magnet:-**

- When a magnet is suspended freely then it always points in a particular direction. North pole of the magnet points towards geographical north and the south pole points towards geographical south.

4> **No existence of isolated magnetic poles**

- The magnetic poles exist only in pairs of opposite nature.

5> **Nature of the force between two poles**

- Nature of force between similar poles is repulsive while that between opposite poles is attractive.

Coulomb's law in magnetism:-

- The magnitude of force between two magnetic poles varies directly as the product of the strengths their poles and inversely as the square of the distance between then

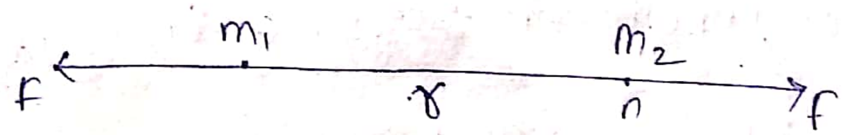


Fig. force between two magnetic poles.

Consider two magnetic poles of similar nature (n) of strength m_1 & m_2 separated by a distance (r) from each other.

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$

$$\Rightarrow F = k\frac{m_1 m_2}{r^2}$$

The force between them

$$F = k\frac{m_1 m_2}{r^2}$$

k = constant of proportionality.

In SI system the unit of k

$$k = \frac{\mu_0}{4\pi}$$

$$F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2}$$

μ = absolute magnetic permeability

$\mu_0 = 4\pi \times 10^{-7}\ WbA^{-1}m^{-1}$

<u>In C.G.S</u>

$$k = 1$$

$$F = 1 \times \frac{m_1 m_2}{r^2}$$

$$\Rightarrow F = \frac{m_1 m_2}{r^2}$$

<u>Unit pole</u> <u>In S.I unit</u>

- A unit pole in S.I system is that pole which when placed in air at a distance of 1m from a similar pole repels it with a force of 10^{-7}N.

- In C.G.S

- A unit pole in C.G.S system is that pole which when placed in air at a distance of 1cm from a similar pole ~~respe~~ repels it with a force of 1 dyne.

$$m = I_1 \begin{pmatrix} S.I \\ C.G.S \end{pmatrix}$$

Magnetic field

- Magnetic field of any magnetic pole is the space around it in which its magnetic influence can be realised.

Magnetic field Intensity or Strength of magnetic field

- Strength of magnetic field at any point is defined as the force experienced by a unit north pole at that point.
- The direction of field is the direction in which the unit north pole would move if it were free to do so.
- Magnetic intensity at any point is a vector quantity.

S.I system

$$F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2}$$

If $m_1 = 1$, $m_2 = 1$.

$$F = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^2} = B$$

C.G.S system

If $m_1 = 1$, $m_2 = 1$

$$F = \frac{m}{r^2}$$

$$B = \frac{m}{r^2}$$

M.K.S unit - Tesla

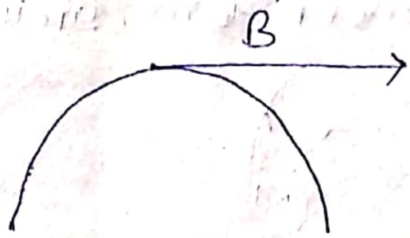
C.G.S unit - Gauss

Magnetic lines of force:-

- Magnetic lines of force is the path along which a unit north pole would move if it were freed to do so

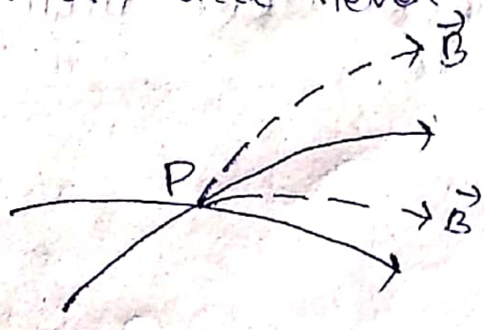
Properties of magnetic lines of force:-

- Magnetic lines of force are directed away from a north pole and are directed towards a south pole.
- Tangent at any points to the magnetic line of force gives the direction of magnetic intensity at the point.



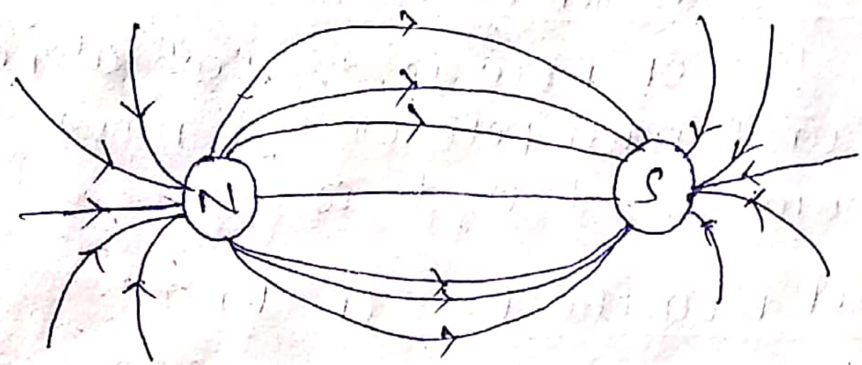
(Direction of magnetic intensity at a point)

- Two lines of force never cross each other.



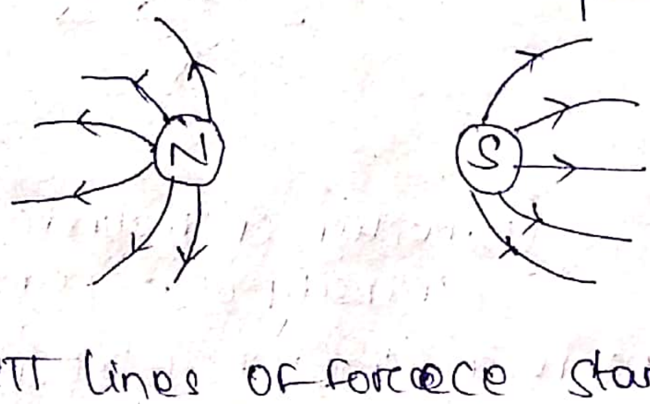
- The number of lines of force ~~per~~ per unit area is proportional to magnitude of strength of field at that point.

- The lines of force tend to contract length wise due to its p this property the two opposite poles attract each other.



(magnetic lines of force)

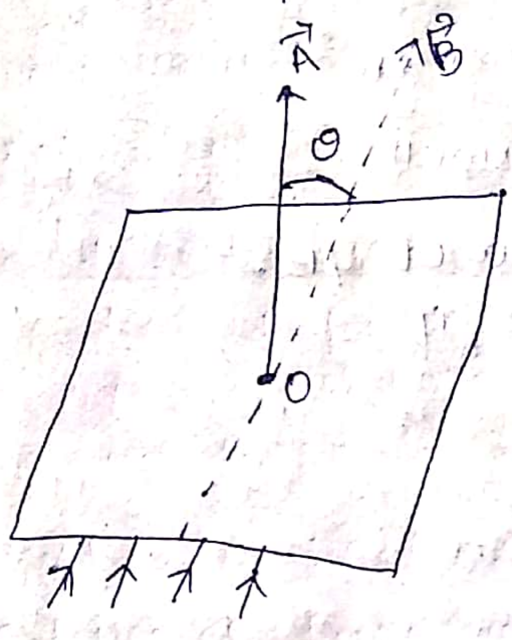
- The lines of force tend to exert lateral pressure that is they repel each other laterally it explains the repulsion between two similar poles.



- 4π lines of force start from a unit magnetic pole.

Magnetic flux (ϕ_B)

Magnetic flux link with a surface is defined as the product of area and the component of 'B' perpendicular to the area.



Where

$B\cos\theta$ = component of B perpendicular to area 'a'.

Case A → If $\theta = 90°$, $\cos\theta = 0$

$\phi_B = 0$

Case-2 → If $\theta = 0°$, $\cos\theta = 1$

then $\phi_B = AB$

Units of magnetic flux

S.I unit

ϕ_B = 1weber (wb)

1weber = 1 tesla meter^2

C.G.S unit

$A = 1cm^2$

ϕ = 1maxwell

1maxwell = 1gauss cm^2

Relation between weber and maxwell

$1 Wb = 1 tesla \times 1 m^2$

$= 10^4 gauss \times (100 cm)^2$

$= 10^8 gauss \ cm^2$

$1 Wb = 10^8 maxwell$

Dimension formula of ϕ_B.

$\phi_B = [M^1 L^1 T^{-2} A^{-1}]$

Current Electricity

Electric Current:-

- Electric current in a conductor is defined as the rate of flow of charge across any cross section of the conductor.
- If a charge 'q' flows across section in 't' second then current (i) is given by

$$i = \frac{q}{t}$$

$$\boxed{q = it}$$

- Electric current flowing through a conductor is associated magnitude as well as direction.

unit- Ampere (A)

Coulomb/sec

Ohm's law:-

- The flow of electric current through a conductor is directly proportional to the potential difference across its two ends.

$$V \propto I$$

$$\Rightarrow V = IR$$

$$\Rightarrow I = \frac{V}{R}$$

Ohmic Conductor

- The conductor which obey ohm's law is called ohmic conductor.
* The graph between voltage and current is a straight line.

Non-ohmic conductor :-

- The conductors which do not obey ohm's law
* The graph between V and I is not a straight line.

Resistance connected in series :-

- If a no. of resistances are connected in series with each other then net resistance of the combination is equal to the sum of their individual resistances.

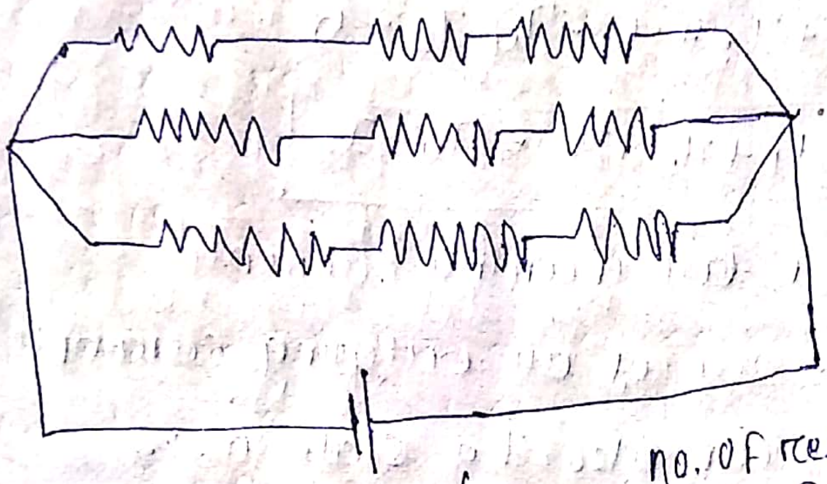
$$R = R_1 + R_2 + R_3$$

- Resistance connected in parallel :-

- If a no. of resistances are connected in parallel with each other then the reciprocal of the resistance of the combination is equal to the sum of the reciprocals of their individual resistances.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistance in grouping



$$\text{Net resistance} = \text{resistance} \times \frac{\text{no. of resistance in on row}}{\text{No. of rows}}$$

Kirchoff's law

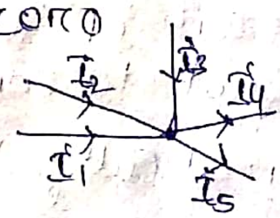
There are two types of kirchhoff's law

(i) KCL (kirchhoff's current law) :-

Statement:-

The algebraic sum of all the current meeting at the junction point (node), in an electrical network is zero

$$\boxed{\Sigma I = 0}$$



- In the above figure I_1, I_2 & I_3 are the incoming currents & I_4 & I_5 are the outgoing currents.
- In KCL all the incoming currents may be considered as positive & all the outgoing currents considered as negative.

~~By applying KCL at node 'O' we get~~

- By applying KCL at node 'O' we get

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

$$\boxed{\Rightarrow I_1 + I_2 + I_3 = I_4 + I_5}$$

i.e (sum of all incoming currents = sum of all outgoing currents)

Hence KCL can also be state as:-

"The sum of all incoming currents are equal to the sum of all outgoing currents at a particular junction in an electrical network".

2) Kirchhoff's Voltage law (KVL)

Statement:-

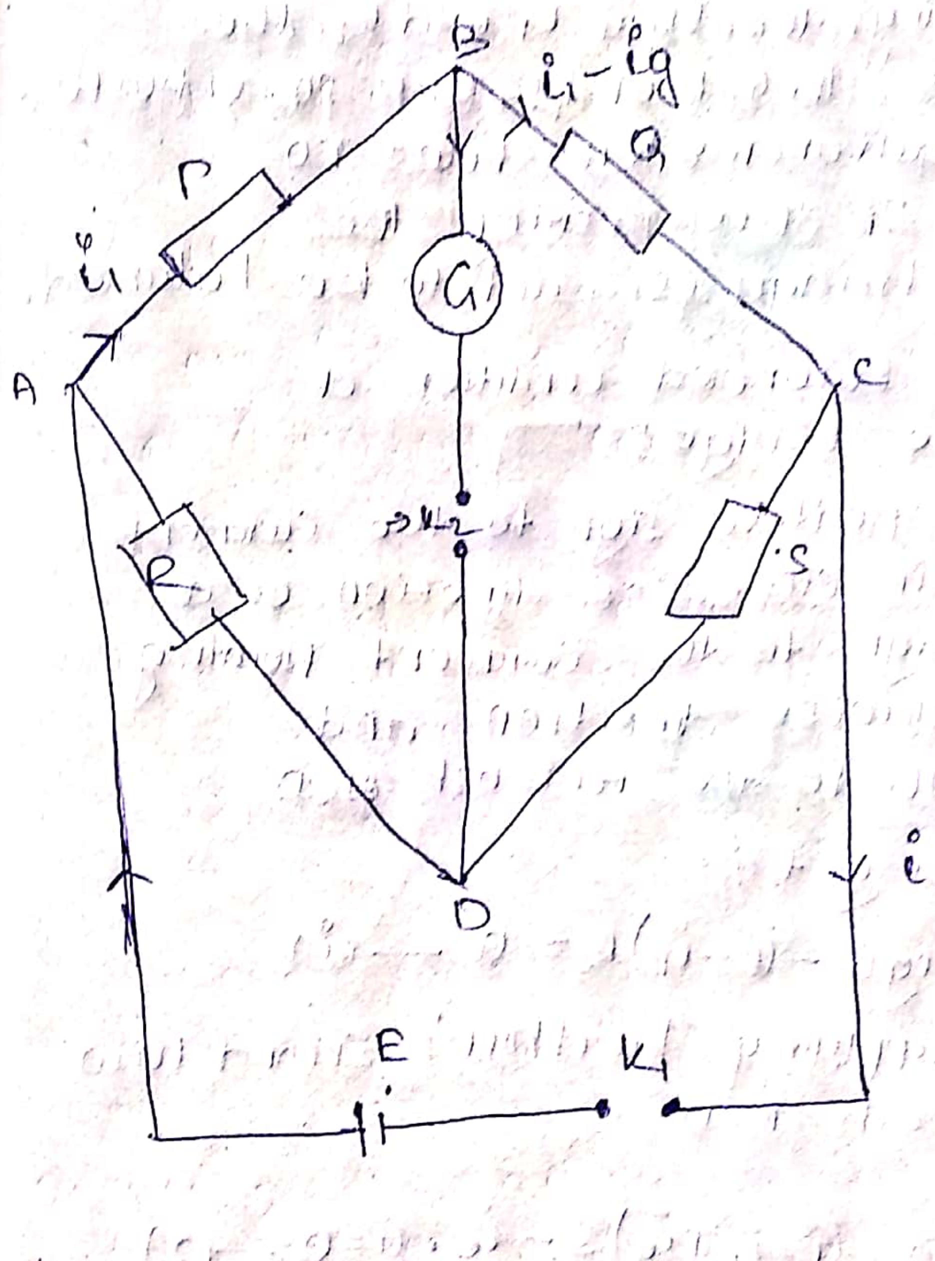
"In a closed path (mesh), the algebraic sum of the products of current and resistances in each conductor plus the algebraic sum of the e.m.f in that path is equal to zero".

Mathematically

$$\Sigma IR + \Sigma e.m.f = 0$$

Application of Kirchhoff's Law
(Wheatstone Bridge)

- Wheatstone bridge is an electrical arrangement which forms the basis of most of the instruments used to determine ~~an~~ an unknown resistance.



- It consist of four resistances P, Q, R, S connected in the four arms of a square ABCD. A cell of e.m.f 'E' is connected between the points A and C through a key K_1 galvanometer (G) is connected between the terminals B and D through a key K_2

- After closing the keys K_1 and K_2 the resistance P, Q, R and S are so, adjusted that the galvanometer shows no deflection. In this position the wheatstone bridge is said to be balanced.

→ Write the balanced condition of wheatstone bridge?

Now, giving positive sign to the current flowing in clockwise direction and negative sign to the current flowing in anticlockwise direction and applying KVL to the ~~re~~ mesh ~~ABD~~ ABD we get

$$i_1P + i_gG - (i - i_1)R = 0 \quad \text{--- (i)}$$

Similarly applying Kirchhoff's second law to the mesh BCD we get

$$(i_1 + i_g)Q - (i - i_1 + i_g)S - i_g \cdot G = 0 \quad \text{--- (2)}$$

Since, the bridge is balanced so, the current 'i_g' flowing through the arm BD is zero or $i_g = 0$

putting the value of i_g in eq (1) & (2)
We get

$$i_1 P - (i - i_1) R = 0 \quad \text{--- (3)}$$

$$i_1 Q - (i - i_1) S = 0 \quad \text{--- (4)}$$

Dividing eq (3) by eq (4) we get

$$\frac{\cancel{i_1} P}{\cancel{i_1} Q} = \frac{\cancel{(i - i_1)} R}{\cancel{(i - i_1)} S}$$

$$\boxed{\frac{P}{Q} = \frac{R}{S}} \quad \text{--- (5)}$$

This eq (5) is the required condition for the bridge to be balanced and gives the principle of wheatstone bridge

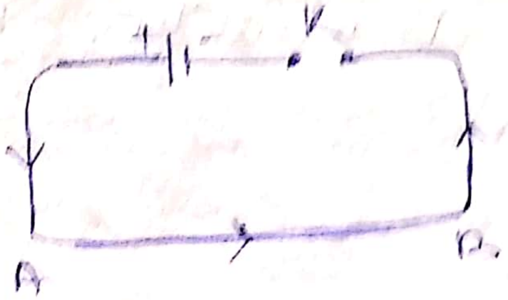
Electromagnetism and Induction

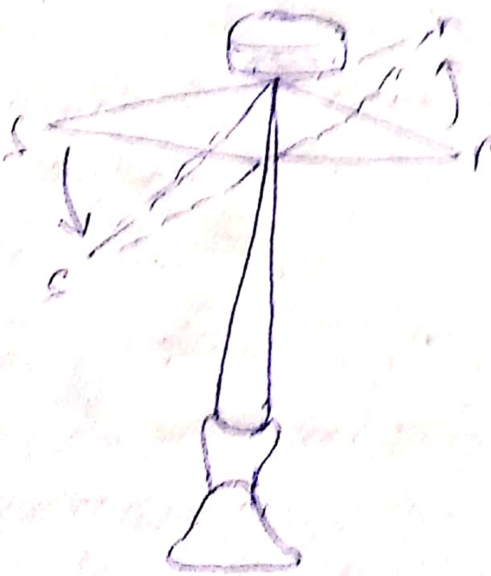
Electromagnetism

- It is a branch of physics which deals with the electromagnetic force, that occurs between electrically charged particles.
- ~~An interaction between electric &~~
- An interaction
- A physical interaction that occurs between electrically charged particles.

Magnetic effect of electric currents

(Oersted's Experiment)





Deflection of magnetic compass due to the magnitude field of a current

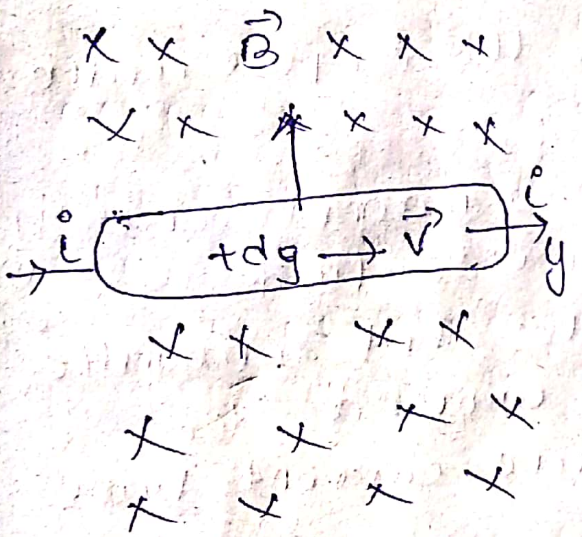
- In 1820, Oersted was able to show that an electric current flowing through a wire produces a magnetic field around it.
- Consider a wire AB connected to a battery in such a way that electric current flows from A to B on closing the key. Place a compass needle just below the wire AB.
- When the key is closed then a current is flowing through the wire.
- Here we can see that the compass needle gets deflected.
- Compass needle which is a magnet can only be effected by a magnetic field.
- This experiment indicates that a magnetic field gets developed around the wire, when an electric current flows through it.
- This process is called magnetic effect of current.

Force acting on a current carrying conductor placed in a uniform magnetic field

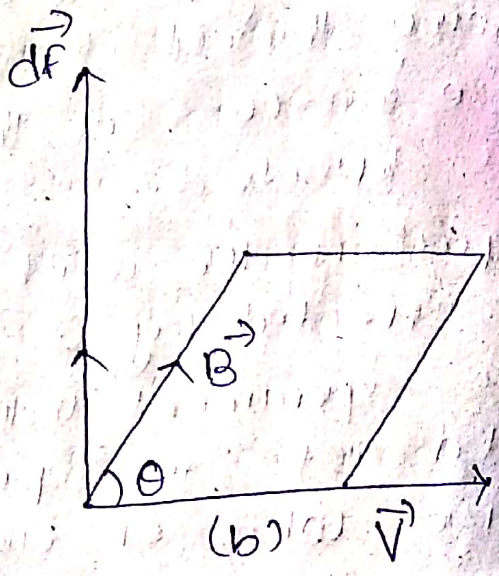
- A conductor has free electrons in it. When a potential difference is maintained across the two ends of the conductor then the electrons drift from lower potential to higher potential with a small velocity. These electrons constitute a current through the conductor.

- When the electron move in a magnetic field they experience a force '$\vec{F}$'.
- Consider a conductor xy placed in a uniform magnetic field '$\vec{B}$' acting inwards at right angle to the plane of paper.

Let, a current 'I' flow through the conductor x to y.



(a)



(b)

(Force on a current carrying conductor placed in the magnetic field.)

* Let 'dq' be a small amount of positive charge moving from 'x' to 'y' with a velocity v. The force $d\vec{F}$ experienced by this charge is given by $\boxed{d\vec{F} = dq\,(\vec{V} \times \vec{B})}$

If the Charge travells a small distance $\vec{dl}$ in time dt then,

$$\vec{v} = \frac{\vec{dl}}{dt}$$

$$\Rightarrow \vec{dF} = dq\left(\frac{dl}{dt} \times \vec{B}\right)$$

$$= \frac{dq}{dt}(\vec{dl} \times \vec{B}) \quad \left(\frac{q}{t} = i\right)$$

$$= i(\vec{dl} \times \vec{B})$$

The force $\vec{dF}$ acts vertically upward.
Net force F acting on the conductor can be obtained by integrating above eq.

$$F = \int d\vec{F} = \int i(\vec{dl} \times \vec{B})$$

$$\vec{F} = i(\vec{l} \times \vec{B})$$

$F = ilB \sin\theta \hat{n}$, where, $\hat{n}$ is a unit vector in a direction perpendicular to the plane containing $\vec{l}$ and $\vec{B}$.

θ = angle between $\vec{l}$ and $\vec{B}$.

Magnitude of force is given by $\rightarrow |\vec{F}| = BIl\sin\theta$.
Direction of force can be obtained by applying the rule of cross product or fleming left hand rule.

Fleming's Left Hand Rule

- Direction of force $\vec{F}$ is obtained by applying fleming's left hand rule.

→ Stretch first finger, middle finger and the thumb of your left hand in mutually perpendicular direction. If the ~~first~~ index finger points towards magnetic field, middle finger points towards electric current then the thumb gives the direction of force acting on the conductor.

- Fleming's left hand rule can only be applied when the direction of motion of charged particle is perpendicular to the lines of magnetic field.

* The direction of force can be obtained by applying the rule of cross product.

Case-I

If the conductor is placed at right angled to the field then.

$\theta = 90°$

$\sin\theta = \sin 90 = 1$

$|\vec{F}| = iLB$ (maximum)

Case-II

If the ~~angle~~ length of the conductor is along the direction of line of force then

$\theta = 0°$ or $180°$

$\sin\theta = 0$

$|\vec{F}| = 0$

So, no force is experienced by current carrying conductor when its length is parallel to the lines of force of magnetic field whatever be the direction of electric current.

Faraday's Law of Electromagnetic Induction

The process of production of electricity due to magnetism is called electro-magnetic induction.

Faraday's law deals with the induction of an emf in an electric circuit when magnetic flux linked with the circuit changes.

1st law:-

- Whenever magnetic flux linked with a circuit changes, then an emf is induced in it.

~~2nd~~ 3rd law:-

The induced emf is directly proportional to the negative rate of change of magnetic flux linked with the circuit.

2nd law:-

The induced emf exists in the circuit so long as the change in magnetic flux linked with it continues.

* If $d\phi_B$ - changes in magnetic flux linked with a circuit and it takes place in a time dt.

Rate of change of magnetic flux $= \frac{d\phi_B}{dt}$

* If E = emf induced in the circuit then

$$E \propto \frac{-d\phi_B}{dt}$$

$$\Rightarrow E = -K \frac{d\phi_B}{dt}$$

By selecting units of E, ϕ_B, t in a proper way we have $K = 1$

$$\boxed{E = \frac{-d\phi_B}{dt}}$$

- Here negative sign is due to direction of induced emf.

Lenz's law :-

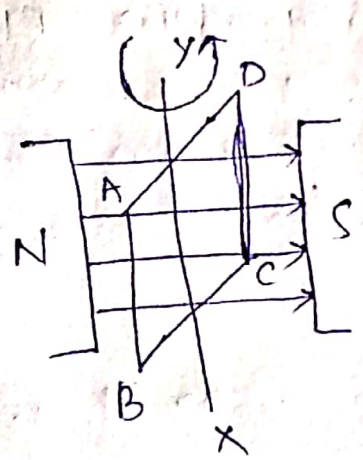
It deals with the direction of emf induced in the circuit due to a change in magnetic flux linked with it.

Statement :-

- It states that the direction of induced emf is such that it tends to oppose the very cause which produces it.

Flemings Right hand rule

- It is a rule to find the direction of induced current in a conductor.
- Stretched first finger, central finger and thumb of your ~~so~~ right hand in three mutually perpendicular direction.
- If the index finger points towards the magnetic field, the thumb points towards the direction of motion of the conductor, the direction of middle finger gives the direction of induced current set up in the conductor.



(A rectangular coil rotating in a uniform magnetic field)

- Consider a coil ABCD turning in between the ~~tou~~ two pole pieces of a magnet. let the direction of rotation of coil be such that AB moves out of the plane of the paper while CD moves into the paper.
- Applying Flemings right hand ~~soto~~ rule separately on AB & CD. It can be seen that direction of induced current is from B to A and D to C

Comparision between Fleming Right hand rule & Fleming Left hand rule.

Fleming Left hand rule	Fleming Right hand rule
- This rule is used to determine the direction of magnetic force.	- This rule is used to determine the direction of electric current.
- Middle finger signifies &	- The middle finger signifies induced current with in the conductor.
- Thumb signifies the direction of magnetic force.	- Thumb points in the direction of motion of the conductor.
- The direction of magnetic field inside the solenoid is determine by this rule.	- The direction of induced current in an electric generator is determine by this rule.

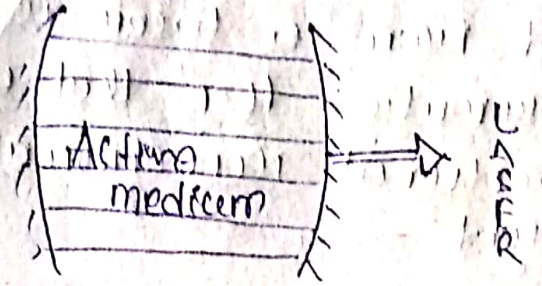
Modern Physics

LASER (Light Amplification by Stimulated emission of Radiation)

- A Laser beam is extremely intensed coherent and highly parallel beam of light.
- A device which produces this kind of beam is called laser.
- The ~~term~~ History Laser has grown out of "maser". "Maser" stands for microwave Amplification by stimulated emission of Radiation. The first successful maser was built by CH. Townes and his associates around 1951 and nine years.
- All lasers are either based upon principle of stimulated emission or 'Bremsstrahlung'. We shall limit our description only to such laser which are based upon former principle.

Principle of Laser :-

- Every laser system consist of an active medium (solid, liquid, gas) having molecules or atom possing at least one meta stable state.
- The active medium is placed in resonating cavity having reflectors at its ends an electrical or optical pump to excite the atoms of the medium.



Electrical or optical pump

(Resonating Cavity)

- The basic principle of all laser is to first bring about population inversion. It means to have more atoms in the metastable state than that in the ground state.
- This is done by supplying suitable energy to the atoms of the active medium with the help of a pump.
- This process of bringing about population inversion is known as pumping

Properties of laser

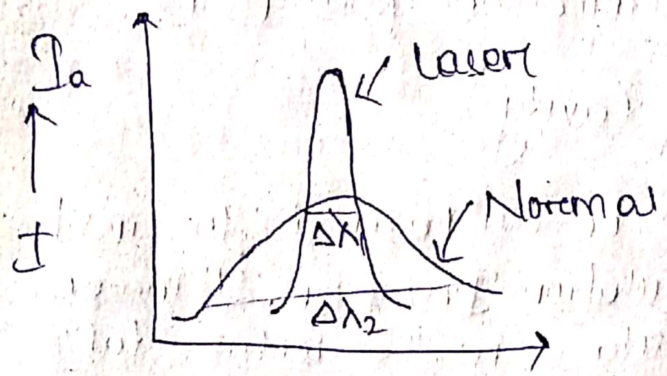
1) Directionality :- Light emitted from conventional sources spread in all directions but the laser beam is highly parallel & directional.

2) Intensity :-

- The laser beam has the ability to focus over a small area $10^{-6} cm^2$
- So it is highly intense beam

3) Monochromaticity

- Light emitted from a laser is vastly more monochromatic than that emitted from a conventional monochromatic sources of light



4) Coherence

- The laser light is highly coherent in space & time.
- This property enables us to realise a tremendous spatial concentration of light power.

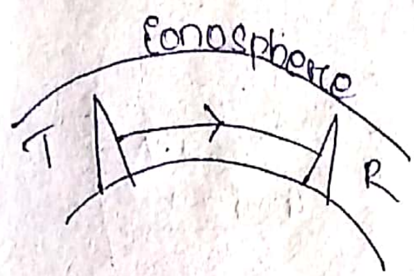
Wireless Transmission

1) Ground ~~Waves~~:-

- Wireless transmission involves no physical link established between two or more devices.

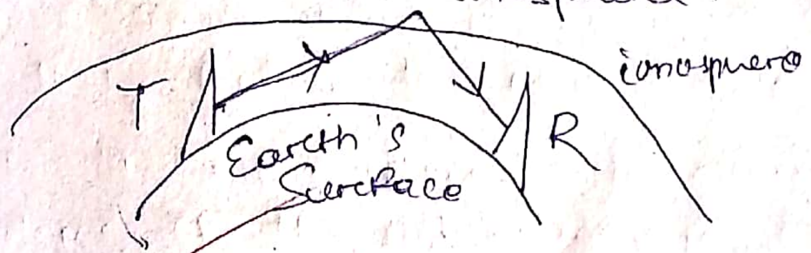
1) Ground waves:-

- A radio wave which reaches a receiver from a transmitter directly without reflection from the ionosphere.



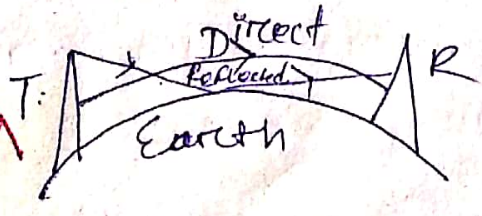
2) Sky waves:-

- In radio Communication sky wave refers to propagation of radio waves reflected or refracted back towards earth from the ionosphere.



3) Spacee Waves

- The spaces waves are the radio waves of very high frequency. The space waves can travel through atmosphere, from transmitter to receiver either directly or ~~indirectly~~ after reflection from ground in the earth troposphere region.



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